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ELECTROMAGNETIC PULSE ENVIRONMENT STUDIES

FINAL REPORT

by

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ABSTRACT

A number of investigations related to the development of a near-surface EMP code were carried out. Important considerations in the development of a near-surface burst EMP code include the choice of coordinate system, the form of the electromagnetic field equations that is chosen for solution, the differencing scheme used to represent the field equations, and the choice of boundary conditions applied to the problem to be solved. Such considerations are also affected by requirements that the conductivity and dielectric constant have spatial structure, and that far-field predictions also be provided, either directly or by extrapolation techniques.

In the investigations carried out, primary emphasis was put on development of a differencing scheme. Several approaches were investigated in detail, with the algorithms considered programmed for actual machine computation. The theoretical basis of each of the algorithms, the differencing approach implemented in detail, and general descriptions of the computational procedure, together with some programming details and computational results, are presented.

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ILLUSTRATIONS

1. INTRODUCTION

In the course of the work to be described in this report, a number of separate studies were undertaken. Because of the common aim of the investigations, certain comments and explanations generally apply to all of the studies. We will, therefore, attempt in this section to summarize a number of such aspects of the problem being investigated.

1.1 Basic Considerations

The overall objective of the study was to investigate a number of algorithms that could be useful in EMP field computations for off-the-ground bursts. The general problem of ground-burst EMP has been extensively discussed in many places[1, 2, 3, 4, 5] and will not be discussed in great detail here. The basic problem of concern is sketched in Figure 1.1, which simply indicates a number of the factors which must be taken into account in computing the EMP environment due to a ground or near-ground burst. From an electromagnetic standpoint, one must simply compute the electromagnetic fields created within a prescribed geometry by a certain set of sources. The relevant geometry is, of course, the air-over-ground geometry shown in Figure 1.1; the electrical properties of both the air and the ground (ϵ, μ, and σ) are prescribed and are part of the problem. In EMP language, the sources are considered to be the driven Compton currents and transient conductivity created by nuclear radiation. Good physical models for these sources are still in the process of refinement. Further, interactions between sources and the fields that they generate are both physically important and a source of some practical difficulty in implementing accurate solutions to the field problem. We have, nevertheless, assumed in this study that field and source problems can be moderately well separated, and have concentrated on the field algorithm problem.

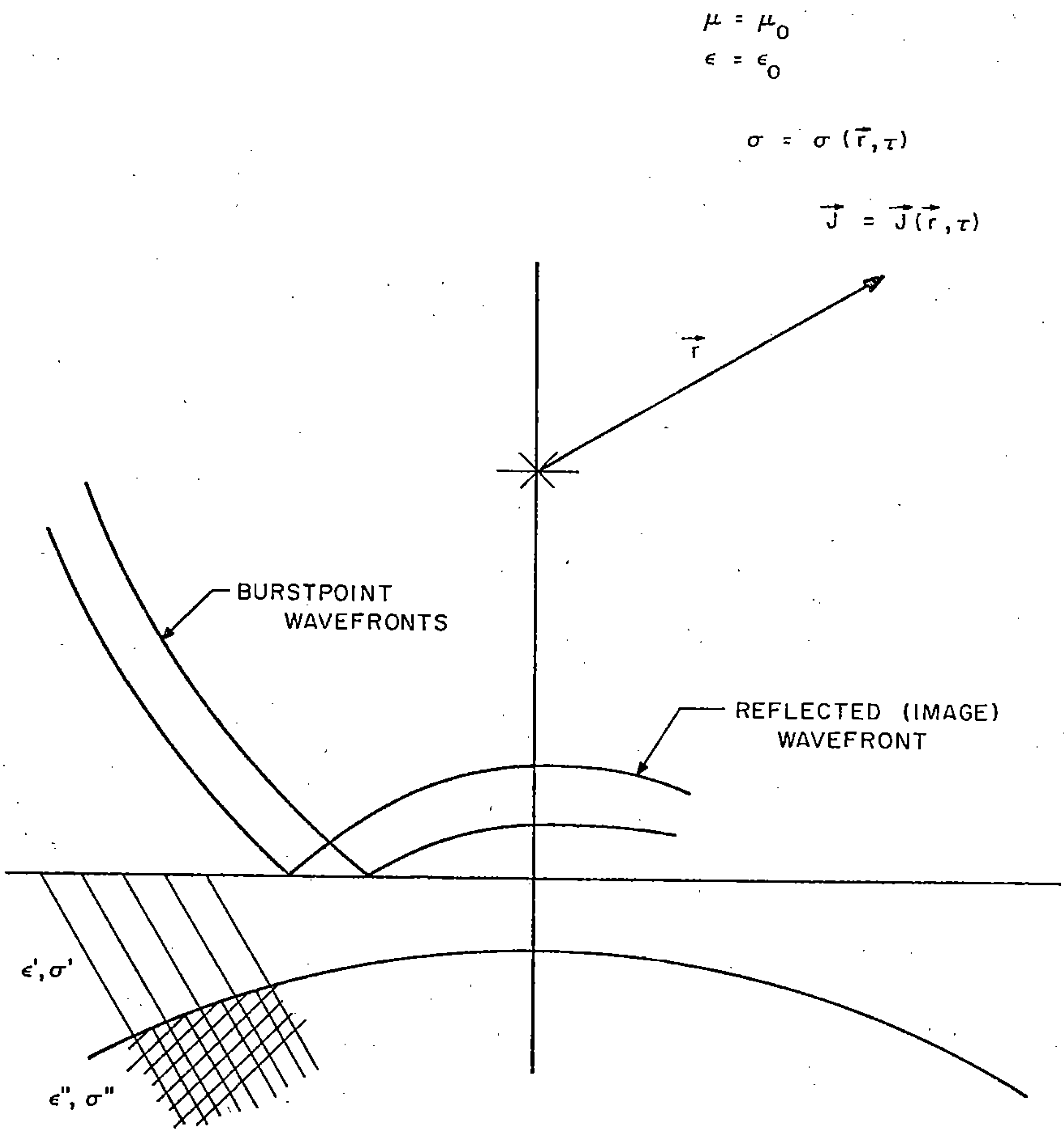


Figure 1.1. Sketch of Near-Ground EMP Problem

A number of ground-burst EMP codes have previously been developed. Many such codes take advantage of various physical approximations and important peculiarities of ground-burst EMP to handle particular aspects of the problem especially well. The G code,[1] ONDINE,[2] and GLANC[3] are examples of such special codes. The ground burst codes LEMP[4] and SC[5] handle solutions to a more general, two-dimensional field problem, but at some expense in other areas. For reasons that are not entirely clear (possible contributing factors are discussed later), neither the LEMP nor the SC field algorithms work well for bursts at a substantial height above the surface of the ground. The field algorithms to be discussed later were investigated for the purpose of providing a more general and adequate treatment of the off-the-ground EMP field problem. Substantial burst heights are presently handled only by one-dimensional field models. The validity of such models (as used in ONDINE or GLANC) is restricted to the close-in region. Some general considerations affecting the finite burst height field problem will be discussed next.

1.2 Major Problem Areas

1.2.1 Choice of Coordinate System

The problem sketched in Figure 1.1 is basically a two-dimensional one: a burst, at some finite height, a, above a ground plane, creates currents and conductivity that are axially symmetric about an axis through the burst and perpendicular to the ground. Ground electrical properties (ϵ, j, and σ) may be stratified and affected by nuclear radiation, but are also assumed to be axially symmetric about the z-axis.

The coordinate system chosen to describe the problem geometry is important because of the convenience that follows from a natural correspondence between coordinate surfaces and physical features of the problem. For example, the LEMP and SC ground-burst geometries use spherical coordinates above the plane containing the burst and cylindrical coordinates below (see Figure 1.2). For a burst directly at ground level, the choice is convenient, since both the sources and the radiated EMP signal are centered at the origin. For bursts above the ground, however, the propagating EMP wavefront (crudely described as originating at the below-ground image-point of the burst) travels obliquely through the composite grid, and is more difficult to conveniently describe.

One very convenient choice of coordinate system for the finite burst-height geometry was found to be the prolate-spheroidal (PS) coordinate system. Many important details of the PS coordinate system have been developed in detail elsewhere,[(6)] and will not be re-derived here. Some basic features will, however, be summarized.

The coordinate system is shown in Figure 1.3. Two points on the z-axis, at $z = \pm a$, are identified; they correspond to the burst point and its image below the $z = 0$ ground-plane. Defining the vectors from burst and image points to some given point P by r and r', respectively, the

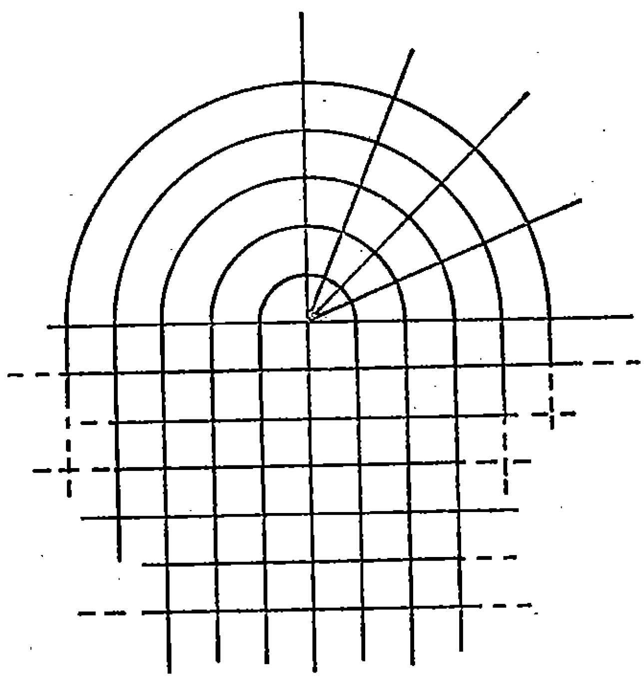


Figure 1.2. Coordinate Grid for Zero-Burst Height Computations

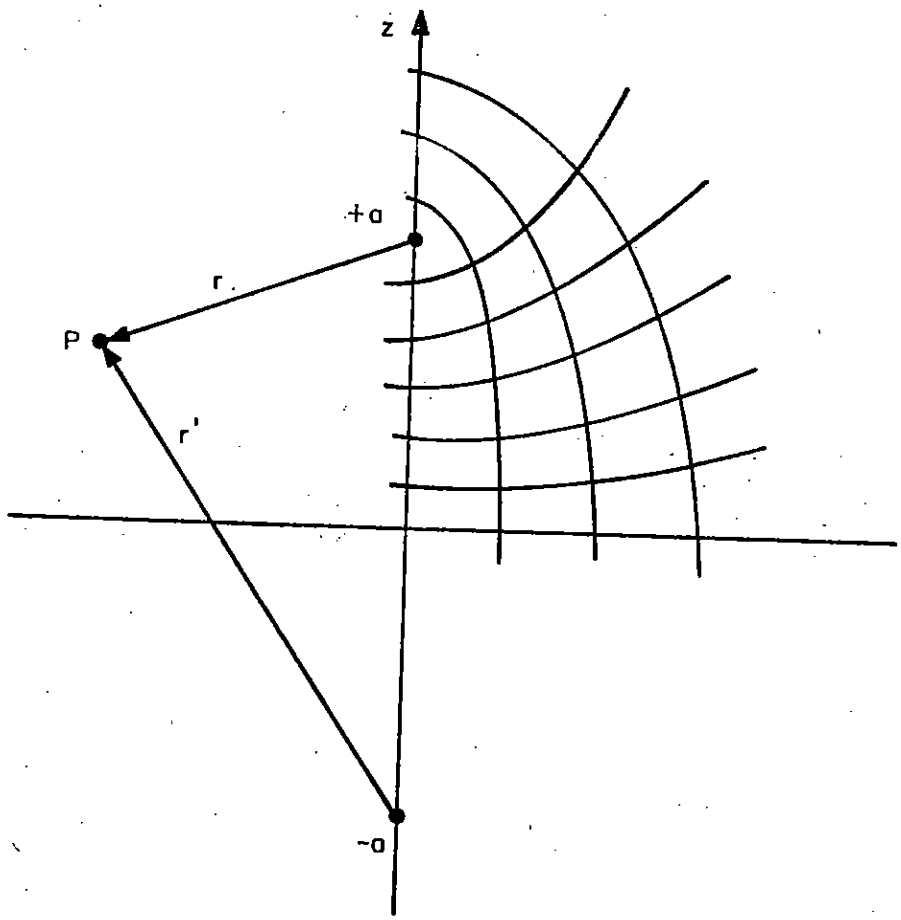


Figure 1.3. Prolate Spheroidal Coordinate Grid

PS coordinates of P are given by ξ, ζ, and φ. φ is the usual azimuthal angle, and

$$\xi = \frac{r' - r}{2a}$$

$$\zeta = \frac{r' + r}{2} \qquad (1.2.1)$$

The surfaces defined by constant values of ζ are a set of confocal ellipsoids with foci at $z = \pm a$; surfaces of constant ξ are an orthogonal set of confocal hyperboloids.

The PS coordinate system is convenient for the following reasons:

1. The burst point (at $z = a$) and the ground surface (at $\xi = 0$) appear naturally in a single coordinate system.
2. Spheres centered about the foci are important auxiliary surfaces, and are easily represented as straight lines in (ξ, ζ)-space, since

$$r = \zeta - a\xi$$

$$r' = \zeta + a\xi \qquad (1.2.2)$$

3. The problem of handling the waves reflected from the ground surface is also somewhat facilitated by PS coordinates. This will be discussed below.

1.2.2 Presence of Reflected Wave

The radiated EM signal is created, in a sense, at the air-ground interface. It resembles a spherical wave centered on the image point of the burst. Another spherical wave (non-propagating, if the atmosphere is homogeneous) is centered about the burst itself, since excitation from the source expands in a wave moving with the velocity of light. Depending upon whether one chooses to do computations in real time t, or retarded time, τ ($\tau=(t - r/c)$), and whether the burst is at zero height or at a finite height, the problem of computing near outgoing wavefronts can be more or less complicated. At zero burst height, both outgoing wavefronts coincide, and computations using τ as a time variable may begin everywhere at once. In real time, computations at Δt only need extend to some range $R_{max} = c\Delta t$. Because large spatial derivatives of the EMP fields occur near R_{max}, spatial regridding may be required there. For bursts at a finite height, burst and burst-image wavefronts do not coincide, and real-time computations must track both. Computations in burst-point retarded time, $\tau = t - (r/c)$, may still need to track and regrid at the image-point wavefront. An additional attraction of PS coordinates in this regard is that surfaces of constant burst-image-point retarded time, $\tau' = t - (r'/c)$, lie (at fixed τ) along hyperboloids $\xi = \xi_o$ in the PS hyperboloidal coordinate surfaces, since $(r' - r) = 2a\xi$,

$$\xi_o = c(\tau_o - \tau'_o)/(2a) \qquad (1.2.3)$$

1.2.3 Space-Structured σ and ϵ

In addition to drivers in the air, an important feature of the problem description is the presence of a finitely-conducting ground. A depth-dependent conductivity, the possible presence of subsurface currents, and radiation-enhanced conductivity, together with stratified dielectric properties, would be extremely important to include in an adequate treatment of the problem. Computations involving imaged sources may be contemplated as a partial solution, but are not strictly adequate in all regimes.

1.2.4 Outer Boundary Conditions

Fields beyond the boundaries of the immediate sources are an important part of the problem. Ideally, fields at extreme ranges are desired; either direct computations or extrapolations based on matching appropriate free-field solutions at the source region boundary are needed. In any event, conditions on the fields at the boundary of the volume of space considered must adequately account for outside fields without serious "mismatch". The kinds of boundary conditions that are adequate depend to some extent on the time duration of interest. For short times, simple 1/r extrapolations of fields have been shown to be adequate; at later times, the source is in a "near-field" region, and more complex extrapolation schemes, such as Babb and Granzow's time-domain treatment of multipoles,[7] are needed.

1.2.5 Choice of Difference Scheme

In the present investigations, numerical field solutions, based on finite difference representations of the Maxwell field equations, are sought. The foregoing physical considerations were found to have considerable impact on the kinds of differencing schemes that were studied. Mathematical questions of accuracy and stability were also extremely important. Detailed aspects of these questions are discussed in the next section.

2. FIELD ALGORITHMS INVESTIGATED

2.1 Algorithm "A"

The field algorithm described in this section employs the first order exponential differencing method and evaluates spatial derivatives analogously to the scheme used in HAPS.[8] The prolate spheroidal geometry and the transverse magnetic field equations are derived in appendix one. The final form of the field equations is

$$\frac{\partial E_\zeta}{\partial \tau} + \frac{z_o \sigma}{\varkappa} E_\zeta = \frac{1}{\varkappa}\left(-z_o j_\zeta + Q_\tau \frac{\partial H_\varphi}{\partial \tau} + \psi_2 \frac{\partial H_\varphi}{\partial v}\right) \tag{2.1.1}$$

$$\frac{\partial E_\xi}{\partial \tau} + \frac{z_o \sigma}{\varkappa} E_\xi = \frac{1}{\varkappa}\left(-z_o j_\xi + \frac{\partial H_\varphi}{\partial \tau} - \psi_1 \frac{\partial H_\varphi}{\partial u}\right) \tag{2.1.2}$$

$$Q_\tau G_2 \frac{\partial E_\zeta}{\partial \tau} + G_1 \frac{\partial E_\xi}{\partial \tau} - \varkappa_m \frac{\partial H_\varphi}{\partial \tau} = G_1 \psi_1 \frac{\partial E_\xi}{\partial u} - G_2 \psi_2 \frac{\partial E_\zeta}{\partial v} \tag{2.1.3}$$

where

$$\tau = ct - r \tag{2.1.4}$$

$$G_1 = \frac{\zeta^2 - 1}{\zeta^2 - \xi^2} \qquad G_2 = \frac{1 - \xi^2}{\zeta^2 - \xi^2} \tag{2.1.5}$$

The fields have been transformed as follows

$$\begin{pmatrix} E_\zeta \\ j_\zeta \end{pmatrix} = \sqrt{(\zeta^2 - \xi^2)(\zeta^2 - 1)} \begin{pmatrix} E'_\zeta \\ j'_\zeta \end{pmatrix} \tag{2.1.6}$$

$$\begin{pmatrix} E_\xi \\ j_\xi \end{pmatrix} = \sqrt{(\zeta^2 - \xi^2)(1 - \xi^2)} \begin{pmatrix} E'_\xi \\ j'_\xi \end{pmatrix} \tag{2.1.7}$$

$$H_\varphi = z_o \sqrt{(\zeta^2 - 1)(1 - \xi^2)} \quad H'_\varphi \tag{2.1.8}$$

in which the primed quantities are in mks units. The functions ψ_1 and ψ_2 in equations (2.1.1) through (2.1.3) are chosen to provide the desired spatial regridding in the code.

Equation (2.1.1) is of the form

$$\frac{\partial E_\zeta}{\partial \tau} + \gamma E_\zeta = \Psi_1 \tag{2.1.9}$$

where

$$\gamma = \frac{z_o \sigma}{\varkappa} \tag{2.1.10}$$

$$\Psi_1 = \frac{1}{\varkappa}\left(-z_o j_\zeta + Q_\tau \frac{\partial H_\varphi}{\partial \tau} + \psi_2 \frac{\partial H_\varphi}{\partial v}\right) \tag{2.1.11}$$

The finite difference approximation of the differential equation is

$$E^k_{\zeta i,j} = E^{k-1}_{\zeta i,j} e^{-\chi} + \Phi_1 \frac{1 - e^{-x}}{x} \tag{2.1.12}$$

where

$$\chi \equiv \overline{\gamma} \Delta\tau = \frac{z_o \Delta\tau}{\varkappa} \sigma^{k-1/2}_{i,j} \tag{2.1.13}$$

and where

$$\Phi_1 = - \frac{z_o \Delta\tau}{\varkappa} j^{k-1/2}_{\zeta i,j} + \frac{Q_\tau}{\varkappa}\left(H^k_{\varphi i,j} - H^{k-1}_{\varphi i,j}\right) + \frac{\Delta\tau}{2\varkappa\Delta v} \psi^j_2 \left(H^k_{\varphi i,j-1} - H^k_{\varphi i,j}\right.$$

$$\left. + H^{k-1}_{\varphi i,j} - H^{k-1}_{\varphi i,j+1}\right) \tag{2.1.14}$$

Equation (2.1.2) is of the form

$$\frac{\partial E_\xi}{\partial \tau} + \gamma E_\xi = \Psi_2 \tag{2.1.15}$$

in which γ is the same as before and

$$\Psi_2 = \frac{1}{\varkappa}\left(-z_o j_\xi + \frac{\partial H_\varphi}{\partial \tau} - \psi_1 \frac{\partial H_\varphi}{\partial u}\right) \tag{2.1.16}$$

The finite difference equation is

$$E^k_{\xi i,j} = E^{k-1}_{\xi i,j} e^{-\chi} + \Phi_2 \frac{1 - e^{-x}}{x} \tag{2.1.17}$$

where

$$\Phi_2 = -\frac{z_o \Delta\tau}{\varkappa} j^{k-1/2}_{\xi i,j} + \frac{1}{\varkappa}\left(H^k_{\varphi i,j} - H^{k-1}_{\varphi i,j}\right) - \frac{\Delta\tau}{2\varkappa\Delta u}\psi^i_1\left(H^k_{\varphi i,j} - H^k_{\varphi i-1,j} + H^{k-1}_{\varphi i+1,j} - H^{k-1}_{\varphi i,j}\right) \tag{2.1.18}$$

Exponential differencing is not appropriate for equation (2.1.3). We simply write

$$Q_\tau G_2\left(E^k_{\zeta i,j} - E^{k-1}_{\zeta i,j}\right) + G_1\left(E^k_{\xi i,j} - E^{k-1}_{\xi i,j}\right) - \varkappa_m\left(H^k_{\varphi i,j} - H^{k-1}_{\varphi i,j}\right)$$
$$= \frac{\Delta\tau}{2\Delta u}\psi_1 G_1\left(E^k_{\xi i,j} - E^k_{\xi i-1,j} + E^{k-1}_{\xi i+1,j} - E^{k-1}_{\xi i,j}\right)$$
$$- \frac{\Delta\tau}{2\Delta v}\psi_2 G_2\left(E^k_{\zeta i,j-1} - E^k_{\zeta i,j} + E^{k-1}_{\zeta i,j} - E^{k-1}_{\zeta i,j+1}\right) \tag{2.1.19}$$

Equations (2.1.12), (2.1.17) and (2.1.19) form a set of explicit difference equations that can be solved simultaneously to obtain fields at the point (i, j, k). The boundary conditions are provided by the azimuthal symmetry at the z-axis, the continuity of E_ζ and H_φ across the air-ground interface, the vanishing of the fields at sufficient depth in the ground, and the radiation condition on the fields at the maximum zeta grid. Presently the latter condition is implemented by simply extrapolating the fields as 1/r. At the air-ground interface, a difficulty arises in obtaining derivatives normal to the surface (i.e., $\frac{\partial H_\varphi}{\partial v}$ and $\frac{\partial E_\zeta}{\partial v}$). These derivatives are evaluated at two points above (or below) the interface and then linearly extrapolated to the interface.

2.2 Algorithm "B"

The second algorithm is set up in analogy to the differencing scheme used in the one-dimensional code, ONDINE. A number of variations on the basic approach were implemented, and will be described in this section. Either explicit or implicit versions are possible; a peculiarity of the implicit scheme requires that the time increment over which the fields are advanced be larger than the grid spacing. For this reason, one may be forced (with the basic approach) to perform early-time computations by an explicit method, and switch to its implicit counterpart at late times, when large time steps are desirable. An attractive feature of the basic scheme includes the possibility of handling a flexible and almost arbitrary stratification of ϵ and σ in the differencing grid without affecting the basic method of advancing the fields.

2.2.1 Theoretical Outline of Algorithm "B"

The "B" algorithm is based on Maxwell's equations as written for PS coordinates (described in Section 1) and retarded time of the burst point. We summarize for completeness the precise form used hereafter; standard RMKS notation is used unless otherwise specified. (For details, see Knight[6, 8].)

The "true" field components of interest (because of azimuthal symmetry) are the ξ- and ζ-components of D and E, and the φ-components of B and H. We define the permittivity and permeability constants by:

$$D = \epsilon\epsilon_0 E$$

and (2.2.1)

$$B = \mu\mu_0 H$$

where, as usual, $c = (\epsilon_0 \cdot \mu_0)^{-1/2}$ and $Z_0 = (\mu_0/\epsilon_0)^{1/2}$. The following field transformations are introduced for convenience (primes now refer to old coordinates):

$$\begin{Bmatrix} E_\xi \\ j_\xi \end{Bmatrix} = \sqrt{(\zeta^2 - \xi^2 a^2)(1 - \xi^2)} \begin{Bmatrix} E'_\xi \\ j'_\xi \end{Bmatrix}$$

$$\begin{Bmatrix} E_\zeta \\ j_\zeta \end{Bmatrix} = \frac{1}{a}\sqrt{(\zeta^2 - \xi^2 a^2)(\zeta^2 - a^2)} \begin{Bmatrix} E'_\zeta \\ j'_\zeta \end{Bmatrix} \qquad (2.2.2)$$

$$H_\varphi = Z_o \cdot \sqrt{(\zeta^2 - a^2)(1 - \xi^2)} \qquad H'_\varphi$$

and the burst-point retarded time is defined by

$$\tau = ct - r \qquad (2.2.3)$$

With these transformations, and with the further definition of the often-used geometrical factors γ_ξ and γ_ζ:

$$\gamma_\xi = \frac{a^2(1 - \xi^2)}{(\zeta^2 - \xi^2 a^2)}$$

$$\gamma_\zeta = \frac{(\zeta^2 - a^2)}{(\zeta^2 - \xi^2 a^2)} \qquad (2.2.4)$$

we find the final form of the field equations to be differenced:

$$\frac{\partial E_\xi}{\partial \tau} = -\frac{Z_o j_\xi}{\epsilon} - \frac{Z_o \sigma}{\epsilon} E_\xi + \frac{1}{\epsilon}\frac{\partial H_\varphi}{\partial \zeta} - \frac{1}{\epsilon}\frac{\partial H_\varphi}{\partial \tau} \tag{2.2.5}$$

$$\frac{\partial E_\zeta}{\partial \tau} = -\frac{Z_o j_\zeta}{\epsilon} - \frac{Z_o \sigma}{\epsilon} E_\zeta - \frac{1}{\epsilon}\cdot\frac{1}{a}\frac{\partial H_\varphi}{\partial \xi} - \frac{1}{\epsilon}\frac{\partial H_\varphi}{\partial \tau} \tag{2.2.6}$$

$$\frac{\partial H_\varphi}{\partial \tau} = -\frac{1}{\mu}\gamma_\xi\left[\frac{1}{a}\cdot\frac{\partial E_\zeta}{\partial \xi} + \frac{\partial E_\zeta}{\partial \tau}\right] + \frac{1}{\mu}\cdot\gamma_\zeta\left[\frac{\partial E_\xi}{\partial \zeta} - \frac{\partial E_\xi}{\partial \tau}\right] \tag{2.2.7}$$

2.2.2 Differencing Scheme for Algorithm "B"

The differencing procedure for equations (2.2.5) through (2.2.7) is based on the grid and fields defined in the sketch of Figure 2.1. The (i, j) grid structure is constructed so that the grid lines fall at $\zeta_j = a + (j - \frac{1}{2})\Delta\zeta$ and $\xi_i = 1 - (i - \frac{1}{2})\Delta\xi$. The fields are defined and indexed at various spatial positions as shown:

$$\begin{aligned} H^1_{\varphi ij} &= H_\varphi(\xi_i, \zeta_j, \tau_1) \\ E^1_{\zeta ij} &= E_\zeta(\xi_{i-1/2}, \zeta_j, \tau_1) \\ E^1_{\xi ij} &= E_\xi(\xi_i, \zeta_{j-1/2}, \tau_1) \end{aligned} \tag{2.2.8}$$

The "1" superscripts refer the fields to their values at the "old" time, τ_1; new values at the time $\tau_2 = \tau_1 + \Delta\tau$ are denoted by 2-superscripts.

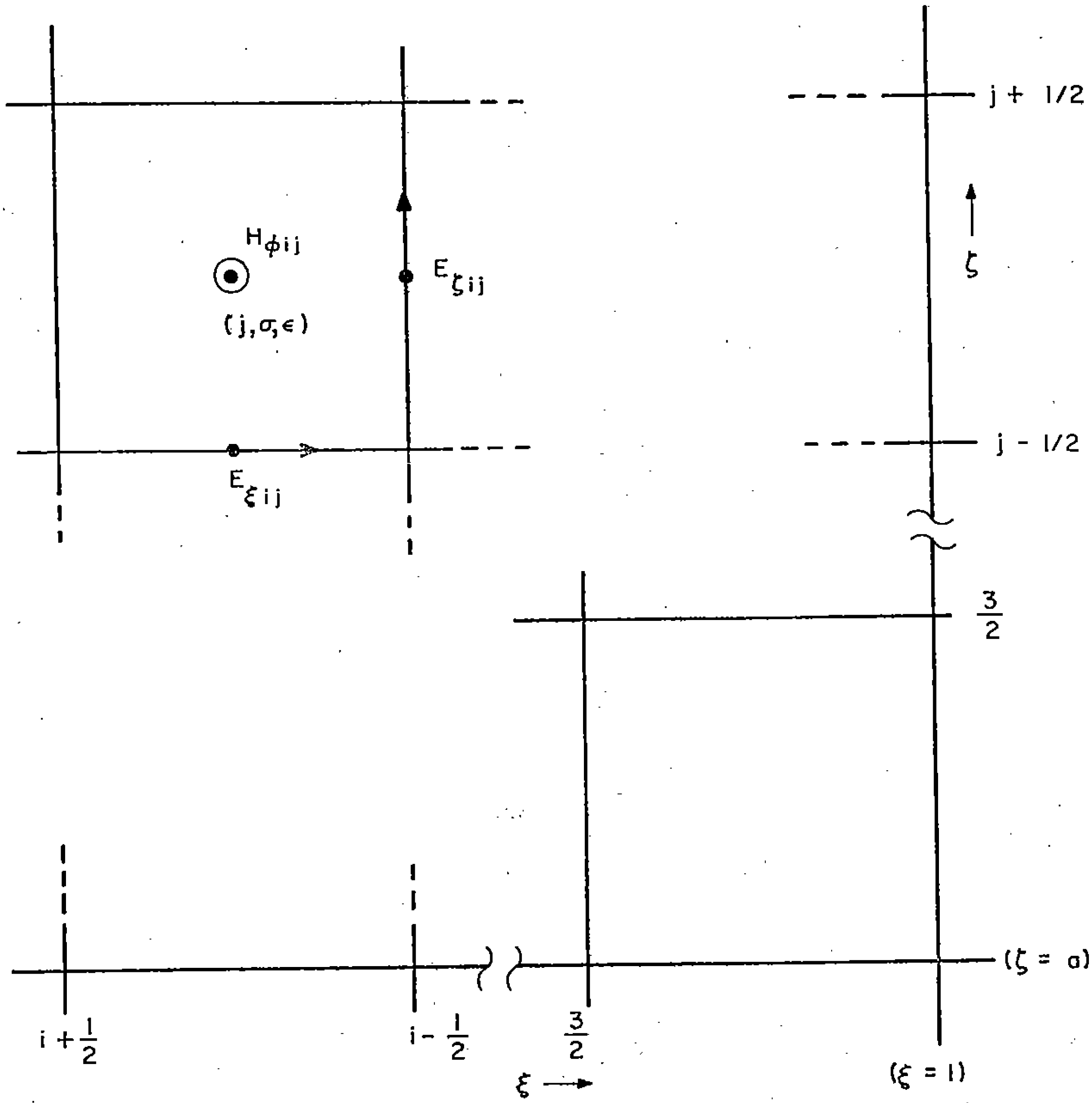


Figure 2.1. Algorithm "B" Differencing Conventions

The spatial cell in which H_{ij} is centered is also indexed by i and j, together with values for the Compton current drivers, J_ξ and J_ζ, the conductivity σ, and the dielectric constant ϵ. The detailed procedure followed in differencing equations (2.2.5) through (2.2.7) is perhaps best explained by reference to Figure 2.2. We will first consider equation (2.2.7) (the $\dot{H}$-equation), which is differenced identically for each of the (i, j)-th cells in the mesh. Figure 2.2 (a) shows the fields involved in the differenced form of the $\dot{H}$-equation, which is centered at $(\xi_i, \zeta_j, \tau_1 + \frac{1}{2}\Delta\tau)$. Straightforward algebraic manipulation leads to an expression for H^2_{ij} of the form

$$H^2_{ij} = H_C + A_0 E^2_{\zeta i,j} + A_1 E^2_{\zeta i+1,j} + B_0 E^2_{\xi i,j} + B_1 E^2_{\xi i,j+1} \tag{2.2.9}$$

where

$$A_0 = -\frac{\gamma_\xi}{2}\left(1 + \frac{\Delta\tau}{a\Delta\xi}\right)$$

$$A_1 = -\frac{\gamma_\xi}{2}\left(1 - \frac{\Delta\tau}{a\Delta\xi}\right)$$

$$B_0 = -\frac{\gamma_\zeta}{2}\left(1 + \frac{\Delta\tau}{\Delta\zeta}\right) \tag{2.2.10}$$

$$B_1 = -\frac{\gamma_\zeta}{2}\left(1 - \frac{\Delta\tau}{\Delta\zeta}\right)$$

and

$$H_C = H^1_{ij} - A_1 E^1_{\zeta ij} - A_0 E^1_{\zeta i+1,j} - B_1 E^1_{\xi i,j} - B_0 E^1_{\xi i,j+1}$$

The $\dot{E}_\xi$- and $\dot{E}_\zeta$-equations (equations (2.2.5) and (2.2.6), respectively) are differenced by putting them into the form:

$$\frac{\partial E}{\partial \tau} + k(\tau)E = S(\tau) \tag{2.2.11}$$

The solution of equation (2.2.11) above (for $k(\tau)$ and $S(\tau)$ approximated by constants evaluated at mid-timestep) is used to advance the E's:

$$E^2 = E^1 e^{-\bar{k}\Delta\tau} + (1 - e^{-\bar{k}\Delta\tau})\frac{\bar{S}}{\bar{k}} \tag{2.2.12}$$

The $\dot{E}_\xi$-equation and $\dot{E}_\zeta$-equation are handled in exactly analogous fashions; we will give the details for the $\dot{E}_\xi$-equation. First, equation (2.2.5) is put into the form of equation (2.2.11). We use equation (2.2.12) to advance $(E_{\xi ij} + E_{\xi i,j+1})/2$ from τ_1 to τ_2. Source terms are centered at mid-time in the (i, j)-th cell, as is the evaluation of $\partial H/\partial\tau$. The spatial derivative of H is handled as indicated in Figure 2.2. (b), explicitly, we use:

$$\left[\frac{\partial H}{\partial \tau} - \frac{\partial H}{\partial \zeta}\right] = \left\{\frac{(H^2_{ij} - H^1_{ij})}{\Delta\tau} - \frac{1}{2}\left[\frac{(H^1_{i,j+1} - H^1_{i,j})}{\Delta\zeta u} + \frac{(H^2_{ij} - H^2_{i,j-1})}{\Delta\zeta L}\right]\right\} \tag{2.2.13}$$

At the z-axis boundary (i.e., j = 1 for the $\dot{E}_\xi$-equation or i = 1 for the $\dot{E}_\zeta$-equation), one may wish to use a form of equation (2.2.13) for which $H_\varphi = 0$ along the z-axis. In that case (see Figure 2.2. (c)):

$$\left[\frac{\partial H}{\partial \tau} - \frac{\partial H}{\partial \zeta}\right] = \left\{\frac{(H^2_{ij} - H^1_{ij})}{\Delta\tau} - \frac{1}{2}\left[\frac{(H^1_{i,j+1} - H^1_{ij})}{\Delta\zeta u} + \frac{H^2_{ij} - (0)}{\left(\frac{\Delta\zeta}{2}\right)}\right]\right\} \tag{2.2.14}$$

Using either of the above forms for the H-field derivative terms in equation (2.2.5), we can manipulate equation (2.2.12) (as applied to the advanced E^2_ξ's) into the form:

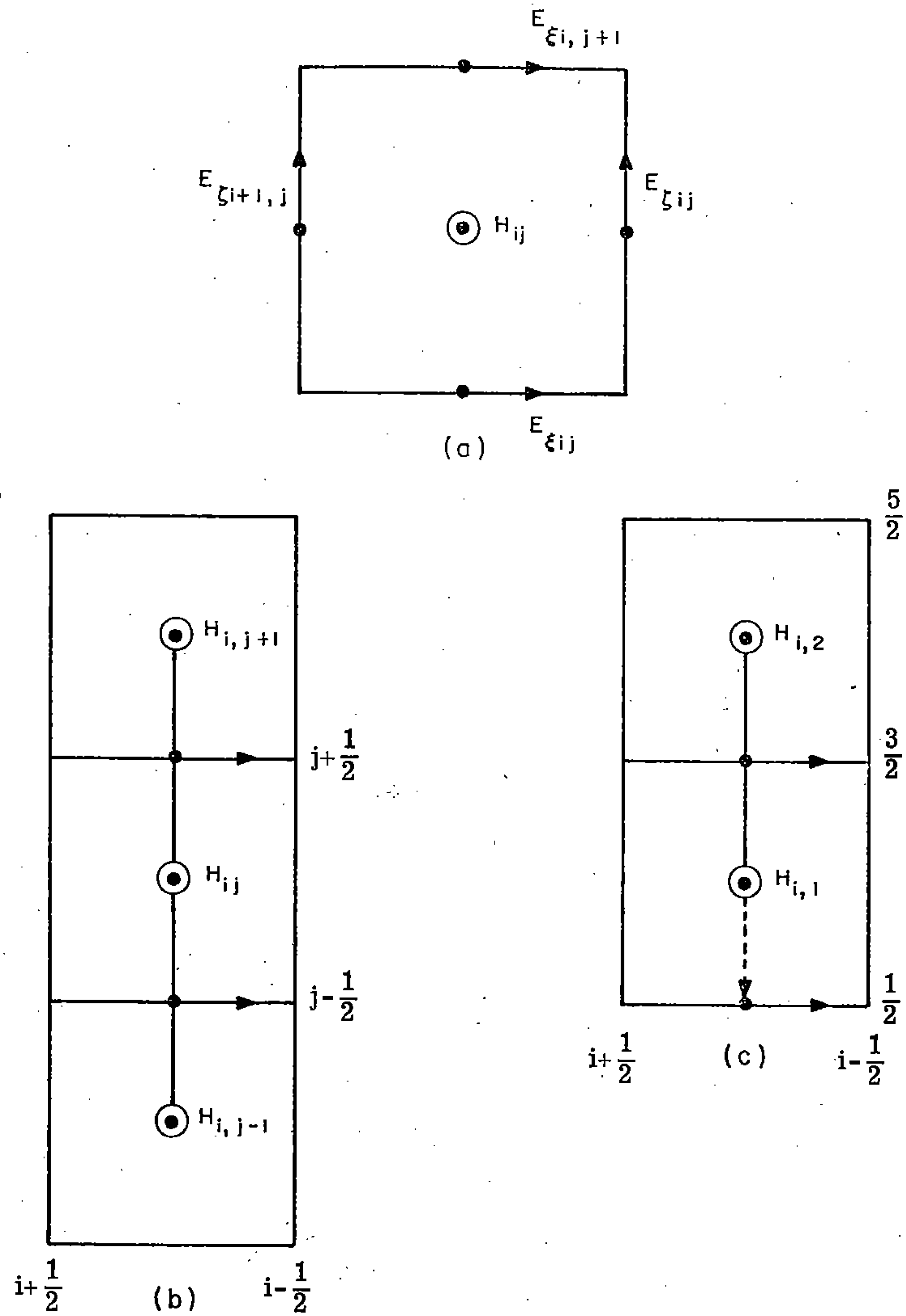


Figure 2.2. Specific Fields Involved in Typical Differencing for Equations 2.2.5 and 2.2.7.

$$E^2_{\xi i,j} + E^2_{\xi i,j+1} = E_{\xi c} + C_{1L} H^2_{ij} + r_{\zeta L} H^2_{i,j-1} \tag{2.2.15}$$

As before, the constants are functions of the old fields and known parameters. In particular, let us define

$$P = -2 \frac{\left(1 - e^{-k\Delta\tau}\right)}{Z_o \sigma \Delta\tau} \tag{2.2.16}$$

Then

$$C_{1L} = \begin{cases} P \cdot \left(1 - \frac{\Delta\tau}{2\Delta\zeta L}\right), & \text{if } j > 1 \\ P \cdot \left(1 - \frac{\Delta\tau}{\Delta\zeta}\right), & \text{if } j = 1 \end{cases}$$

$$C_{1u} = P \cdot \left(1 - \frac{\Delta\tau}{2\Delta\zeta_u}\right)$$

$$r_{\zeta L} = \begin{cases} P \cdot \left(\frac{\Delta\tau}{2\Delta\zeta_L}\right), & \text{if } j > 1 \\ 0 \quad , & \text{if } j = 1 \end{cases}$$

$$r_{\zeta u} = P \cdot \left(\frac{\Delta\tau}{2\Delta\zeta_u}\right)$$

$$E_{\xi C} = \left(E^1_{\xi ij} + E^1_{\xi i,j+1}\right) e^{-k\Delta\tau} - \frac{\left(1 - e^{-k\Delta\tau}\right)}{\sigma} \cdot 2j_\xi - r_{\zeta u} H^1_{i,j+1} - C_{1u} H^1_{ij} \tag{2.2.17}$$

Exactly analogous results are obtained for the E^2_ζ's, so that we must consider a set of equations which, for a typical cell, has the following appearance:

$$H_{ij} = H_c + A_0 E^2_{\zeta ij} + A_1 E^2_{\zeta i+1, j} + B_0 E^2_{\xi ij} + B_1 E^2_{\xi i, j+1} \tag{2.2.18}$$

$$E^2_{\zeta ij} + E^2_{\zeta i+1, j} = E_{\zeta c} + D_{1L} H^2_{ij} + r_{\xi L} H^2_{i-1, j} \tag{2.2.19}$$

$$E^2_{\xi i, j} + E^2_{\xi ij+1} = E_{\xi c} + C_{1L} H^2_{ij} + r_{\zeta L} H^2_{i, j-1} \tag{2.2.20}$$

2.2.3 Summary of Computational Procedure

The above equations can be used in an explicit manner, if $H^2_{i,j-1}$, $H^2_{i-1,j}$, $E^2_{\zeta i,j}$, and $E^2_{\xi i,j}$ are known, since equations (2.2.18) through (2.2.20) then become three equations in the three unknown fields $H^2_{i,j}$, $E^2_{\zeta i+1,j}$ and $E^2_{\xi i,j+1}$. The process is started from values of E and H along the z-axis and proceeds to some outer boundary, where outermost values of either E or H are assumed known from some boundary condition.

A variation can be made implicit in either direction (we will choose the ξ-direction). We disregard equation (2.2.19) and $H^2_{i-1,j}$ for i = 1 (rather than using a form of (2.2.19) with $H^2_{1/2,j}$ set at zero, as it is for the explicit method), and obtain a tridiagonal set of simultaneous equations for the E_ζ's. One can readily obtain such a set by using equation (2.2.18) (for H_{ij} and $H_{i-1,j}$) and (2.2.20) (for $E^2_{\xi i,j+1}$ and $E^2_{\xi i+1,j+1}$) in equation (2.2.19), eliminating unknown E_ξ's and H_φ's to obtain a relation involving E_ζ's at i-1, i, and i+1. The computational procedure thus involves stepping from one j-value to another; at each value of j, one sweeps through a range of i to find the new fields at points along a given ellipsoidal coordinate surface.

2.2.4 Implementation of Algorithm "B"

Algorithm "B" was implemented by adaptation of another two-dimensional code, ARIADNE. ARIADNE was originally constructed to solve the prolate-spheroidal Maxwell's equations for a high-altitude burst situation. The new auxiliary machinery (necessary to set up problem geometry parameters, initialize source and field computations, furnish peripheral functions necessary to supply updated sources to the field algorithm, and output results) was readily programmed for the ground-burst situation by minor changes in ARIADNE, so that the field algorithm subroutine was the most significant aspect of the programming problems.

The algorithm was checked by the device of providing "invented" sources that corresponded to known fields. The procedure involved an initially assumed analytical form for $E_\zeta(\xi, \zeta, \tau)$ and $E_\xi(\xi, \zeta, \tau)$. Equation (2.2.7) may be used to find $H(\xi, \zeta, \tau)$, and equations (2.2.5) and (2.2.6) then solved (together with assumptions for $\sigma(\xi, \zeta, \tau)$ to give explicit expressions for the sources J_ξ and J_ζ. Using such invented sources, the algorithm should then numerically produce fields that closely correspond to the analytical form initially assumed. Results of such check comparisons for moderately-realistic geometry, sources, and time variation of the fields showed agreement to within about 5% over approximately 5 orders of magnitude of field variation. Detailed results will be discussed in Section 2.2.5.

Consistent with experience with ground-burst codes now in use, it was found that poor computational results were obtained unless source representations were smoothly varying in space and time. For this reason, analytic representations of current and conductivity were used as a convenient means to drive the field algorithm. Results for a sample problem show fields that vary with space and time in a physically reasonable way. Detailed results will be presented in the following section.

The details of the algorithm, as programmed, are given in Appendix B, which contains a FORTRAN listing of the field subroutine MARCH, together with a brief explanation of the subroutine structure and the variables used.

2.2.5 Some Sample Computational Results

Some computations were carried out for a rather simple, "toy" problem; a burst at HOB = 40 meters (near I = 1, J = 1) was considered, with purely radial Compton currents (and associated conductivity) given by a simple analytic form. Sources were considered to be zero beyond the surface ζ = 640 meters; the ground was assumed to have constant conductivity (.01 mho/m) and dielectric constant $\epsilon = 5$. The computation grid in (ξ_I, ζ_J)-space is sketched in Figure 2.3 , where the positions of points at which output was requested are indicated. The source region is restricted to the gridpoints (I, J) for which $1 \leq I \leq 40$ and $1 \leq J \leq 30$. The ground lies in the interval $40 < I \leq 45$ for all J in the grid. Field intensities and sources are tabulated, as a function of time, in Figures 2.4 through 2.8, the positions are indicated in the grid. The point labelled "9" is far up in the source region, and only a purely radial E-field is found there. Closer to the ground, the image-point wavefront can be seen moving upward perpendicular to the I-surfaces, as retarded time progresses.

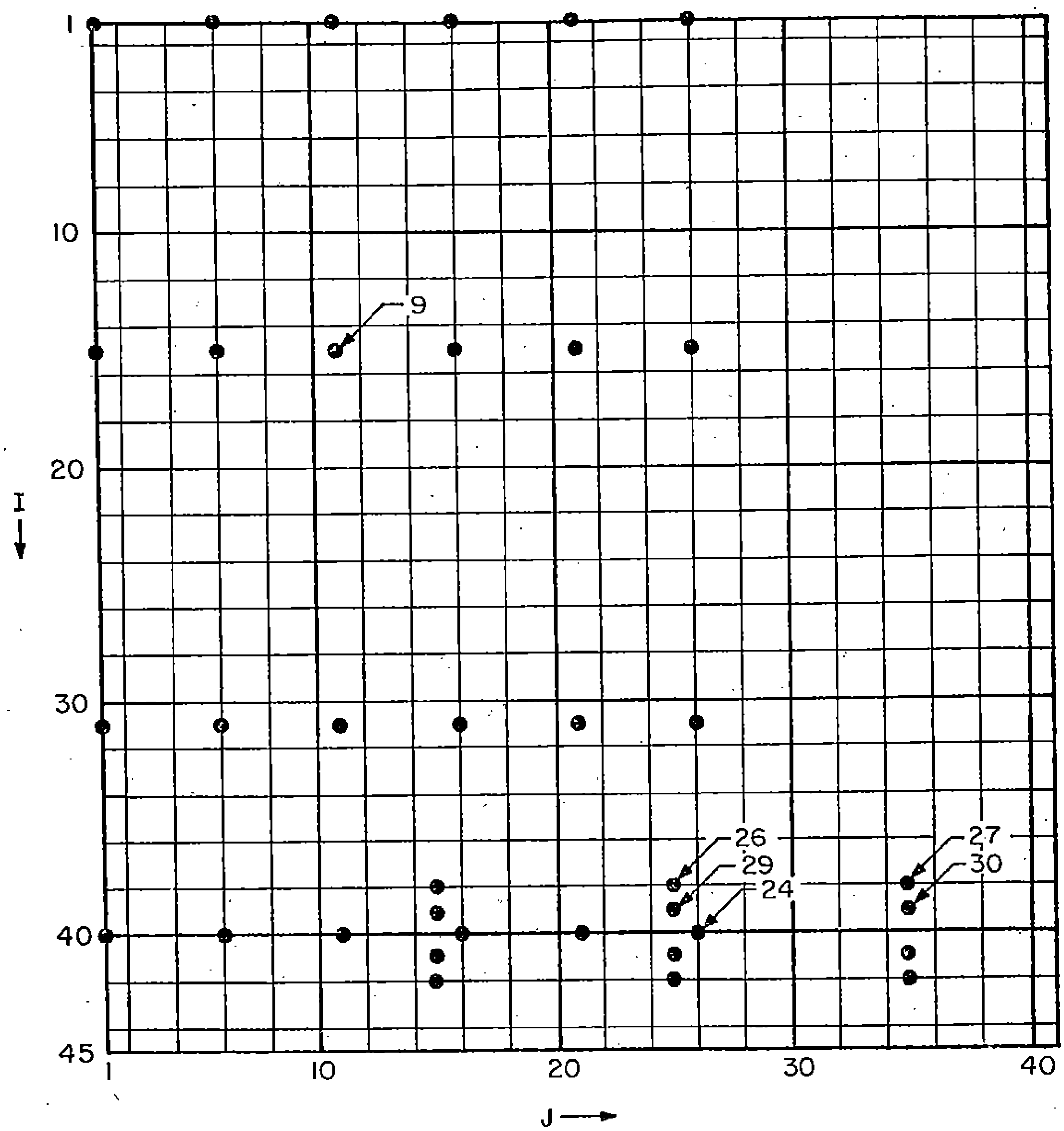


Figure 2.3. (I, J) Computation Grid for Sample Problem

CUMULATIVE TIME HISTORY AT 25-TH SAMPLE POINT

TIME	E-R	E-T	B-F	JC-R	JC-T	SIGMA
-4.600E-08	1.277E-10	-1.458E-07	-1.458E-07	2.003E-13	-1.010E-28	2.003E-16
-4.200E-08	2.025E-09	-1.792E-06	-1.792E-06	2.204E-12	8.078E-23	2.204E-15
-3.800E-08	2.358E-08	-1.985E-05	-1.985E-05	2.426E-11	-1.292E-26	2.426E-14
-3.400E-08	2.620E-07	-2.187E-04	-2.187E-04	2.669E-10	-1.034E-25	2.669E-13
-3.000E-08	2.883E-06	-2.400E-03	-2.400E-03	2.938E-09	-1.654E-24	2.938E-12
-2.600E-08	3.094E-05	-2.566E-02	-2.566E-02	3.233E-08	0.	3.233E-11
-2.200E-08	2.622E-04	-2.170E-01	-2.170E-01	3.558E-07	-1.059E-22	3.558E-10
-1.800E-08	5.096E-04	-8.611E-01	-8.611E-01	3.915E-06	0.	3.915E-09
-1.400E-08	-1.091E-02	4.802E-01	4.802E-01	4.308E-05	-2.711E-20	4.308E-08
-1.000E-08	-1.169E-01	-3.349E-02	-3.326E-02	4.737E-04	-4.337E-19	4.737E-07
-6.000E-09	-1.280E+00	1.037E-01	1.053E-01	5.148E-03	-1.735E-18	5.148E-06
-2.000E-09	-1.278E+01	2.397E-02	2.657E-02	4.817E-02	0.	4.817E-05
2.000E-09	-6.735E+01	4.823E-03	8.593E-03	1.713E-01	-1.665E-16	1.713E-04
6.000E-09	-1.423E+02	5.989E-03	1.106E-02	1.801E-01	-5.551E-17	1.801E-04
1.000E-08	-2.016E+02	6.575E-03	1.280E-02	1.452E-01	0.	1.452E-04
1.400E-08	-2.455E+02	8.251E-03	1.589E-02	1.144E-01	-5.551E-17	1.144E-04
1.800E-08	-2.784E+02	4.613E-02	5.820E-02	9.003E-02	-2.776E-17	9.003E-05
2.200E-08	-3.031E+02	2.790E-01	3.080E-01	7.084E-02	-5.551E-17	7.084E-05
2.600E-08	-3.217E+02	1.173E+00	1.254E+00	5.573E-02	-5.551E-17	5.573E-05
3.000E-08	-3.352E+02	3.780E+00	3.994E+00	4.385E-02	-2.776E-17	4.385E-05
3.400E-08	-3.444E+02	1.026E+01	1.056E+01	3.450E-02	-2.776E-17	3.450E-05
3.800E-08	-3.494E+02	2.308E+01	2.411E+01	2.714E-02	0.	2.714E-05
4.200E-08	-3.531E+02	4.595E+01	4.886E+01	2.135E-02	-6.939E-18	2.135E-05
4.600E-08	-3.462E+02	8.627E+01	8.948E+01	1.680E-02	0.	1.680E-05
5.000E-08	-3.375E+02	1.451E+02	1.501E+02	1.322E-02	-1.388E-17	1.322E-05

Figure 2.4. Fields Near 530 Meters Ground Range and 33 Meters Above the Ground

CUMULATIVE TIME HISTORY AT 29-TH SAMPLE POINT

TIME	E-R	E-T	B-P	JC-R	JC-T	SIGMA
-4.600E-08	5.561E-11	-1.397E-07	-1.397E-07	1.985E-13	-1.010E-28	1.985E-16
-4.200E-08	1.631E-09	-1.723E-06	-1.723E-06	2.185E-12	-8.078E-28	2.185E-15
-3.800E-08	2.279E-08	-1.909E-05	-1.909E-05	2.404E-11	-6.462E-27	2.404E-14
-3.400E-08	2.623E-07	-2.101E-04	-2.101E-04	2.646E-10	1.034E-25	2.646E-13
-3.000E-08	2.898E-06	-2.306E-03	-2.306E-03	2.912E-09	-1.654E-24	2.912E-12
-2.600E-08	3.124E-05	-2.466E-02	-2.466E-02	3.205E-08	-1.323E-23	3.205E-11
-2.200E-08	2.832E-04	-2.093E-01	-2.093E-01	3.527E-07	-1.059E-22	3.527E-10
-1.800E-08	1.364E-03	-8.446E-01	-8.445E-01	3.881E-06	0.	3.881E-09
-1.400E-08	-1.367E-03	4.198E-01	4.205E-01	4.271E-05	0.	4.271E-08
-1.000E-08	-5.608E-02	2.581E-01	2.606E-01	4.696E-04	-2.168E-19	4.695E-07
-6.000E-09	-1.252E+00	1.443E-01	1.472E-01	5.102E-03	-1.735E-18	5.102E-06
-2.000E-09	-1.266E+01	9.732E-03	1.314E-02	4.774E-02	0.	4.774E-05
2.000E-09	-5.678E+01	-7.356E-03	-4.425E-03	1.598E-01	0.	1.598E-04
6.000E-09	-1.412E+02	-9.638E-03	-5.782E-03	1.785E-01	-5.551E-17	1.785E-04
1.000E-08	-1.999E+02	3.334E-02	4.378E-02	1.439E-01	0.	1.439E-04
1.400E-08	-2.433E+02	3.995E-01	4.411E-01	1.134E-01	-1.110E-16	1.134E-04
1.800E-08	-2.753E+02	1.862E+00	2.037E+00	8.925E-02	0.	8.925E-05
2.200E-08	-2.983E+02	5.957E+00	6.356E+00	7.022E-02	-2.776E-17	7.022E-05
2.600E-08	-3.141E+02	1.516E+01	1.606E+01	5.524E-02	-2.776E-17	5.524E-05
3.000E-08	-3.236E+02	3.283E+01	3.460E+01	4.346E-02	-2.776E-17	4.346E-05
3.400E-08	-3.273E+02	6.280E+01	6.590E+01	3.419E-02	0.	3.419E-05
3.800E-08	-3.256E+02	1.086E+02	1.136E+02	2.690E-02	-1.388E-17	2.690E-05
4.200E-08	-3.191E+02	1.727E+02	1.799E+02	2.117E-02	0.	2.117E-05
4.600E-08	-3.086E+02	2.551E+02	2.649E+02	1.665E-02	6.939E-18	1.665E-05
5.000E-08	-2.950E+02	3.533E+02	3.659E+02	1.319E-02	-6.939E-18	1.319E-05

Figure 2.5. Fields Near 530 Meters Ground Range and 20 Meters Above the Ground

CUMULATIVE TIME HISTORY AT 24-TH SAMPLE POINT

TIME	E-R	E-T	B-P	JC-R	JC-T	SIGMA
-4.500E-08	1.092E-09	-1.347E-07	-1.346E-07	1.654E-13	0.	1.654E-16
-4.200E-08	2.073E-08	-1.606E-06	-1.605E-06	1.820E-12	8.678E-28	1.820E-15
-3.800E-08	2.440E-07	-1.771E-05	-1.770E-05	2.003E-11	1.292E-26	2.003E-14
-3.400E-08	2.704E-06	-1.949E-04	-1.947E-04	2.204E-10	1.034E-25	2.204E-13
-3.000E-08	2.975E-05	-2.139E-03	-2.137E-03	2.425E-09	8.272E-25	2.425E-12
-2.600E-08	3.240E-04	-2.285E-02	-2.283E-02	2.669E-08	1.323E-23	2.669E-11
-2.200E-08	3.216E-03	-1.921E-01	-1.919E-01	2.937E-07	0.	2.937E-10
-1.800E-08	2.055E-02	-7.326E-01	-7.312E-01	3.232E-06	8.470E-22	3.232E-09
-1.400E-08	4.338E-02	8.358E-01	8.396E-01	3.557E-05	1.355E-20	3.557E-08
-1.000E-08	-7.211E-02	1.611E-01	1.634E-01	3.911E-04	0.	3.911E-07
-6.000E-09	-1.054E+01	-3.329E-02	-3.176E-02	4.249E-03	1.735E-18	4.249E-06
-2.000E-09	-1.055E+01	-4.823E-02	-4.661E-02	3.976E-02	0.	3.976E-05
2.000E-09	-5.581E+01	2.537E-02	3.501E-02	1.414E-01	0.	1.414E-04
6.000E-09	-1.184E+02	5.737E-01	6.376E-01	1.487E-01	5.551E-17	1.487E-04
1.000E-08	-1.675E+02	3.174E+00	3.431E+00	1.199E-01	0.	1.199E-04
1.400E-08	-2.028E+02	1.008E+01	1.079E+01	9.446E-02	8.327E-17	9.446E-05
1.800E-08	-2.274E+02	2.374E+01	2.529E+01	7.433E-02	2.776E-17	7.433E-05
2.200E-08	-2.435E+02	4.551E+01	4.932E+01	5.848E-02	0.	5.848E-05
2.600E-08	-2.529E+02	8.020E+01	8.470E+01	4.601E-02	0.	4.601E-05
3.000E-08	-2.572E+02	1.257E+02	1.322E+02	3.620E-02	2.776E-17	3.620E-05
3.400E-08	-2.576E+02	1.828E+02	1.915E+02	2.848E-02	1.388E-17	2.848E-05
3.800E-08	-2.554E+02	2.499E+02	2.608E+02	2.241E-02	6.939E-18	2.241E-05
4.200E-08	-2.518E+02	3.243E+02	3.370E+02	1.763E-02	0.	1.763E-05
4.600E-08	-2.480E+02	4.025E+02	4.167E+02	1.387E-02	6.939E-18	1.387E-05
5.000E-08	-2.448E+02	4.838E+02	4.958E+02	1.091E-02	0.	1.091E-05

Figure 2.6. Fields Near 550 Meters Ground Range and 7 Meters Above the Ground

CUMULATIVE TIME HISTORY AT 27-TH SAMPLE POINT

TIME	E-R	E-T	B-F	JC-R	JC-T	SIGMA
-4.600E-08	9.227E-11	-1.093E-07	-1.093E-07	0.	0.	0.
-4.200E-08	1.392E-09	-1.340E-06	-1.340E-06	0.	0.	0.
-3.800E-08	1.587E-08	-1.487E-05	-1.487E-05	0.	0.	0.
-3.400E-08	1.762E-07	-1.637E-04	-1.637E-04	0.	0.	0.
-3.000E-08	1.939E-06	-1.797E-03	-1.797E-03	0.	0.	0.
-2.600E-08	2.089E-05	-1.920E-02	-1.920E-02	0.	0.	0.
-2.200E-08	1.879E-04	-1.619E-01	-1.619E-01	0.	0.	0.
-1.800E-08	7.855E-04	-6.324E-01	-6.324E-01	0.	0.	0.
-1.400E-08	-3.162E-04	3.757E-01	3.757E-01	0.	0.	0.
-1.000E-08	1.344E-03	1.915E-02	1.931E-02	0.	0.	0.
-6.000E-09	8.261E-03	3.692E-02	3.737E-02	0.	0.	0.
-2.000E-09	1.150E-02	-6.997E-02	-6.910E-02	0.	0.	0.
2.000E-09	1.642E-02	-7.406E-01	-7.395E-01	0.	0.	0.
6.000E-09	9.422E-03	-1.565E+00	-1.564E+00	0.	0.	0.
1.000E-08	9.254E-03	-2.410E+00	-2.409E+00	0.	0.	0.
1.400E-08	1.473E-02	-2.968E+00	-2.966E+00	0.	0.	0.
1.800E-08	4.415E-02	-3.319E+00	-3.316E+00	0.	0.	0.
2.200E-08	1.410E-01	-3.281E+00	-3.272E+00	0.	0.	0.
2.600E-08	3.855E-01	-2.376E+00	-2.351E+00	0.	0.	0.
3.000E-08	9.032E-01	3.132E-01	3.740E-01	0.	0.	0.
3.400E-08	1.853E+00	5.249E+00	6.376E+00	0.	0.	0.
3.800E-08	3.451E+00	1.742E+01	1.765E+01	0.	0.	0.
4.200E-08	5.886E+00	3.614E+01	3.652E+01	0.	0.	0.
4.600E-08	9.262E+00	6.459E+01	6.517E+01	0.	0.	0.
5.000E-08	1.359E+01	1.642E+02	1.650E+02	0.	0.	0.

Figure 2.7. Fields Near 730 Meters Ground Range and 45 Meters Above the Ground

CUMULATIVE TIME HISTORY AT 30-TH SAMPLE POINT

TIME	E-R	E-T	B-P	JC-R	JC-T	SIGMA
-4.600E-08	5.481E-11	-1.045E-07	-1.046E-07	0.	0.	0.
-4.200E-08	1.149E-09	-1.291E-06	-1.291E-06	0.	0.	0.
-3.800E-08	1.550E-08	-1.430E-05	-1.430E-05	0.	0.	0.
-3.400E-08	1.765E-07	-1.574E-04	-1.574E-04	0.	0.	0.
-3.000E-08	1.949E-06	-1.727E-03	-1.727E-03	0.	0.	0.
-2.600E-08	2.109E-05	-1.846E-02	-1.846E-02	0.	0.	0.
-2.200E-08	1.985E-04	-1.563E-01	-1.563E-01	0.	0.	0.
-1.800E-08	1.228E-03	-6.222E-01	-6.222E-01	0.	0.	0.
-1.400E-08	5.160E-03	3.491E-01	3.494E-01	0.	0.	0.
-1.000E-08	1.647E-02	1.569E-01	1.679E-01	0.	0.	0.
-6.000E-09	1.814E-02	1.629E-01	1.641E-01	0.	0.	0.
-2.000E-09	1.201E-02	-1.350E-01	-1.341E-01	0.	0.	0.
2.000E-09	9.216E-03	-7.850E-01	-7.843E-01	0.	0.	0.
6.000E-09	1.172E-02	-1.680E+00	-1.679E+00	0.	0.	0.
1.000E-08	3.521E-02	-2.341E+00	-2.339E+00	0.	0.	0.
1.400E-08	1.186E-01	-2.486E+00	-2.479E+00	0.	0.	0.
1.800E-08	3.235E-01	-1.562E+00	-1.542E+00	0.	0.	0.
2.200E-08	7.295E-01	1.352E+00	1.395E+00	0.	0.	0.
2.600E-08	1.422E+00	7.519E+00	7.598E+00	0.	0.	0.
3.000E-08	2.475E+00	1.842E+01	1.854E+01	0.	0.	0.
3.400E-08	3.922E+00	3.552E+01	3.568E+01	0.	0.	0.
3.800E-08	5.723E+00	5.993E+01	6.008E+01	0.	0.	0.
4.200E-08	7.741E+00	9.213E+01	9.212E+01	0.	0.	0.
4.600E-08	9.722E+00	1.312E+02	1.311E+02	0.	0.	0.
5.000E-08	1.131E+01	1.756E+02	1.751E+02	0.	0.	0.

Figure 2.8. Fields Near 730 Meters Ground Range and 27 Meters Above the Ground

2.3 Real-time Fields Algorithm in Cylindrical Coordinates for an Infinitely Conducting Ground: Algorithm "C"

2.3.1 Theoretical Considerations

The Maxwell curl equations in MKS units are

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{2.3.1}$$

$$\nabla \times \vec{B} = \mu \vec{J} + \mu\sigma \vec{E} + \mu\epsilon \frac{\partial \vec{E}}{\partial t} \tag{2.3.2}$$

with the implied constituent relations

$$\vec{B} = \mu \vec{H} \qquad \vec{D} = \epsilon \vec{E}$$

for μ, ϵ constant.

Figure 2.9 shows the coordinate system which is to be considered.

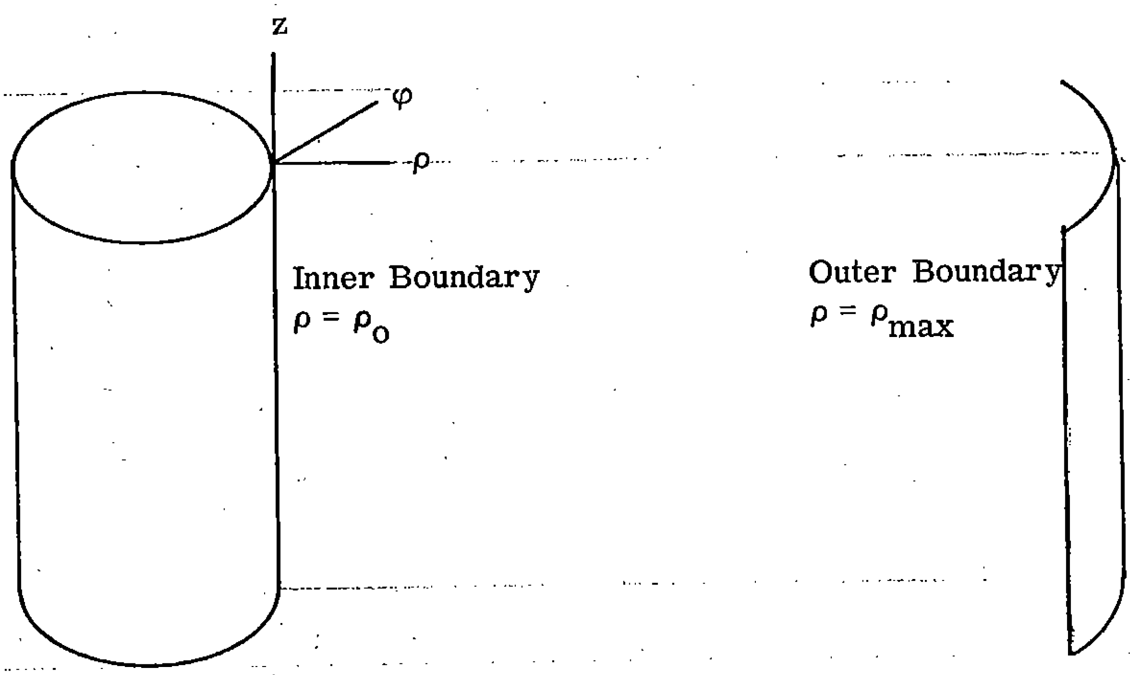


Figure 2.9. Coordinate Grid for Algorithm "C"

The near surface burst problem is azimuthally symmetric if the source is. In this case none of the variables are a function of $\hat{\varphi}$ and the curl equation becomes

$$\frac{-\partial E_\varphi}{\partial z}\hat{\rho} + \left(\frac{\partial E_p}{\partial z} - \frac{\partial E_z}{\partial \rho}\right)\hat{\varphi} + \frac{1}{\rho}\frac{\partial(\rho E_\rho)}{\partial \rho} = -\frac{\partial B_\rho}{\partial t}\hat{\rho} - \frac{\partial B_\varphi}{\partial t}\hat{\varphi} - \frac{\partial B_z}{\partial t}\hat{z} \qquad (2.3.3)$$

$$-\frac{\partial B_\varphi}{\partial z}\hat{\rho} + \left(\frac{\partial B_\rho}{\partial z} - \frac{\partial B_z}{\partial \rho}\right)\hat{\varphi} + \frac{1}{\rho}\frac{\partial(\rho B_\varphi)}{\partial \rho}\hat{z} = \left(\mu J_\rho + \mu\sigma E_\rho + \mu\epsilon\frac{\partial E_\rho}{\partial t}\right)\hat{\rho}$$

$$+\left(\mu J_\varphi + \mu\sigma E_\varphi + \mu\epsilon\frac{\partial E_\varphi}{\partial t}\right)\hat{\varphi} + \left(\mu J_z + \mu\sigma E_z + \mu\epsilon\frac{\partial E_z}{\partial t}\right)\hat{z} \qquad (2.3.4)$$

It is to be noted that due to azimuthal symmetry the equation may be separated into two sets which are coupled only by the dependence of the air chemistry parameter on the total electric field.

The TM equations are

$$\frac{\partial E_\rho}{\partial t} = -\frac{J_\rho}{\epsilon} - \frac{\sigma}{\epsilon}E_\rho - \frac{1}{\sqrt{\mu\epsilon}}\frac{\partial B_\varphi}{\partial z} \qquad (2.3.5)$$

$$\frac{\partial E_z}{\partial t} = -\frac{J_z}{\epsilon} - \frac{\sigma}{\epsilon}E_z - \frac{1}{\rho\sqrt{\mu\epsilon}}\frac{\partial(\rho B_\varphi)}{\partial \rho} \qquad (2.3.6)$$

$$\frac{\partial B_\varphi}{\partial t} = \frac{\partial E_z}{\partial \rho} - \frac{\partial E_\rho}{\partial z} \qquad (2.3.7)$$

The TE equations are

$$\frac{\partial B_\rho}{\partial t} = \frac{\partial E_\varphi}{\partial z} \qquad (2.3.8)$$

$$\frac{\partial B_z}{\partial t} = -\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho E_\varphi) \qquad (2.3.9)$$

$$\frac{\partial E_\varphi}{\partial t} = -\frac{J_\varphi}{\epsilon} - \frac{\sigma}{\epsilon} E_\varphi + \frac{1}{\sqrt{\mu\epsilon}}\left(\frac{\partial B_\rho}{\partial z} - \frac{\partial B_z}{\partial \rho}\right) \qquad (2.3.10)$$

If B_ρ, B_z and E_φ are initially zero and J_φ is zero for all time, then these components of the fields will remain zero. Only the TM set of equations needs to be solved.

Make the substitution $z = -n$ in the TM equation

$$\frac{\partial E_\rho}{\partial t} = -\frac{J_\rho}{\epsilon} - \frac{\sigma}{\epsilon} E_\rho + \frac{1}{\sqrt{\mu\epsilon}}\frac{\partial B_\varphi}{\partial n} \qquad (2.3.11)$$

$$\frac{\partial E_z}{\partial t} = -\frac{J_z}{\epsilon} - \frac{\sigma}{\epsilon} E_z + \frac{1}{\rho\sqrt{\mu\epsilon}}\frac{\partial(\rho B_\rho)}{\partial \rho} \qquad (2.3.12)$$

$$\frac{\partial B_\varphi}{\partial t} = \frac{\partial E_z}{\partial \rho} + \frac{\partial E_\rho}{\partial n} \qquad (2.3.13)$$

Let

$B_\rho = B'_\varphi$

$E_\rho = -E'_\rho$ $\qquad$ $J_\rho = -J'_\rho$

$E_z = E'_z$ $\qquad$ $J_z = J'_z$

Then we obtain

$$\frac{\partial E'_\rho}{\partial t} = -\frac{J'_\rho}{\epsilon} - \frac{\sigma}{\epsilon} E'_\rho - \frac{1}{\sqrt{\mu\epsilon}} \frac{\partial B'_\varphi}{\partial n} \tag{2.3.14}$$

$$\frac{\partial E'_z}{\partial t} = -\frac{J'_z}{\epsilon} + \frac{1}{\sqrt{\mu\epsilon}\,\rho} \frac{\partial(\rho B'_\varphi)}{\partial \rho} \tag{2.3.15}$$

$$\frac{\partial B'_\varphi}{\partial t} = \frac{\partial E'_z}{\partial \rho} - \frac{\partial E'_\rho}{\partial n} \tag{2.3.16}$$

The boundary condition on E'_ρ is the same as that on E_ρ, namely

$$E'_\rho \to 0 \text{ as } \rho \to \infty$$

The field E'_ρ, E'_z, B'_φ produced by the source J'_ρ J'_z at the point ρ, n, t is identical to the field E_ρ, E_z, B_φ produced by the source J_ρ J_z at r, z, t.

To obtain fields above the earth due to a perfectly conducting earth, add the two sets of fields calculated subject to the condition that $J = 0$ for $z < 0$ and $J' = 0$ for $z > 0$. Then at the plane $z = 0$, the ρ component of the electric field is zero, which is precisely the required boundary condition, subject to the condition

$$\mu'(\rho, z, t) = \mu(\rho, -z, t)$$

$$\epsilon'(\rho, z, t) = \epsilon(\rho, -z, t)$$

$$\sigma'(\rho, z, t) = \sigma(\rho, -z, t)$$

For ease in later implementing a varying grid size, make the following transformations

$$x = f(\rho) \qquad \rho = F(x)$$

$$y = y(z) \qquad Z = G(\mu)$$

so that the corresponding partial derivatives transform according to

$$\frac{\partial}{\partial \rho} \rightarrow \frac{\partial f}{\partial \rho} \frac{\partial}{\partial x}$$

$$\frac{\partial}{\partial z} \rightarrow \frac{\partial g}{\partial z} \frac{\partial}{\partial y}$$

With FP = $\partial f/\partial \rho$ and GP = $\partial g/\partial z$ the TM set of equations becomes

$$\frac{\partial E_\rho}{\partial t} = -\frac{J_\rho}{\epsilon} - \frac{\sigma}{\epsilon} E_\rho - \frac{GP}{\sqrt{\mu\epsilon}} \frac{\partial B_\varphi}{\partial y} \tag{2.3.17}$$

$$\frac{\partial E_z}{\partial t} = -\frac{J_z}{\epsilon} - \frac{\sigma}{\epsilon} E_z - \frac{1}{\sqrt{\mu\epsilon}} \left(FP \frac{\partial B_\varphi}{\partial x} + \frac{B_\varphi}{F} \right) \tag{2.3.18}$$

$$\frac{\partial B_\varphi}{\partial t} = FP \frac{\partial E_z}{\partial x} - GP \frac{\partial E_\rho}{\partial y} \tag{2.3.19}$$

The above equations are the ones to be finite differenced as described in the next section.

2.3.2 Finite Difference Methods

The following notation will be used on all the diagrams. The values of E_ρ used at the i, j step are denoted by ◊, values of E_z by △ and values of B_φ by ○. It is to be noted that E_z and B_φ are defined at the grid points, E_ρ midway between the grid points. The superscripts 1 refer to time (t) and superscripts 2 refer to time (t+Δt).

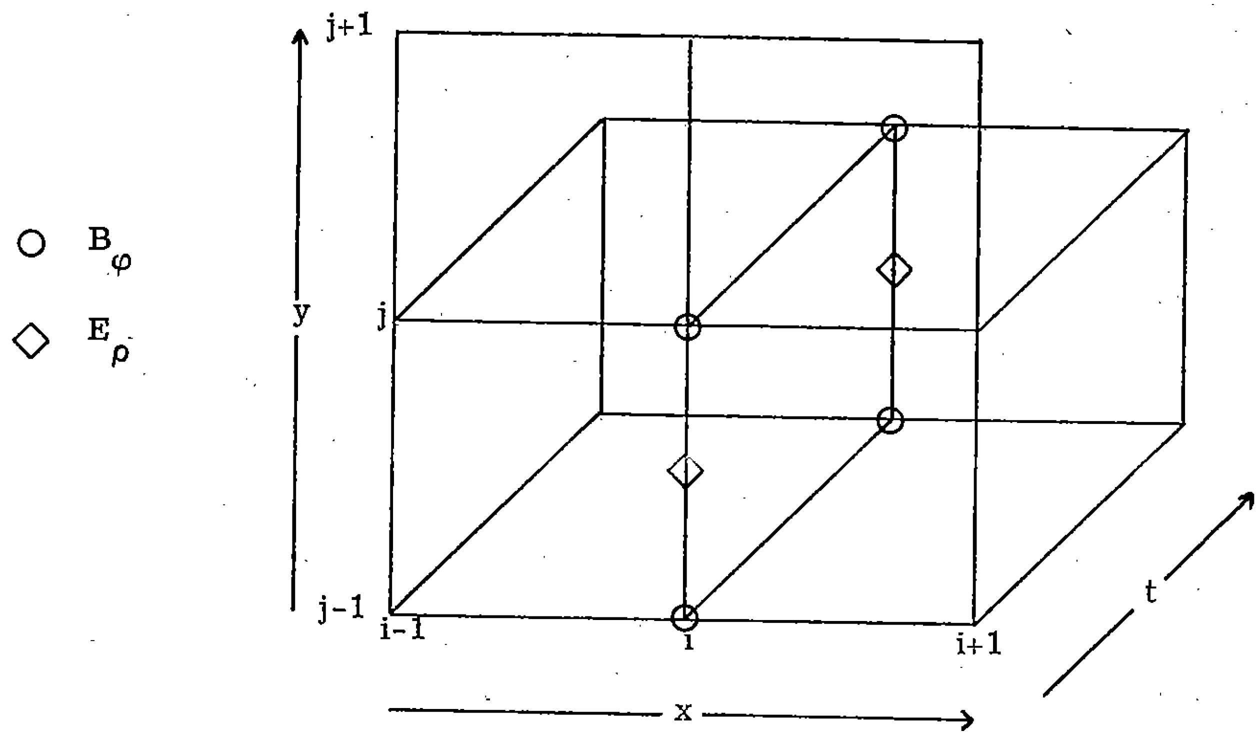


Figure 2.10. Differencing for Equation 2.3.17

Equation (2.3.17)

$$\frac{\partial E_\rho}{\partial t} + \frac{\sigma}{\epsilon} E_\rho = - \frac{J_\rho}{\epsilon} - \frac{GP}{\sqrt{\mu\epsilon}} \frac{\partial B_\varphi}{\partial y}$$

will be differenced centered at $(i,\ j-1/2,\ t+\Delta t/2)$. Both equations (2.3.17) and (2.3.18) are of the form

$$\frac{dF}{d\ell} + AF = B \tag{2.3.20}$$

This equation has an exact solution given by

$$F(\ell) = F_o e^{-\int_o^{\ell} A(\ell')d\ell'} + e^{-\int_o^{\ell} A(\ell')d\ell'} \int_o^{\ell} e^{\int_o^{\ell'} A(\ell'')d\ell''} B(\ell')\,d\ell'$$

A first order approximation to solution valid for variations in A and B is

$$F(\ell) = F(o)\,e^{-A(\ell/2)\ell} + \left(1 - e^{-A(\ell/2)\ell}\right) \frac{B(\ell/2)}{A(\ell/2)}$$

Equation (2.3.17) differenced in this form is

$$E^2_{\rho\, ij-1} = E^1_{\rho\, ij-1}\, e^{-\theta} + (1 - e^{-\theta_1}) \left[\frac{\psi_1}{\gamma_1}\right]^{1/2}_{ij-1/2} \tag{2.3.21}$$

where

$$\theta_1 = \gamma_1 \Delta t$$

$$\gamma_1 = \sigma/\epsilon \,\Big|_{t=t+\Delta t/2}$$

$$\psi_1 = -\frac{J^{1/2}_{rij-1}}{\epsilon} - \frac{GP_{ij-1}}{2\sqrt{\mu\epsilon}\,\Delta y}\left[B^1_{\varphi\, ij} - B^1_{\varphi\, ij-1} + B^2_{\varphi\, ij} - B^2_{\varphi\, ij-1}\right]$$

Define the following constant

$$A_1 = \frac{\left(1 - e^{-\theta_1}\right)}{\gamma_1} \quad \frac{GP_{ij-1}}{2\sqrt{\mu\epsilon}\,\Delta y}$$

$$C_1 = E^1_{\rho ij-1}\, e^{-\theta_1} + \frac{\left(1 - e^{-\theta_1}\right)}{\gamma_1}\left[-\frac{J_{\rho\, ij-1}}{\epsilon} - \frac{GP_{ij}}{2\sqrt{\mu\epsilon}\,\Delta y}\left(B^1_{\varphi\, ij} - B^1_{\varphi ij-1}\right)\right]$$

Equation (2.3.21) can be written

$$E^2_{\rho\, ij-1} + A_1 B^2_{\varphi\, ij} - A_1 B^2_{\varphi\, ij-1} = C_1 \qquad (2.3.22)$$

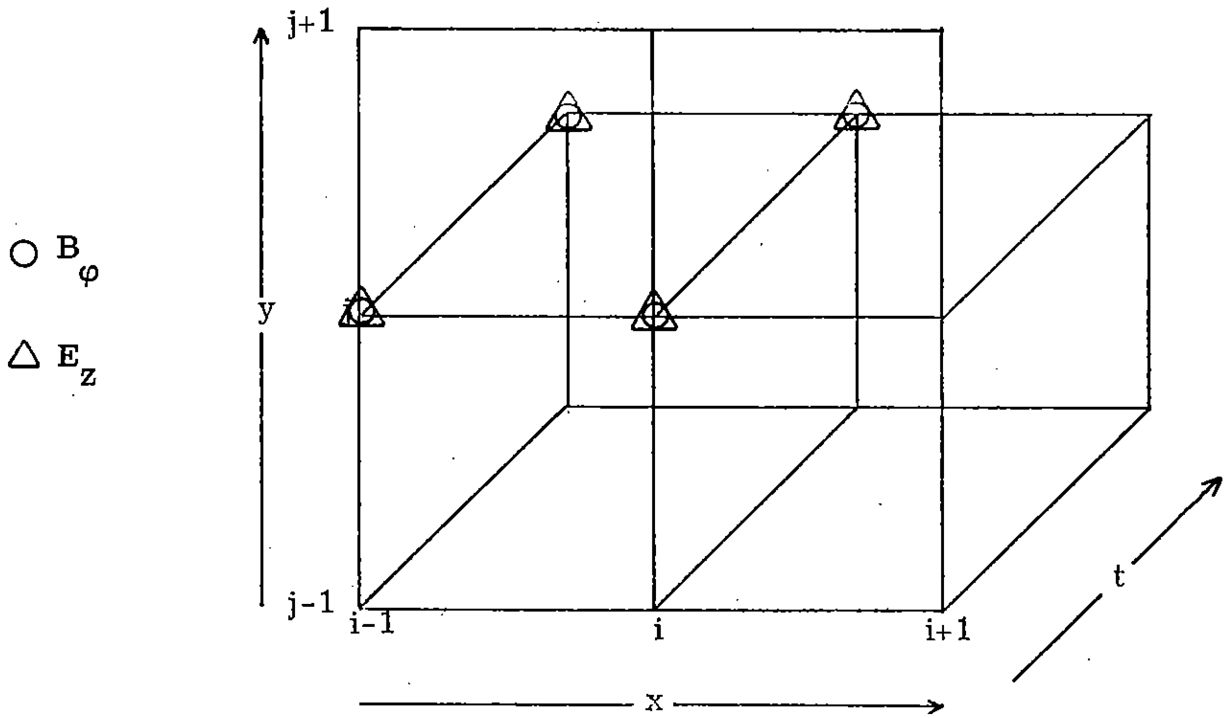


Figure 2.11. Differencing for Equation 2.3.18

Equation (2.3.18)

$$\frac{\partial E_z}{\partial t} + \frac{\sigma}{\epsilon} E_z = -\frac{J_z}{\epsilon} - \frac{FP}{\sqrt{\mu\epsilon}} \frac{\partial B_\varphi}{\partial x} - \frac{1}{F\sqrt{\mu\epsilon}} B_\varphi$$

will be centered differenced at (i-1/2, j, t+Δt/2). The finite difference form of equation (2.3.18) is

$$\frac{E^2_{zi-1j} + E^2_{zij}}{2} = \frac{\left(E^1_{zi-1j} + E^1_{zij}\right)}{2} e^{-\theta} + (1 - e^{-\theta})\left[\frac{\psi_2}{\gamma}\right]^{t+\Delta t/2}_{i-1/2j} \tag{2.3.23}$$

where

$$\theta_2 = \gamma_2 \Delta t$$

$$\gamma_2 = \sigma / \epsilon$$

$$\psi_2 = - \frac{J^{1/2}_{zi-1j}}{\epsilon} - \frac{FP_{i-1j}}{2\sqrt{\mu\epsilon}\,\Delta x}\left[B^1_{\varphi ij} - B^1_{\varphi i-1j} + B^2_{\varphi ij} - B^2_{\varphi i-1j}\right]$$

$$- \frac{1}{4F_{i-1/2j}\sqrt{\mu\epsilon}}\left[B^2_{\varphi i-1j} + B^1_{\varphi i-1j} + B^2_{\varphi ij} + B^1_{\varphi ij}\right]$$

Define the following constants

$$A_2 = \left[\frac{\left(1 - e^{-\theta_2}\right)}{\gamma_2}\; 2\left(\frac{FP_{i-1j}}{2\sqrt{\mu\epsilon}\,\Delta x} + \frac{1}{2F_{i-1j}\sqrt{\mu\epsilon}}\right)\right]$$

$$C_2 = E^2_{zi-1j} + \left(E^1_{zi-1j} + E^1_{zij}\right)e^{-\theta_2} + \frac{2\left(1 - e^{-\theta_2}\right)}{\gamma_2}\left[\frac{-J_{zi-1j}}{\epsilon} - \frac{FP_{i-1j}}{2\sqrt{\mu\epsilon}\,\Delta x}\right.$$

$$\left.\left(B^1_{\varphi ij} - B^1_{\varphi i-1j} - B^2_{\varphi i-1j}\right) - \frac{1}{2F_{i-1j}\sqrt{\mu\epsilon}}\left(B^2_{\varphi i-1j} + B^1_{\varphi i-1j} + B^1_{\varphi ij}\right)\right]$$

Equation (2.3.23) becomes

$$E^2_{zij} + A_2 B^2_{\varphi ij} = C_2 \tag{2.3.24}$$

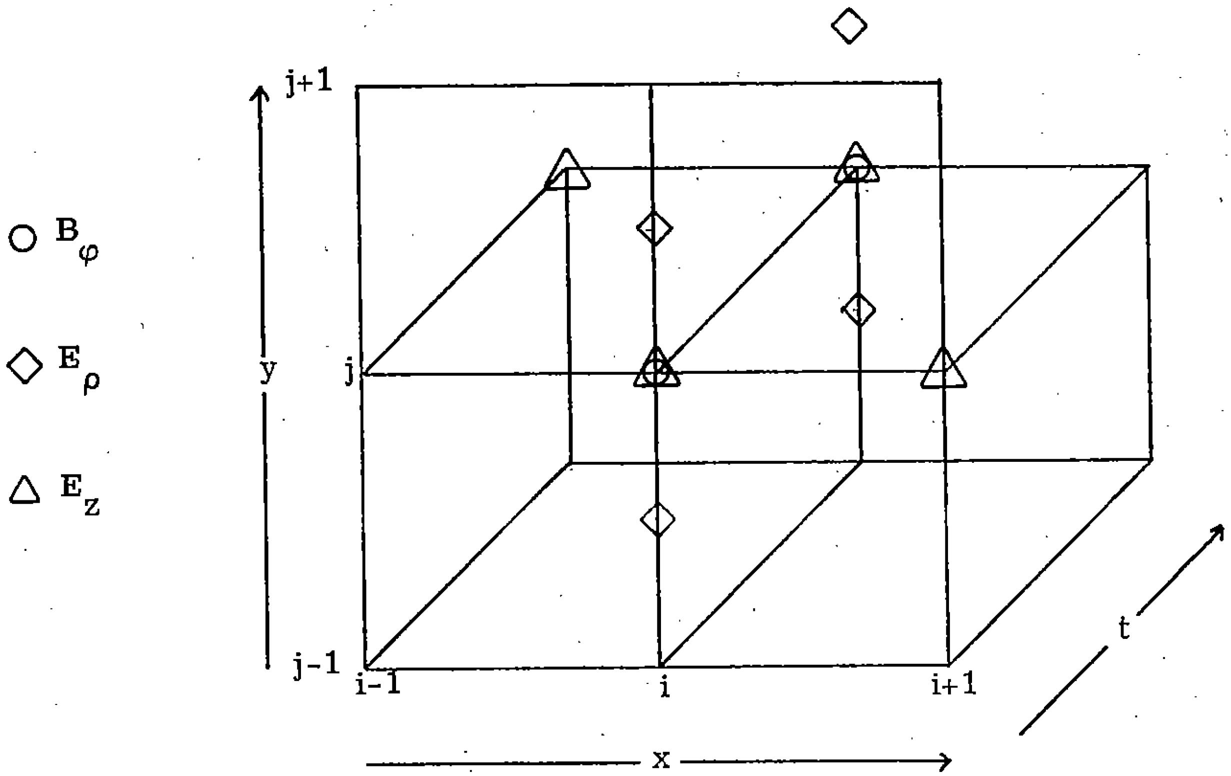


Figure 2.12. Differencing for Equation 2.3.19

Equation (2.3.19)

$$\frac{\partial B_\varphi}{\partial t} = FP\frac{\partial E_z}{\partial x} - GP\frac{\partial E_\rho}{\partial y}$$

will be differenced, centered at (i, j, t+Δt/2). The finite difference form of equation (2.3.19) is

$$\frac{B^2_{ij} - B^1_{\varphi ij}}{\Delta t} = \frac{FP_{ij}}{2\Delta x}\left[E^2_{zij} - E^2_{i-1j} + E^1_{zi+1j} - E^1_{zij}\right]$$

$$- \frac{GP}{2\Delta y}\left[E^1_{\rho ij} - E^1_{\rho ij-1} + E^2_{\rho ij} - E^2_{\rho ij-1}\right] \qquad (2.3.25)$$

Define the following constants

$$A_3 = \frac{FP_{ij}\,\Delta t}{2\Delta x}$$

$$A_4 = \frac{GP_{ij}\,\Delta t}{2\Delta y}$$

$$C_3 = B^1_{\rho\, ij} + \frac{FP_{ij}\,\Delta t}{2\Delta x}\left[E^1_{zi+1j} - E^1_{ij} - E^2_{zi-1j}\right] - \frac{GP_{ij}\,\Delta t}{2\Delta y}\left[E^1_{\rho\, ij} - E^1_{\rho\, ij-1}\right]$$

With these definitions equation (2.3.25) becomes

$$B^2_{\varphi\, ij} + A_3 E^2_{zij} + A_4 E^2_{\rho\, ij} - A_4 E^2_{\rho\, ij-1} = C_3 \qquad (2.3.26)$$

Substitute the expression for E^2_{zij} from equation (2.3.24) into (2.3.26) and collect the terms

$$B^2_{\varphi\, ij}(1 - A_3A_2) + A_4 E^2_{\rho\, ij} - A_4 E^2_{\rho\, ij-1} = C_3 - A_3 C_2 \qquad (2.3.27)$$

Now substitute the expression for $E^2_{\rho\, ij-1}$ from equation (2.3.22) into (2.3.27) and collect

$$(1 - A_3 A_2 + A_1 A_4) B^2_{\varphi\, ij} + A_4 E^2_{\gamma\, ij} - A_1 A_4 B^2_{\varphi\, ij-1} = C_3 - A_3C_2 + A_4C_1$$

(2.3.28)

Rewrite equation (2.3.22) to center the equation at (i, j+1/2, t+Δt/2)

$$E^2_{\rho\, ij} - a_1 B^2_{\varphi\, ij+1} + a_1 B^2_{\varphi\, ij} = c_1 \qquad (2.3.29)$$

Substitute equation (2.3.29) into (2.3.28) and collect like terms

$$(1 - A_3 A_2 + A_1 A_4 - a_1 A_4) B^2_{\varphi ij} - A_1 A_4 B^2_{\varphi ij-1} - A_4 a_1 B^2_{\varphi ij+1}$$

$$= C_3 - A_3 C_2 + A_4 C_1 - A_4 c_1$$

Define the following constants

$$B_1 = + A_4 a_1$$

$$B_2 = 1 - A_3 A_2 + A_1 A_4 - a_1 A_4$$

$$B_3 = - A_1 A_4$$

$$B_4 = C_3 - A_4 C_2 + A_4 C_1 - A_1 c_1$$

$$B_1 B^2_{\varphi ij+1} + B_2 B^2_{\varphi ij} + B_3 B^2_{\varphi ij-1}$$

Assume that the B_φ can be calculated recursively, that is

$$B^2_{\varphi j} = E_j B^2_{\varphi j+1} + FF_j$$

or

$$B^2_{\varphi j-1} = E_{j-1} B^2_{\varphi j} + FF_{j-1}$$

then

$$B_1 B^2_{\varphi ij+1} + (B_2 + B_3 E_{j-1}) B^2_{\varphi ij} = B_4 - B_3 FF_{j-1}$$

We define the E_j's and FF_j's as

$$E_j = \frac{B_1}{B_2 + B_3 E_{j-1}}$$

$$FF_j = \frac{B_4 - B_3 FF_{j-1}}{B_2 + B_3 E_{j-1}}$$

A possible set of boundary conditions for the recursive relation is $B_\varphi = 0$ at $j = 1$ and $j = j_{max}$. This implies $E_1 = FF_1 = 0$.

2.3.3 Calculational Procedure

The computer code is composed of a main routine, which is responsible for supervising activity of a number of subroutines. The main routine IG2D reads input data, defines calculational grids and some constants. IG2D then calls SETPT which defines the constants for the simple analytical sources. RDS is then called by IG2D to print a summary of the input data and constants for this run.

The calculation is stepped in time a Δt, SIGMA is called to calculate the conductivity and CURRENT is called to calculate the current densities. The calculation is then stepped in range and the fields are calculated over all altitudes explicitly as described in the section on finite differencing. At the end of the time loop, the fields are extrapolated in range to the last range grid point. OVTPVT is called for all output functions. PROFILE is called by OVTPVT to graph the sources and fields at a constant time and range as a function of altitude. RANGER is called to graph the source and field at a constant time and altitude as a function of range.

2.3.4 General Considerations

Since this algorithm is a real time algorithm, the grid spacing must be kept rather small to resolve the fast rise of the probe as it propagates through the grid. To make a real time code run to very late times, it would be necessary to incorporate into the code some form of regridding.

The particular implicit scheme used to advance the calculation over altitude has been used with good success in several retarded time codes, such as SCX and ONDINE. The scheme seen to be very stable as long as the Courant condition is used to select the rate of the sizes of the grid steps.

Since the algorithm assumed an infinite conductivity for the ground, the results can be expected to agree well with other calculations when the air conductivity is much less than the ground conductivity. For close-in, early time regions, however, the code can not be expected to perform as well as other codes which assume a finite conductivity. Since the pulse which is observed far from the sources is produced on the outer edge of the source region where the air conductivity is low, the code should be able to predict the radiated signal fairly accurately.

2.3.5 Conclusion

Representative data is given for two ground ranges, 500 meters and 1000 meters with a source height of 50 meters. The sources are not for a particular weapon but were chosen as values which might typically be encountered. The time axes on the plots are in retarded time of the burst point. The two observers plotted are located just above the infinitely conducting ground. It is to be noted that the jitter in the vertical electric field is due to the lack of transverse currents and can be expected to go away if the transverse is installed.

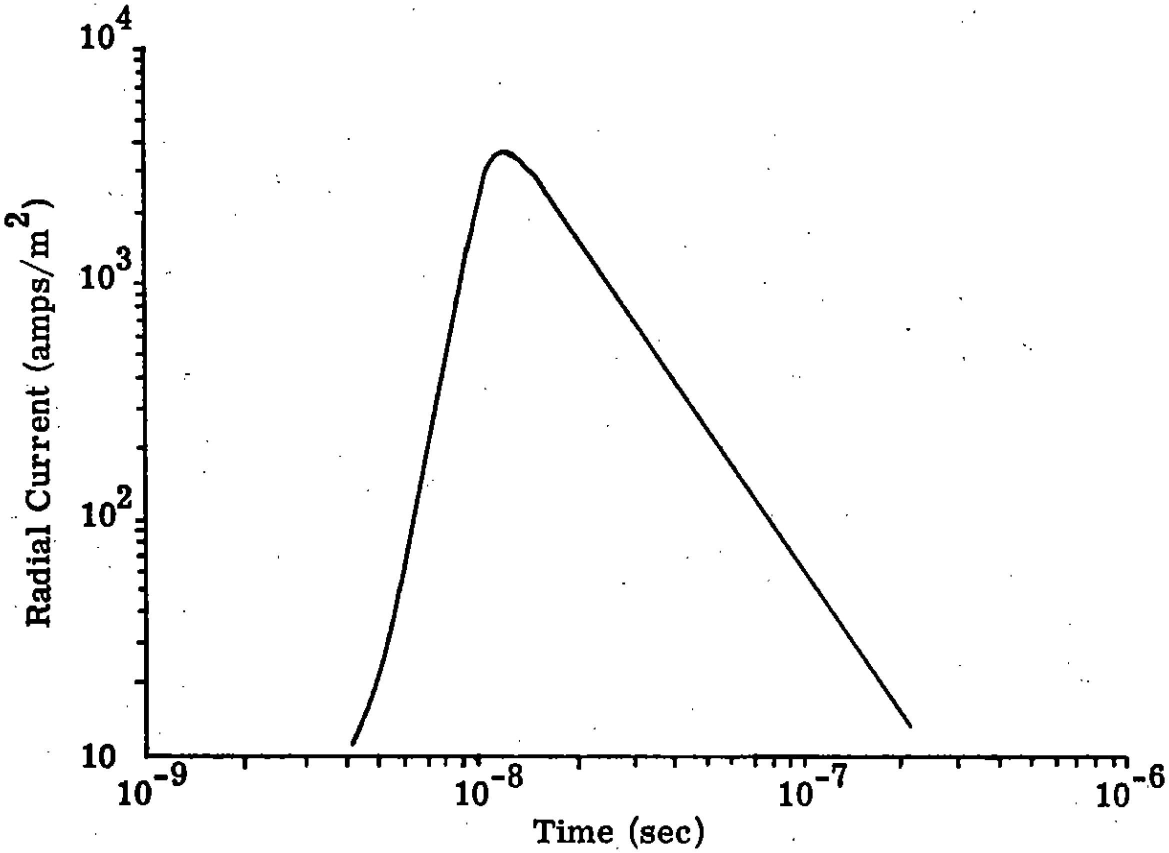


Figure 2.13. Radial Current at 500 Meters

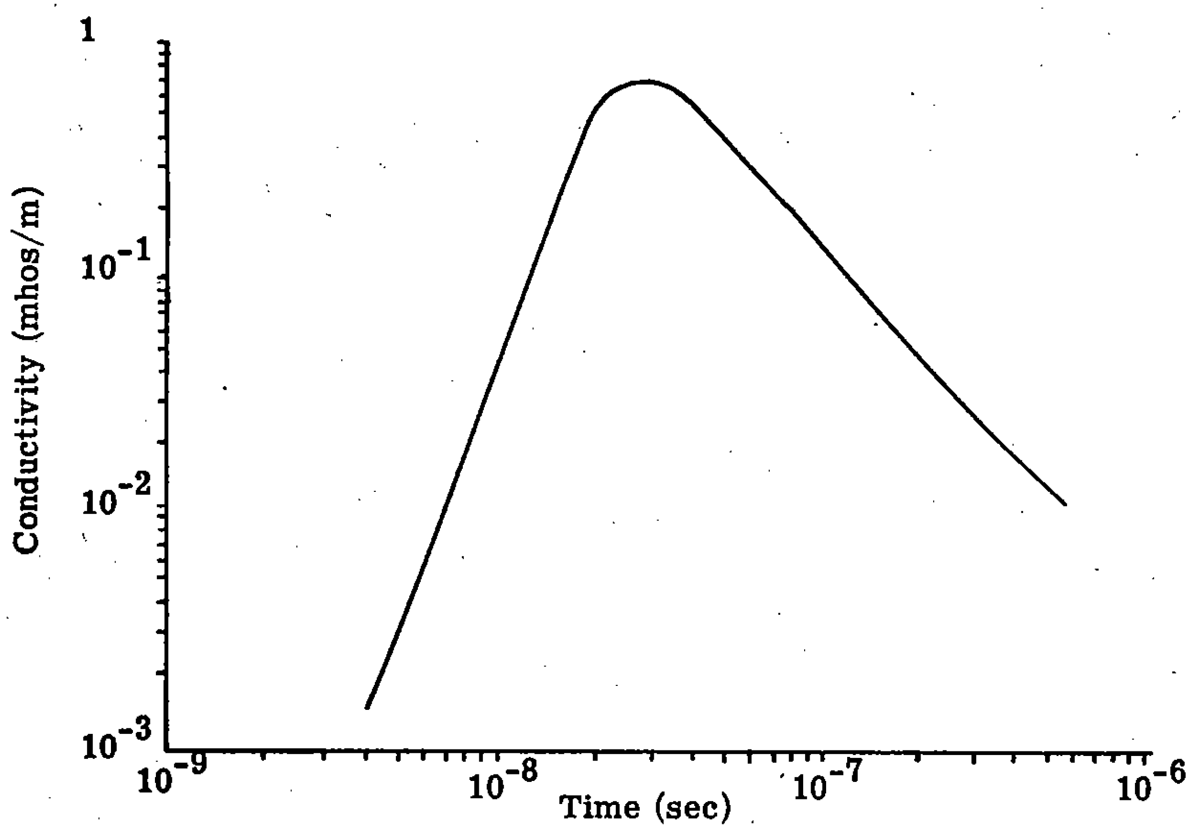


Figure 2.14. Conductivity at 500 Meters

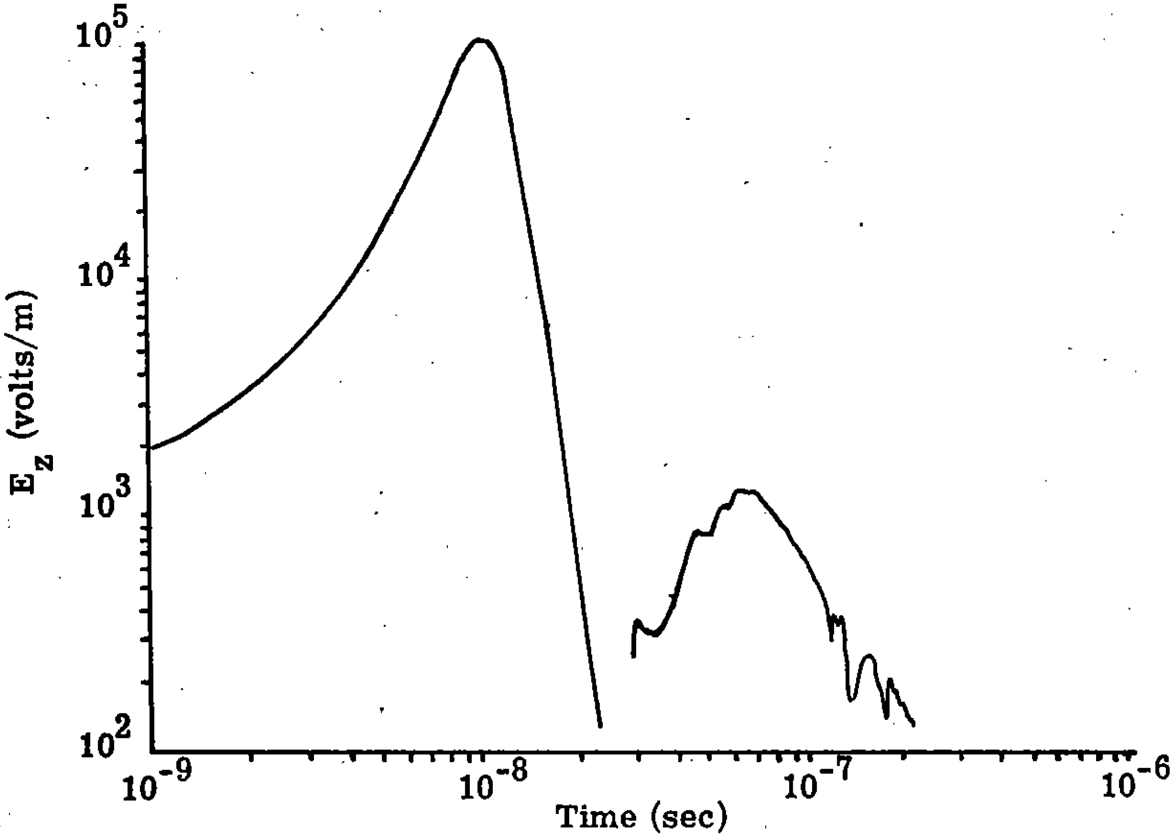


Figure 2.15. Vertical Electric Field at 500 Meters

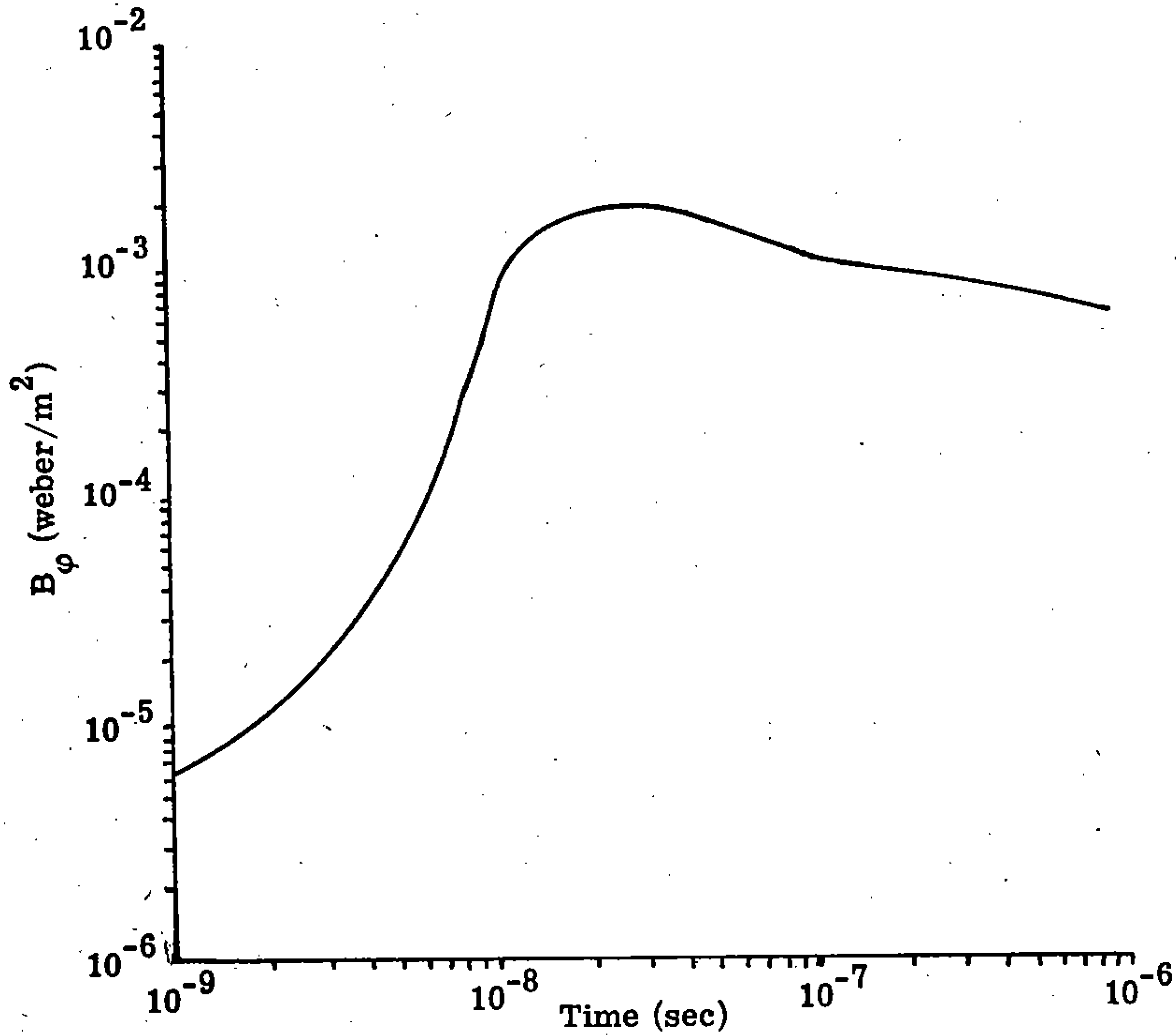


Figure 2.16. Magnetic Field at 500 Meters

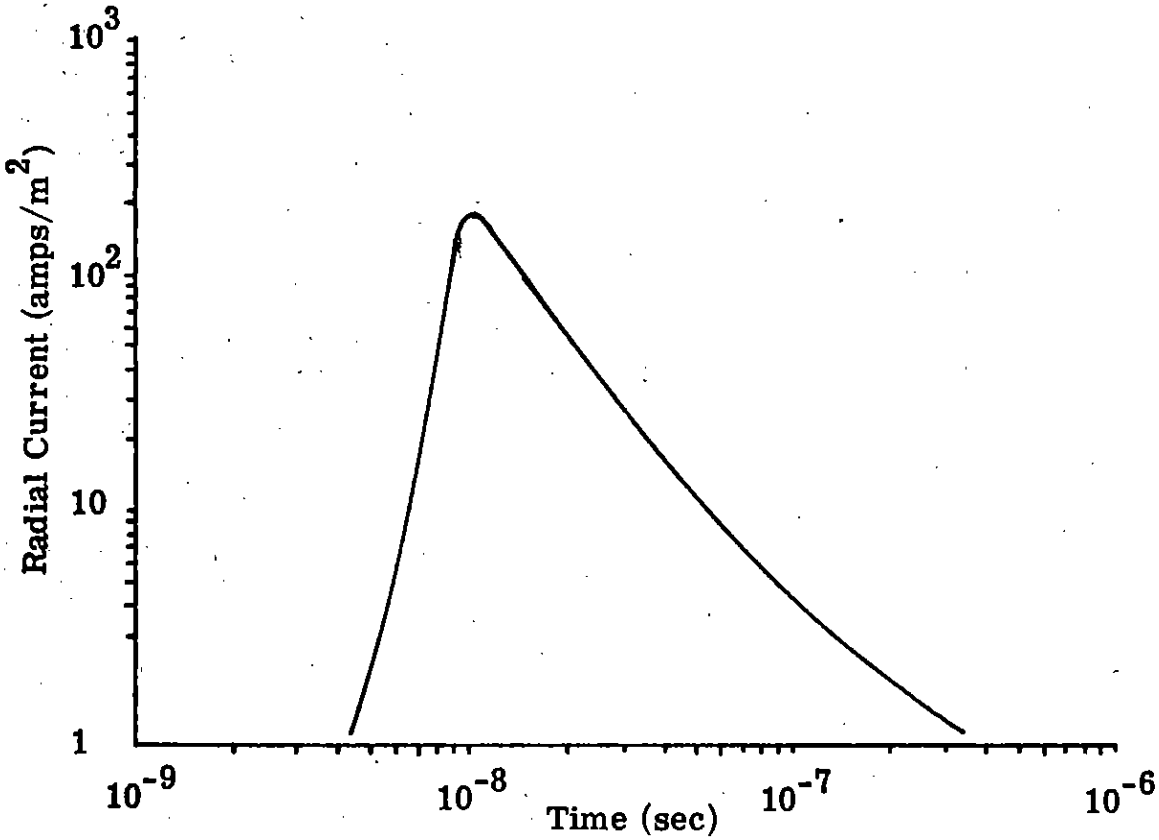


Figure 2.17. Radial Current at 1000 Meters

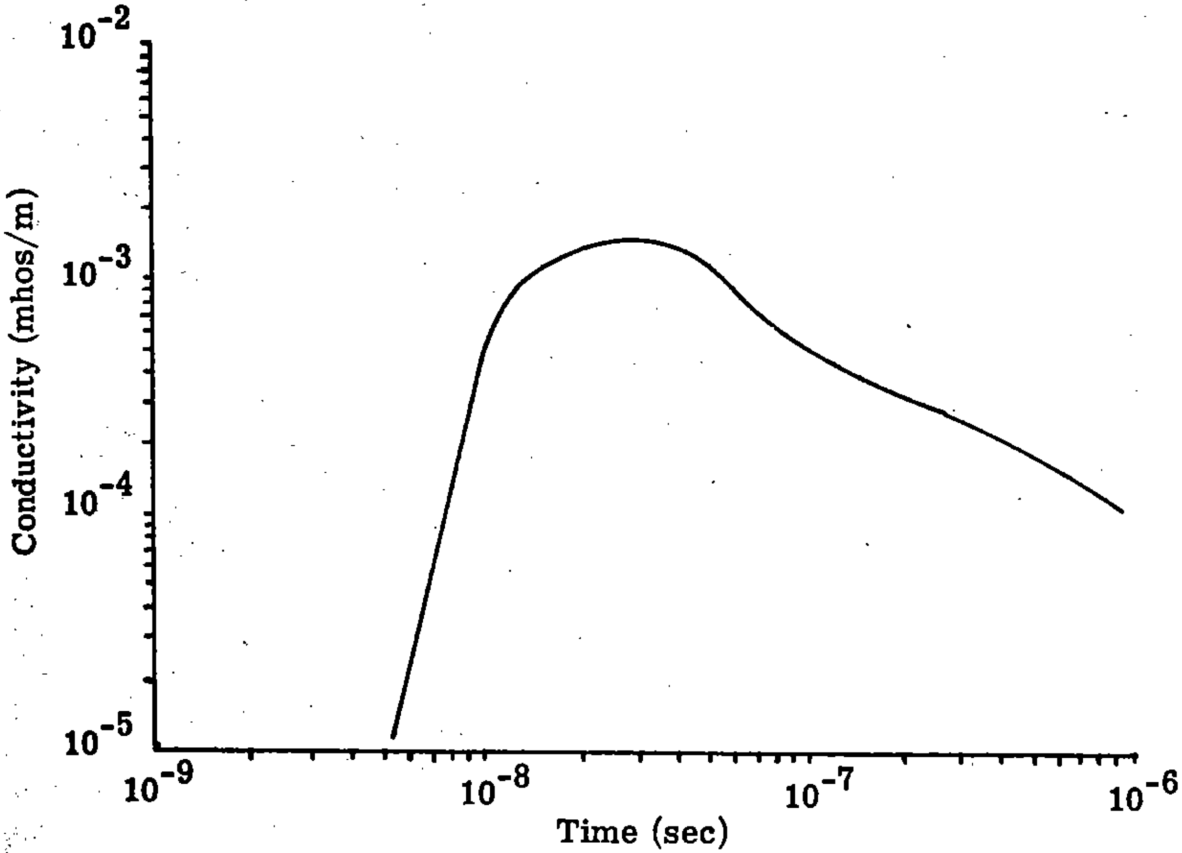


Figure 2.18. Conductivity at 1000 Meters

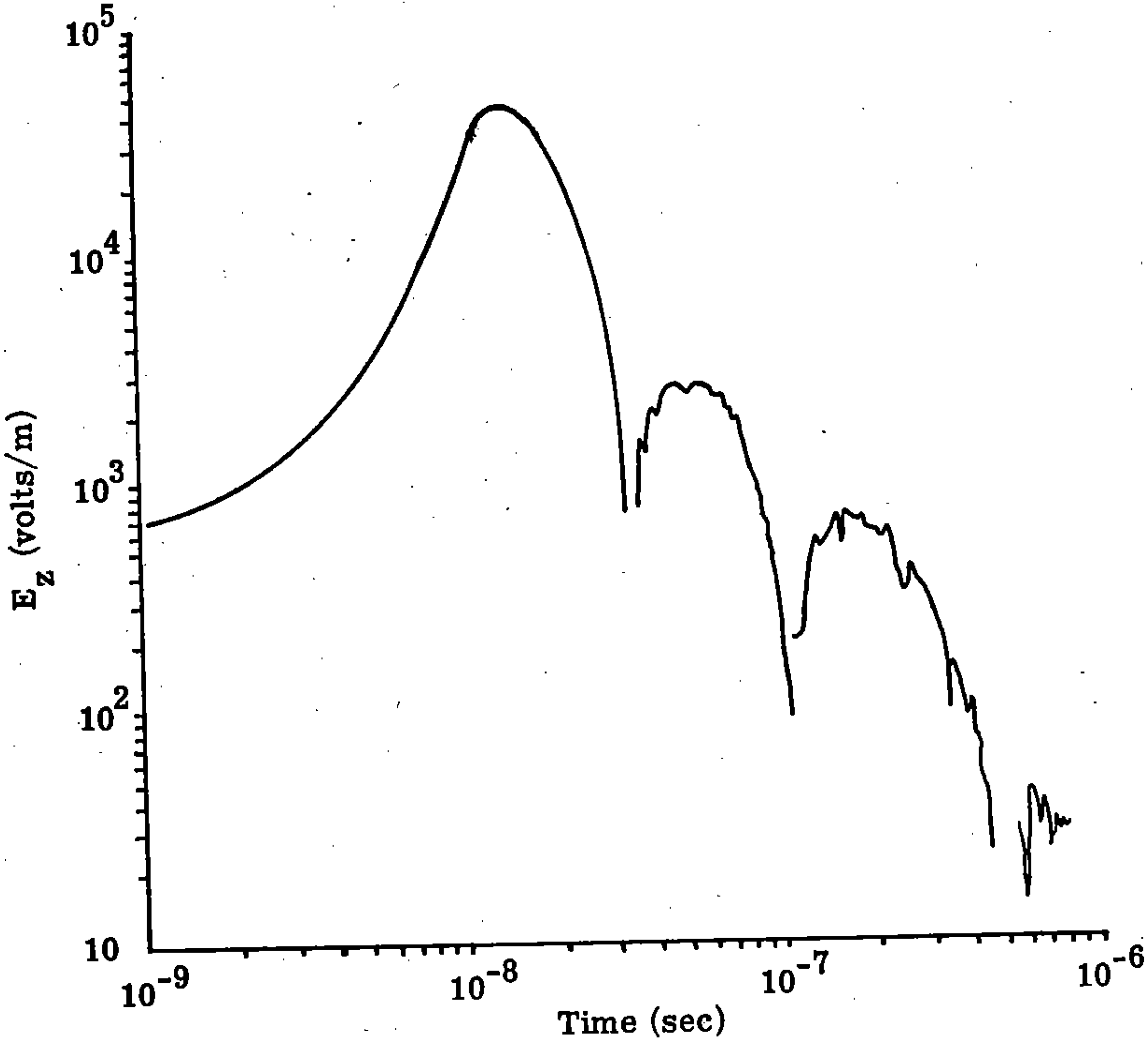
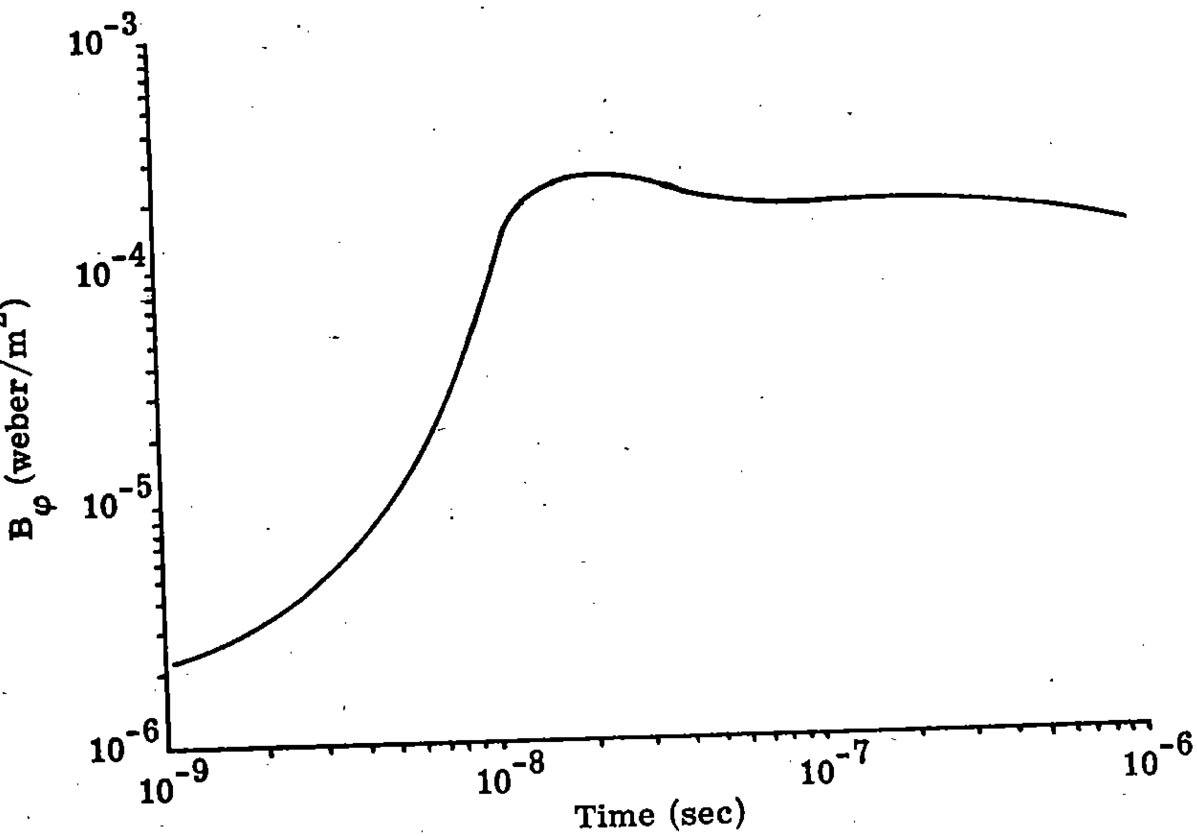


Figure 2.19. Vertical Electric Field at 1000 Meters


B_φ (weber/m^2)
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
Time (sec)

2.4 Algorithm "D"

An approach that was briefly investigated near the end of the effort covered by this report involved the development of an algorithm requiring integration of the field equations along characteristics. Unfortunately, there was insufficient time to program an algorithm for machine computation, but the approach nevertheless appeared to be quite promising; a brief description of the general scheme was therefore felt to be justified.

The use of characteristics methods has been extensively described in the literature,[9, 10, 11] and will not be presented in detail here. Briefly, characteristics for systems of hyperbolic partial differential equations are curves or surfaces associated with the propagation of wavefronts. The process of "integrating along characteristics" is attractive from a physical standpoint because it is connected with notions of causality, and attractive from a numerical standpoint because the original partial differential equations take on the form of very convenient relations (called "characteristic conditions") for the variations of fields along the directions of characteristics. Peebles[12] has presented a solution technique using characteristics for an EMP problem with two independent variables: one space dimension, $\underline{r}$, and time, $\underline{t}$. In that case, the characteristics were families of straight lines ($r \pm ct$ = constant, and r = constant), and the characteristic conditions had the form of ordinary differential equations. An analogous result appears for the three-independent-variable EMP problem, except that the characteristics then become surfaces, and the characteristic conditions involve derivatives in two directions along the characteristic surfaces. We can summarize the detailed results of a characteristics analysis of Maxwell's equations, as written in prolate-spheroidal coordinates and retarded time, as follows:

We may rewrite equations (2.2.5) through (2.2.7) in the following form:

$$\epsilon \frac{\partial E_\xi}{\partial \tau} + \left[\frac{\partial H_\varphi}{\partial \tau} - \frac{\partial H_\varphi}{\partial \zeta}\right] = - J_\xi \tag{2.4.1}$$

$$\epsilon \frac{\partial E_\zeta}{\partial \tau} + \left[\frac{\partial H_\varphi}{\partial \tau} + \frac{1}{a} \frac{\partial H_\varphi}{\partial \xi}\right] = - J_\zeta \tag{2.4.2}$$

$$\gamma_\zeta \left[\frac{\partial E_\xi}{\partial \tau} - \frac{\partial E_\xi}{\partial \zeta}\right] + \gamma_\xi \left[\frac{\partial E_\zeta}{\partial \tau} + \frac{1}{a} \frac{\partial E_\zeta}{\partial \xi}\right] + \frac{\partial H_\varphi}{\partial \tau} = 0 \tag{2.4.3}$$

For $\epsilon = 1$ (as an example) we can identify three convenient characteristic surfaces:

<u>A.</u> τ = cons't

<u>B.</u> $\tau - 2a\xi$ = cons't

<u>C.</u> ξ = cons't

The characteristic conditions associated with each of the above surfaces are differential relations amongst the field variables <u>within</u> the surfaces A., B., and C.

For <u>A.</u>,

$$\gamma_\zeta D_+ (E_\xi - H_\varphi) - \gamma_\xi D_A (E_\zeta - H_\varphi) = - \gamma_\zeta J_\xi - \gamma_\xi J_\zeta \tag{2.4.4}$$

For <u>B.</u>,

$$\gamma_\zeta D_+ (E_\xi - H_\varphi) - \gamma_\xi D_B (E_\zeta + H_\varphi) = - \gamma_\zeta J_\xi + \gamma_\xi J_\zeta \tag{2.4.5}$$

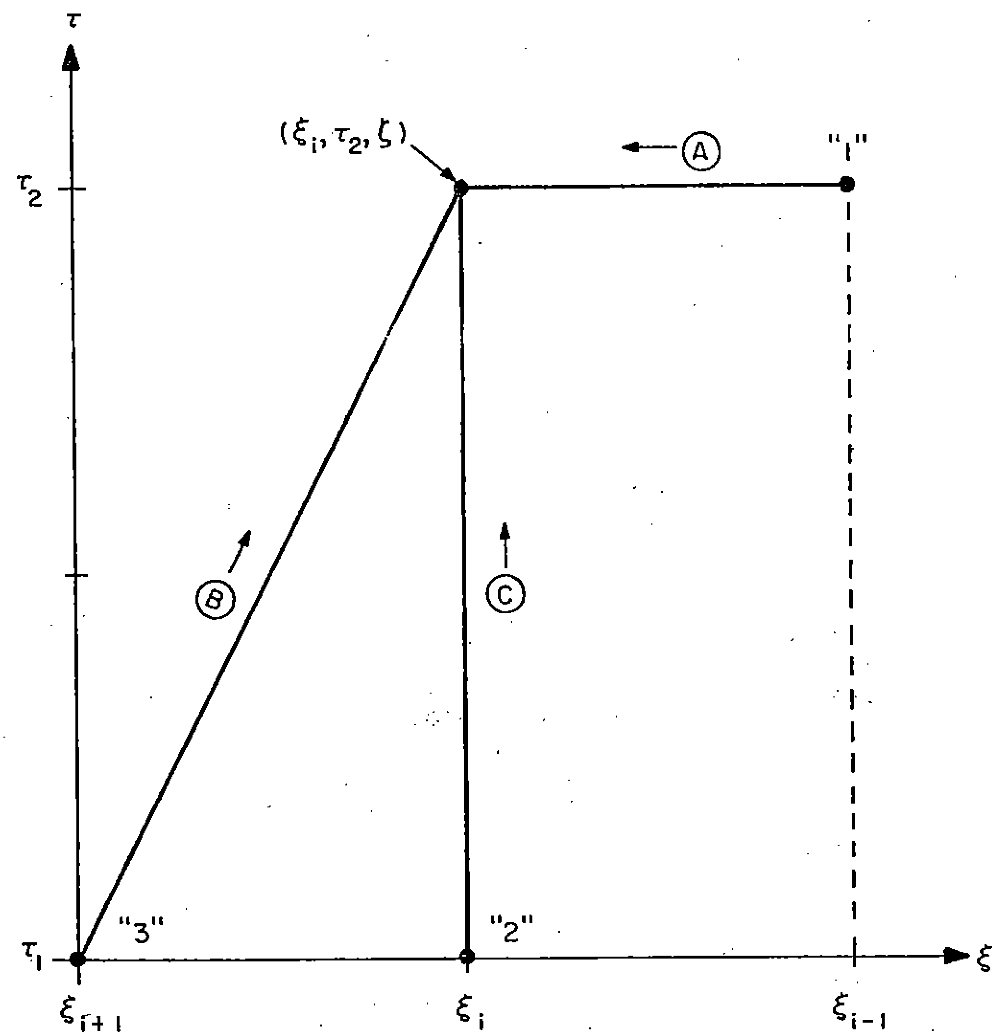


Figure 2.21. Integration in Characteristic Surfaces for Algorithm "D"

and for C.,

$$- D_{\perp} H_{\varphi} + D_C(E_{\xi} + H_{\varphi}) = - J_{\xi} \tag{2.4.6}$$

The D-operators in equations (2.4.4) through (2.4.6) are readily defined:

$$D_{\perp} = \frac{\partial}{\partial \zeta} \tag{2.4.7}$$

$$D_A = \frac{1}{a} \frac{\partial}{\partial \xi} \tag{2.4.8}$$

$$D_B = \frac{1}{a} \frac{\partial}{\partial \xi} + 2 \frac{\partial}{\partial \tau} \tag{2.4.9}$$

$$D_C = \frac{\partial}{\partial \tau} \tag{2.4.10}$$

D_A, D_B, and D_C simply represent derivatives taken parallel to the directions of the A, B, and C-characteristics in the (ξ, τ)-plane.

The three characteristic conditions (2.4.4) through (2.4.6) thus can replace the original set of field equations, and can be used, as suggested in Figure 2.21 , to find fields along the line through ξ_i and τ_2, once the fields along the lines "1", "2", and "3" are known. The sets of implicit equations obtained from this procedure can be solved by standard methods, and are likely to be more satisfactory, because of the causality considerations implicit in their development, than equations generated by more straightforward difference representations of the original field equations. It is felt that this approach is attractive enough to warrant more detailed consideration.

3. SUMMARY

Several finite difference algorithms for the calculation of low altitude EMP environments were investigated. These algorithms were programmed and run on a CDC 6600 computer. The results were checked for two test problems. The first test was to input a radial current and then to determine how nearly radial the resultant fields were. The second test problem employed the Maxwell equations to calculate currents for an assumed set of fields, then these currents were put into the codes and the resulting fields were compared with the assumed ones. A third test problem was developed. This consisted of a real-time air burst code and an image calculation to incorporate the effects of a perfectly conducting earth. This code is complete and is running. However, comparison of it and the low altitude environment codes has not yet been made.

Comparison of the algorithms studied with the test problems indicated that the two explicit algorithms described in Sections 2.1 and 2.2 above worked considerably better than any of the several implicit methods studied. Details of a few of the implicit calculations are given in Appendix D.

A new approach to the problem using the method of characteristics is discussed in Section 2.4. Although this method appears very promising, it was developed near the end of the contract and has not yet been programmed for the computer.

APPENDIX A

I. FIELD EQUATIONS

Define modified MKS units by redefining $\vec{B}$ and $\vec{H}$ according to

$$\vec{B} = c\,\vec{B}\ \text{MKS}, \qquad \vec{H} - \mu_o c\,\vec{H}\ \text{MKS} \tag{1}$$

Maxwell's equations become

$$\nabla \cdot \vec{D} = \rho \tag{2}$$

$$\nabla \cdot \vec{B} = 0 \tag{3}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{4}$$

$$\nabla \times \vec{H} = z_o\left(\vec{j} + \frac{\partial \vec{D}}{\partial t}\right) \tag{5}$$

where the definition

$$z_o = \sqrt{\frac{\mu_o}{\epsilon_o}} = \mu_o c \tag{6}$$

has been used. The constitutive equations become

$$\vec{D} = \epsilon \vec{E}, \quad \vec{B} = \varkappa_m \vec{H}, \quad \epsilon = \varkappa \epsilon_o, \quad \mu = \varkappa_m \mu_o \tag{7}$$

Then, define

$$z = \sqrt{\frac{\varkappa_m}{\varkappa}}\; z_o \tag{8}$$

We transform to retarded time defined by

$$\tau = ct - r = ct - a(\zeta - \xi) \tag{9}$$

The transformations of derivatives are

$$\frac{\partial}{\partial t} = c \frac{\partial}{\partial \tau} \tag{10}$$

$$\nabla_t = \nabla_\tau - \hat{r} \frac{\partial}{\partial \tau} \tag{11}$$

where ∇_t and ∇_τ are the gradient operators at constant t and constant τ. Maxwell's equations become

$$(\nabla_\tau - \hat{r} \frac{\partial}{\partial \tau}) \cdot \vec{E} = \frac{\rho}{\varkappa \epsilon_o} \tag{12}$$

$$(\nabla_\tau - \hat{r} \frac{\partial}{\partial \tau}) \cdot \vec{H} = 0 \tag{13}$$

$$(\nabla_\tau - \hat{r} \frac{\partial}{\partial \tau}) \times \vec{E} = - \varkappa_m \frac{\partial \vec{H}}{\partial \tau} \tag{14}$$

$$(\nabla_\tau - \hat{r} \frac{\partial}{\partial \tau}) \times \vec{H} = z_o \vec{j} + \varkappa \frac{\partial \vec{E}}{\partial \tau} \tag{15}$$

The divergence equations are initial conditions for the problem. The fields are thus determined by the curl equations. Now:

$$\hat{r} = \alpha_{11} \hat{\zeta} + \alpha_{21} \hat{\xi} \tag{16}$$

The coordinates are cyclic in the order (ζ, ξ, φ), thus:

$$\hat{r} \times \frac{\partial \vec{E}}{\partial \tau} = \alpha_{21} \frac{\partial E_\varphi}{\partial \tau} \hat{\zeta} - \alpha_{11} \frac{\partial E_\varphi}{\partial \tau} \hat{\xi} + \left(\alpha_{11} \frac{\partial E_\xi}{\partial \tau} - \alpha_{21} \frac{\partial E_\zeta}{\partial \tau} \right) \hat{\varphi} \tag{17}$$

A similar equation holds for the magnetic field. For an azimuthally symmetric problem, Maxwell's equations become

$$\alpha_{11}\frac{\partial E_\zeta}{\partial \tau} + \alpha_{21}\frac{\partial E_\xi}{\partial \tau} = -\frac{\rho}{\varkappa\epsilon_o} + \frac{1}{a(\zeta^2-\xi^2)}\left[\frac{\partial}{\partial\zeta}\left(\sqrt{(\zeta^2-\xi^2)(\zeta^2-1)}\,E_\zeta\right) + \frac{\partial}{\partial\xi}\left(\sqrt{(\zeta^2-\xi^2)(1-\xi^2)}\,E_\xi\right)\right] \quad (18)$$

$$\alpha_{11}\frac{\partial H_\zeta}{\partial \tau} + \alpha_{21}\frac{\partial H_\xi}{\partial \tau} = \frac{1}{a(\zeta^2-\xi^2)}\left[\frac{\partial}{\partial\zeta}\left(\sqrt{(\zeta^2-\xi^2)(\zeta^2-1)}\,H_\zeta\right) + \frac{\partial}{\partial\xi}\left(\sqrt{(\zeta^2-\xi^2)(1-\xi^2)}\,H_\xi\right)\right] \quad (19)$$

$$\alpha_{21}\frac{\partial E_\varphi}{\partial \tau}\hat{\zeta} - \alpha_{11}\frac{\partial E_\varphi}{\partial \tau}\hat{\xi} + \left(\alpha_{11}\frac{\partial E_\xi}{\partial \tau} - \alpha_{21}\frac{\partial E_\zeta}{\partial \tau}\right)\hat{\varphi} - \varkappa_m\frac{\partial}{\partial\tau}(H_\zeta\hat{\zeta} + H_\xi\hat{\xi} + H_\varphi\hat{\varphi})$$

$$= \frac{\hat{\zeta}}{a\sqrt{(\zeta^2-\xi^2)(\zeta^2-1)}}\frac{\partial}{\partial\xi}\left(\sqrt{(\zeta^2-1)(1-\xi^2)}\,E_\varphi\right) - \frac{\hat{\xi}}{a\sqrt{(\zeta^2-\xi^2)(1-\xi^2)}}$$

$$\frac{\partial}{\partial\zeta}\left(\sqrt{(\zeta^2-1)(1-\xi^2)}\,E_\varphi\right) + \frac{\hat{\varphi}\sqrt{(\zeta^2-1)(1-\xi^2)}}{a(\zeta^2-\xi^2)}$$

$$\left[\frac{\partial}{\partial\zeta}\left(\sqrt{\frac{\zeta^2-\xi^2}{1-\xi^2}}\,E_\xi\right) - \frac{\partial}{\partial\xi}\left(\sqrt{\frac{\zeta^2-\xi^2}{\zeta^2-1}}\,E_\zeta\right)\right] \quad (20)$$

$$\alpha_{21}\frac{\partial H_\varphi}{\partial \tau}\hat{\zeta} - \alpha_{11}\frac{\partial H_\varphi}{\partial \tau}\hat{\xi} + \left(\alpha_{11}\frac{\partial H_\xi}{\partial \tau} - \alpha_{21}\frac{\partial H_\zeta}{\partial \tau}\right)\hat{\varphi} + \varkappa\frac{\partial}{\partial \tau}\left(E_\zeta\hat{\zeta} + E_\xi\hat{\xi} + E_\varphi\hat{\varphi}\right)$$

$$= - z_o\left(j_\zeta\hat{\zeta} + j_\xi\hat{\xi} + j_\varphi\hat{\varphi}\right) + \frac{\hat{\zeta}}{a\sqrt{(\zeta^2 - \xi^2)(\zeta^2 - 1)}}\frac{\partial}{\partial \xi}\left(\sqrt{(\zeta^2 - 1)(1 - \xi^2)}\, H_\varphi\right)$$

$$- \frac{\hat{\xi}}{a\sqrt{(\zeta^2 - \xi^2)(1 - \xi^2)}}\frac{\partial}{\partial \zeta}\left(\sqrt{(\zeta^2 - 1)(1 - \xi^2)}\, H_\varphi\right)$$

$$+ \frac{\hat{\varphi}\sqrt{(\zeta^2 - 1)(1 - \xi^2)}}{a(\zeta^2 - \xi^2)}\left[\frac{\partial}{\partial \zeta}\left(\sqrt{\frac{\zeta^2 - \xi^2}{1 - \xi^2}}\, H_\xi\right) - \frac{\partial}{\partial \xi}\left(\sqrt{\frac{\zeta^2 - \xi^2}{\zeta^2 - 1}}\, H_\zeta\right)\right] \qquad (21)$$

In component form, these separate into two sets of equations, the TM and TE field equations.

TM Equations

$$\varkappa\frac{\partial E_\zeta}{\partial \tau} + \alpha_{21}\frac{\partial H_\varphi}{\partial \tau} = - z_o j_\zeta + \frac{1}{a\sqrt{(\zeta^2 - \xi^2)(\zeta^2 - 1)}}\frac{\partial}{\partial \xi}\left(\sqrt{(\zeta^2 - 1)(1 - \xi^2)}\, H_\varphi\right) \qquad (22)$$

$$\varkappa\frac{\partial E_\xi}{\partial \tau} - \alpha_{11}\frac{\partial H_\varphi}{\partial \tau} = - z_o j_\xi - \frac{1}{a\sqrt{(\zeta^2 - \xi^2)(1 - \xi^2)}}\frac{\partial}{\partial \zeta}\left(\sqrt{(\zeta^2 - 1)(1 - \xi^2)}\, H_\varphi\right) \qquad (23)$$

$$\alpha_{11}\frac{\partial E_\xi}{\partial \tau} - \alpha_{21}\frac{\partial E_\zeta}{\partial \tau} - \varkappa_m\frac{\partial H_\varphi}{\partial \tau} = \frac{\sqrt{(\zeta^2 - 1)(1 - \xi^2)}}{a(\zeta^2 - \xi^2)}\left[\frac{\partial}{\partial \zeta}\left(\sqrt{\frac{\zeta^2 - \xi^2}{1 - \xi^2}}\, E_\xi\right) - \frac{\partial}{\partial \xi}\left(\sqrt{\frac{\zeta^2 - \xi^2}{\zeta^2 - 1}}\, E_\zeta\right)\right] \qquad (24)$$

TE Equations

$$\alpha_{21}\frac{\partial E_\varphi}{\partial \tau} - \varkappa_m \frac{\partial H_\zeta}{\partial \tau} = \frac{1}{a\sqrt{(\zeta^2 - \xi^2)(\zeta^2 - 1)}}\frac{\partial}{\partial \xi}\left(\sqrt{(\zeta^2 - 1)(1 - \xi^2)}\, E_\varphi\right) \tag{25}$$

$$\alpha_{11}\frac{\partial E_\varphi}{\partial \tau} + \varkappa_m \frac{\partial H_\xi}{\partial \tau} = \frac{1}{a\sqrt{(\zeta^2 - \xi^2)(1 - \xi^2)}}\frac{\partial}{\partial \zeta}\left(\sqrt{(\zeta^2 - 1)(1 - \xi^2)}\, E_\varphi\right) \tag{26}$$

$$\alpha_{11}\frac{\partial H_\xi}{\partial \tau} - \alpha_{21}\frac{\partial H_\zeta}{\partial \tau} + \varkappa\frac{\partial E_\varphi}{\partial \tau} = - z_o j_\varphi + \frac{\sqrt{(\zeta^2 - 1)(1 - \xi^2)}}{a(\zeta^2 - \xi^2)}\left[\frac{\partial}{\partial \zeta}\left(\sqrt{\frac{\zeta^2 - \xi^2}{1 - \xi^2}}\, H_\xi\right) - \frac{\partial}{\partial \xi}\left(\sqrt{\frac{\zeta^2 - \xi^2}{\zeta^2 - 1}}\, H_\zeta\right)\right] \tag{27}$$

For the low altitude burst problem j_φ is zero. Thus the TE mode is not excited and we confine our investigation to the TM fields. The matrix elements in equation (16) are

$$\alpha_{11} = \frac{a}{h_\zeta} = \sqrt{\frac{\zeta^2 - 1}{\zeta^2 - \xi^2}} \tag{28}$$

$$\alpha_{21} = \frac{-a}{h_\xi} = -\sqrt{\frac{1 - \xi^2}{\zeta^2 - \xi^2}} \tag{29}$$

The TM equations become

$$\varkappa\frac{\partial E_\zeta}{\partial \tau} - \sqrt{\frac{1 - \xi^2}{\zeta^2 - \xi^2}}\frac{\partial H_\varphi}{\partial \tau} = - z_o j_\zeta + \frac{1}{a\sqrt{(\zeta^2 - \xi^2)(\zeta^2 - 1)}}\frac{\partial}{\partial \xi}\left(\sqrt{(\zeta^2 - 1)(1 - \xi^2)}\, H_\varphi\right) \tag{30}$$

$$\varkappa \frac{\partial E_\xi}{\partial \tau} - \sqrt{\frac{\zeta^2 - 1}{\zeta^2 - \xi^2}} \frac{\partial H_\varphi}{\partial \tau} = - z_o j_\xi - \frac{1}{a\sqrt{(\zeta^2 - \xi^2)(1 - \xi^2)}} \frac{\partial}{\partial \zeta} \left(\sqrt{(\zeta^2 - 1)(1 - \xi^2)}\, H_\varphi \right) \tag{31}$$

$$\sqrt{\frac{\zeta^2 - 1}{\zeta^2 - \xi^2}} \frac{\partial E_\xi}{\partial \tau} + \sqrt{\frac{1 - \xi^2}{\zeta^2 - \xi^2}} \frac{\partial E_\zeta}{\partial \tau} - \varkappa_m \frac{\partial H_\varphi}{\partial \tau} = \frac{\sqrt{(\zeta^2 - 1)(1 - \xi^2)}}{a(\zeta^2 - \xi^2)} \left[\frac{\partial}{\partial \zeta} \left(\sqrt{\frac{\zeta^2 - \xi^2}{1 - \xi^2}}\, E_\xi \right) - \frac{\partial}{\partial \xi} \left(\sqrt{\frac{\zeta^2 - \xi^2}{\zeta^2 - 1}}\, E_\zeta \right) \right] \tag{32}$$

The field equations are simplified by the following transformation of dependent variables

$$\begin{pmatrix} E_\zeta \\ j_\zeta \end{pmatrix} = \sqrt{(\zeta^2 - \xi^2)(\zeta^2 - 1)} \begin{pmatrix} E'_\zeta \\ j'_\zeta \end{pmatrix} \tag{33}$$

$$\begin{pmatrix} E_\xi \\ j_\xi \end{pmatrix} = \sqrt{(\zeta^2 - \xi^2)(1 - \xi^2)} \begin{pmatrix} E'_\xi \\ j'_\xi \end{pmatrix} \tag{34}$$

$$H_\varphi = \sqrt{(\zeta^2 - 1)(1 - \xi^2)} \quad H'_\varphi \tag{35}$$

where the primed variables are those used previously. We obtain

$$\varkappa \frac{\partial E_\zeta}{\partial \tau} - \frac{\partial H_\varphi}{\partial \tau} = - z_o j_\zeta + \frac{1}{a} \frac{\partial H_\varphi}{\partial \xi} \tag{36}$$

$$\varkappa \frac{\partial E_\xi}{\partial \tau} - \frac{\partial H_\varphi}{\partial \tau} = - z_o j_\xi - \frac{1}{a} \frac{\partial H_\varphi}{\partial \zeta} \tag{37}$$

$$G_2 \frac{\partial E_\zeta}{\partial \tau} + G_1 \frac{\partial E_\xi}{\partial \tau} - \varkappa_m \frac{\partial H_\varphi}{\partial \tau} = \frac{G_1}{a} \frac{\partial E_\xi}{\partial \zeta} - \frac{G_2}{a} \frac{\partial E_\zeta}{\partial \xi} \tag{38}$$

where

$$G_1 = \frac{\zeta^2 - 1}{\zeta^2 - \xi^2} = \frac{a^2}{h_1^2} \tag{39}$$

$$G_2 = \frac{1 - \xi^2}{\zeta^2 - \xi^2} = \frac{a^2}{h_2^2} \tag{40}$$

To facilitate regridding in the three coordinate directions, (ζ, ξ, τ) we introduce the following transformations

$$\frac{1}{a} \frac{\partial}{\partial \zeta} = \psi_1 \frac{\partial}{\partial u} \tag{41}$$

$$\frac{1}{a} \frac{\partial}{\partial \xi} = \psi_2 \frac{\partial}{\partial v} \tag{42}$$

We assume that the conduction current can be described by a scalar conductivity, then

$$\vec{j} \Longrightarrow \vec{j} + \sigma \vec{E} \tag{43}$$

A derivation similar to the one above can be done in which the retarded time of the image point is used. The resulting differential equations differ only by sign changes on some terms. Let $Q_\tau = -1$ signify retarded time of the image point, then the final form of the field equations is

$$\varkappa \frac{\partial E_\zeta}{\partial \tau} - Q_\tau \frac{\partial H_\varphi}{\partial \tau} = - z_o j_\zeta - z_o \sigma E_\zeta + \psi_2 \frac{\partial H_\varphi}{\partial v} \tag{44}$$

$$\varkappa \frac{\partial E_\xi}{\partial \tau} - \frac{\partial H_\varphi}{\partial \tau} = - z_o j_\xi - z_o \sigma E_\xi - \psi_1 \frac{\partial H_\varphi}{\partial u} \tag{45}$$

$$Q_\tau G_2 \frac{\partial E_\zeta}{\partial \tau} + G_1 \frac{\partial E_\xi}{\partial \tau} - \varkappa_m \frac{\partial H_\varphi}{\partial \tau} = G_1 \psi_1 \frac{\partial E_\xi}{\partial u} - G_2 \psi_2 \frac{\partial E_\zeta}{\partial v} \tag{46}$$

II. GEOMETRY

Define:

$$\zeta \equiv \frac{1}{2a}(r' + r); \qquad \xi \equiv \frac{1}{2a}(r' - r) \tag{1}$$

then

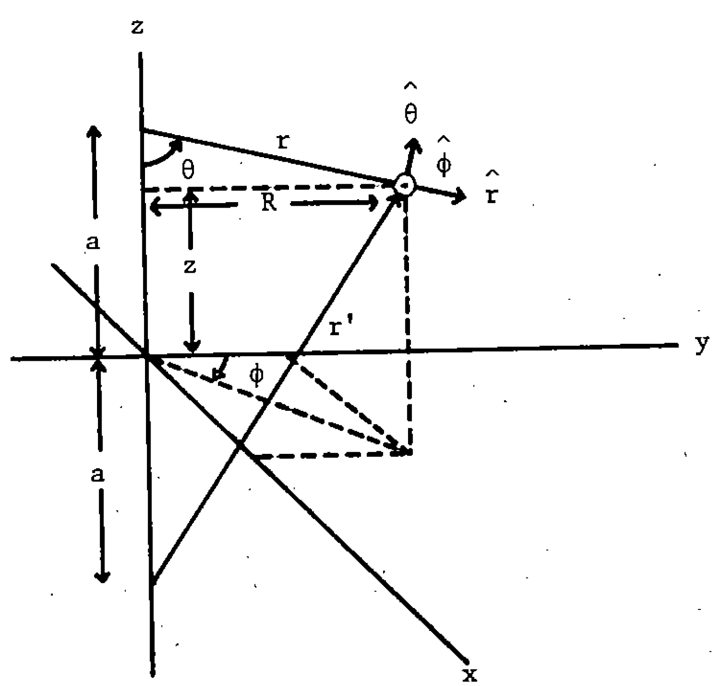
$$r' = a(\zeta + \xi); \qquad r = a(\zeta - \xi) \tag{2}$$

Choose the prolate spheroidal coordinates to be cyclic in the order (ζ, ξ, φ). Note that:

$$\cos\theta = \frac{1 - \xi\zeta}{\zeta - \xi} \tag{3}$$

$$\sin\theta = \frac{\sqrt{(\zeta^2 - 1)(1 - \xi^2)}}{\zeta - \xi} \tag{4}$$

$$R = a\sqrt{(\zeta^2 - 1)(1 - \xi^2)} \tag{5}$$



The transformations to Cartesian coordinates are

$$x = R \sin\varphi = a\sqrt{(\zeta^2 - 1)(1 - \xi^2)} \sin\varphi \qquad (6)$$

$$y = R \cos\varphi = a\sqrt{(\zeta^2 - 1)(1 - \xi^2)} \cos\varphi \qquad (7)$$

$$z = a\xi\zeta \qquad (8)$$

The inverse transformations are:

$$\zeta = \frac{1}{2a}\left[\sqrt{x^2 + y^2 + (a + z)^2} + \sqrt{x^2 + y^2 + (a - z)^2}\right] \qquad (9)$$

$$\xi = \frac{1}{2a}\left[\sqrt{x^2 + y^2 + (a + z)^2} - \sqrt{x^2 + y^2 + (a - z)^2}\right] \qquad (10)$$

$$\varphi = \tan^{-1}\left(\frac{x}{y}\right) \qquad (11)$$

The radicals in equations (9) and (10) are just:

$$r = \sqrt{x^2 + y^2 + (a - z)^2} \qquad (12)$$

$$r' = \sqrt{x^2 + y^2 + (a + z)^2} \qquad (13)$$

the transformation to the spherical polar system in Figure 1 is given by

$$r = a(\zeta - \xi) \qquad (14)$$

$$\theta = \cos^{-1}\left(\frac{1 - \xi\zeta}{\zeta - \xi}\right) \qquad (15)$$

The inverse transformations are

$$\zeta = \frac{1}{2a}\left[r + \sqrt{r^2 + 4a(a - r\cos\theta)}\right] \tag{16}$$

$$\xi = \frac{1}{2a}\left[-r + \sqrt{r^2 + 4a(a - r\cos\theta)}\right] \tag{17}$$

where

$$r' = \sqrt{r^2 + 4a(a - r\cos\theta)} \tag{18}$$

We now wish to determine scale factors h_ζ, h_ξ, h_φ such that

$$ds^2 = h_\zeta^2 d_\zeta^2 + h_\xi^2 d_\xi^2 + h_\varphi^2 d_\varphi^2 \tag{19}$$

To obtain these, differentiate equations (14) and (15) and substitute into

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2 \tag{20}$$

We find

$$ds^2 = a^2 \frac{\zeta^2 - \xi^2}{\zeta^2 - 1} d\zeta^2 + a^2 \frac{\zeta^2 - \xi^2}{1 - \xi^2} d\xi^2 + a^2(\zeta^2 - 1)(1 - \xi^2) d\varphi^2 \tag{21}$$

The following relations were used

$$\frac{\partial\theta}{\partial\zeta} = \frac{1}{\zeta - \xi}\sqrt{\frac{1 - \xi^2}{\zeta^2 - 1}} \tag{22}$$

$$\frac{\partial\theta}{\partial\xi} = \frac{1}{\zeta - \xi}\sqrt{\frac{\zeta^2 - 1}{1 - \xi^2}} \tag{23}$$

Finally,

$$h_\zeta = a\sqrt{\frac{\zeta^2 - \xi^2}{\zeta^2 - 1}} \tag{24}$$

$$h_\xi = a\sqrt{\frac{\zeta^2 - \xi^2}{1 - \xi^2}} \tag{25}$$

$$h_\varphi = a\sqrt{(\zeta^2 - 1)(1 - \xi^2)} \tag{26}$$

Some useful vector operators are defined by:

$$\nabla f = \frac{1}{a}\sqrt{\frac{\zeta^2 - 1}{\zeta^2 - \xi^2}}\,\frac{\partial f}{\partial \zeta}\,\hat{\zeta} + \frac{1}{a}\sqrt{\frac{1 - \xi^2}{\zeta^2 - \xi^2}}\,\frac{\partial f}{\partial \xi}\,\hat{\xi} + \frac{1}{a\sqrt{(\zeta^2 - 1)(1 - \xi^2)}}\,\frac{\partial f}{\partial \varphi}\,\hat{\varphi} \tag{27}$$

$$\nabla \cdot \vec{f} = \frac{1}{a(\zeta^2 - \xi^2)}\left[\frac{\partial}{\partial \zeta}\left(\sqrt{(\zeta^2 - \xi^2)(\zeta^2 - 1)}\, f_\zeta\right) + \frac{\partial}{\partial \xi}\left(\sqrt{(\zeta^2 - \xi^2)(1 - \xi^2)}\, f_\xi\right) + \frac{\partial}{\partial \varphi}\left(\frac{\zeta^2 - \xi^2}{\sqrt{(\zeta^2 - 1)(1 - \xi^2)}}\, f_\varphi\right)\right] \tag{28}$$

$$\begin{aligned}\nabla \times f = {} & \frac{\hat{\zeta}}{a\sqrt{(\zeta^2 - \xi^2)(\zeta^2 - 1)}}\left[\frac{\partial}{\partial \xi}\left(\sqrt{(\zeta^2 - 1)(1 - \xi^2)}\, f_\varphi\right) - \frac{\partial}{\partial \varphi}\left(\sqrt{\frac{\zeta^2 - \xi^2}{1 - \xi^2}}\, f_\xi\right)\right] \\ & + \frac{\hat{\xi}}{a\sqrt{(\zeta^2 - \xi^2)(1 - \xi^2)}}\left[\frac{\partial}{\partial \varphi}\left(\sqrt{\frac{\zeta^2 - \xi^2}{\zeta^2 - 1}}\, f_\zeta\right) - \frac{\partial}{\partial \zeta}\left(\sqrt{(\zeta^2 - 1)(1 - \xi^2)}\, f_\varphi\right)\right] \\ & + \frac{\hat{\varphi}\sqrt{(\zeta^2 - 1)(1 - \xi^2)}}{a(\zeta^2 - \xi^2)}\left[\frac{\partial}{\partial \zeta}\left(\sqrt{\frac{\zeta^2 - \xi^2}{1 - \xi^2}}\, f_\xi\right) - \frac{\partial}{\partial \xi}\left(\sqrt{\frac{\zeta^2 - \xi^2}{\zeta^2 - 1}}\, f_\zeta\right)\right]\end{aligned} \tag{29}$$

We now consider the transformation of vectors. In the following, the unprimed variables are in the prolate spheroidal system and the primed ones are in another orthogonal system. Define a transformation matrix (α_{ij}) by:

$$\hat{e}_i = \sum_j \alpha_{ij} \hat{e}'_j \tag{30}$$

The inverse transformation is

$$\hat{e}'_i = \sum_j \alpha_{ji} \hat{e}_j \tag{31}$$

The orthogonality relations associated with the transformation are

$$\sum_k \alpha_{ki} \alpha_{kj} = \delta_{ij} \tag{32}$$

$$\sum_k \alpha_{ik} \alpha_{jk} = \delta_{ij} \tag{33}$$

For any vector, $\vec{f}$, we have

$$f_i = \sum_j \alpha_{ij} f'_j \tag{34}$$

$$f'_i = \sum_j \alpha_{ji} f_j \tag{35}$$

To determine the matrix elements (α_{ij}), we note that

$$\hat{\zeta} = h_\zeta \nabla \zeta , \qquad \hat{\xi} = h_\xi \nabla \xi \tag{36}$$

Now, for an azimuthally symmetric problem

$$\nabla\zeta = \frac{\partial\zeta}{\partial r}\,\hat{r} + \frac{1}{r}\,\frac{\partial\zeta}{\partial\theta}\,\hat{\theta} \tag{37}$$

or,

$$\nabla\zeta = \frac{a}{h_\zeta^2}\,\hat{r} + \frac{a}{h_\xi h_\zeta}\,\hat{\theta} \tag{38}$$

$\therefore$

$$\hat{\zeta} = \frac{a}{h_\zeta}\,\hat{r} + \frac{a}{h_\xi}\,\hat{\theta} \tag{39}$$

Similarly,

$$\nabla\xi = \frac{\partial\xi}{\partial r}\,\hat{r} + \frac{1}{r}\frac{\partial\xi}{\partial\theta}\,\hat{\theta} \tag{40}$$

or,

$$\nabla\xi = \frac{-a}{h_\xi^2}\,\hat{r} + \frac{a}{h_\zeta h_\xi}\,\hat{\theta} \tag{41}$$

$\therefore$

$$\hat{\xi} = -\frac{a}{h_\xi}\,\hat{r} + \frac{a}{h_\zeta}\,\hat{\theta} \tag{42}$$

Comparing equations (30), (39), and (42) yields the desired matrix elements

$$(\alpha_{ij}) = a\begin{pmatrix} \frac{1}{h_\zeta} & \frac{1}{h_\xi} \\ -\frac{1}{h_\xi} & \frac{1}{h_\zeta} \end{pmatrix} \tag{43}$$

The transformation is orthogonal. Its inverse is:

$$(\alpha_{ij})^{-1} = a \begin{pmatrix} \frac{1}{h_\zeta} & -\frac{1}{h_\xi} \\ \frac{1}{h_\xi} & \frac{1}{h_\zeta} \end{pmatrix} \tag{44}$$

The equations inverse to equations (39) and (42) are:

$$\hat{r} = \frac{a}{h_\zeta} \hat{\zeta} - \frac{a}{h_\xi} \hat{\xi} \tag{45}$$

$$\hat{\theta} = \frac{a}{h_\xi} \hat{\zeta} + \frac{a}{h_\zeta} \hat{\xi} \tag{46}$$

Note that

$$\frac{1}{h_\xi^2} + \frac{1}{h_\zeta^2} = \frac{1}{a^2} \tag{47}$$

The transformation of vectors to the spherical polar system centered at the image point of the weapon is given by

$$(\alpha'_{ij}) = \begin{pmatrix} \frac{a}{h_\zeta} & -\frac{a}{h_\xi} \\ \frac{a}{h_\xi} & \frac{a}{h_\zeta} \end{pmatrix} \tag{48}$$

The inverse transformation is:

$$(\alpha'_{ij})^{-1} = \begin{pmatrix} \frac{a}{h_\zeta} & \frac{a}{h_\xi} \\ \\ -\frac{a}{h_\xi} & \frac{a}{h_\zeta} \end{pmatrix} \tag{49}$$

The equations analogous to equations (45) and (46) are:

$$\hat{r}' = \frac{a}{h_\zeta}\hat{\zeta} + \frac{a}{h_\xi}\hat{\xi} \tag{50}$$

$$\theta' = -\frac{a}{h_\xi}\hat{\zeta} + \frac{a}{h_\zeta}\hat{\xi} \tag{51}$$

APPENDIX B: PROGRAMMING DETAILS OF ALGORITHM "B"

B.1 General Description

A good deal of machinery is required to operate prior to the exercising of Subroutine MARCH, as it is used to implement Algorithm "B". As noted in Section 2.2.4, the overall computational procedure is organized much as in ARIADNE,[13] with MARCH performing the specific function of advancing the fields by one timestep. The total procedure, in brief, may be assumed to consist of repeated cycles (one for each field-timestep) in which current and conductivity sources are updated and supplied to MARCH via a set of labelled COMMON data blocks. MARCH is called to complete a timestep cycle by advancing the fields to their "new" values, using the sources supplied from elsewhere in the procedure. Section B.2 contains a summary of important physical variables in MARCH, together with their FORTRAN equivalents, and references to the appropriate equations in Section 2.2. Section B.3 contains a FORTRAN listing of the MARCH subroutine itself. Details of the MARCH internal computational procedure are felt to be best explained by the fairly complete commentary that is supplied as part of the FORTRAN listing.

B.2 Important Variables In MARCH

B.2.1 Input/Output Data

Data is exchanged with MARCH via the five labelled COMMON blocks /PHYS1/, /PHYS2/, /PHYS3/, /DRIVR3/, and /DRIVR4/. Vital elements of these data are listed and briefly explained in the table below.

Table B.1

COMMUNICATION WITH MARCH

Variable	Analytic Equivalent	Description
TAUED1	τ_1	Value of time at timestep beginning
TAUED2	τ_2	Value of time at timestep end
AAA	a	PS coordinate system parameter, equation (1.2)
DXI	$\Delta\xi_i$	---
DZETA	$\Delta\zeta_j$	---
NXIS, NZETAS, IM, IM1, JM, JM1, etc. are indexing limits on subscripts		
XI	ξ_i	---
X2A2	$\xi^2 a^2$	---
XNUM	$a^2(1 - \xi^2)$	---
SRXNUM	---	$XNUM^{1/2}$
ZETA	ζ_j	---
Z2	ζ^2	---
ZNUM	$(\zeta^2 - a^2)$	---

Table B.1 (Cont'd.)

Variable	Analytic Equivalent	Description
SRZNUM	---	$ZNUM^{1/2}$
EPS	ϵ	(See 2.2.1)
EXI	E_ξ	TM Components of the EM field. (See equation (2.2.2))
EZETA	E_ζ	
BPHI	H_φ	
XJXI	J_ξ	Compton current sources. (See equation (2.2.2))
XJZETA	J_ζ	
SIGMA	σ	
XBJX	J_ξ	Sources along z-axis, at $\zeta = a$
XBSIG	σ	
ZBJZ	J_ζ	Sources along z-axis, at $\xi = 1$
ZBSIG	σ	

B.2.2 Internal Variables

Geometrical parameters, source input data, and field output data are listed in the table above. Most of the remainder of the variables are internally-defined in an obvious way. Important exceptions are the basic coefficients mentioned in Section 2.2 and the variables used in the implicit variation of the field solution mentioned in Section 2.2.3.

Table B.2

INTERNAL MARCH VARIABLES

Variable	Analytic Equivalent	Description
A0	A_0	See equations (2.2.10)
A1	A_1	
B0	B_0	
B1	B_1	
C1L	C_{1L}	See equations (2.2.17)
C1U	C_{1U}	
RZL	$r_{\zeta L}$	
RZU	$r_{\zeta U}$	
D1L	D_{1L}	Analogues of equations (2.2.17) for
D1U	D_{1U}	
RXL	$r_{\xi L}$	
RXU	$r_{\xi U}$	
HPA	---	See equations (2.2.9) and (2.2.10)
EXA	---	See equations (2.2.17) and (2.2.20)
EZA	---	Analogous to EXA. See equation (2.2.19)
EU	E	Recursion coefficients used in implicit solution of difference equations for E_ζ
FU	F	

Table B.2 (Cont'd.)

Variable	Analytic Equivalent	Description
Q____	---	A variety of variables are used as intermediate storage of constants needed to obtain remaining fields from E_ζ
EXAS	---	
C1S	---	

B.3 FORTRAN Listing of MARCH

```
                SUBROUTINE MARCH                                                   ACH      348
          C                                                                        ACH      349
          C     MARCH IS THE ELECTRODYNAMIC ALGORITHM.  IT SOLVES THE C.M.F.E.     ACH      350
          C     FROM THE SOURCES THAT HAVE BEEN PREPARED AND INSERTED INTO         ACH      351
          C     COMMON /DRIVR3/.                                                   ACH      352
          C                                                                        ACH      353
          C                                                                        MRCH       7
  000001        COMMON /PHYS1/    TAUED1, TAUED2, TSRC0, TSRC1, TSRC2              MRCH       8
  000001        COMMON /PHYS2/    AAA, DXI(200), DZETA(200), NXIS, NZETAS,         ACH      354
               1                  IM, IM1, IM2, IM3, JM, JM1, JM2, JM3,            MRCH      10
               2                  XI(200), X2A2(200), XNUM(200), SRXNUM(200),      MRCH      11
               3                  ZETA(100), Z2(100), ZNUM(100), SRZNUM(100)       MRCH      12
               4                 ,EPS(200),SIG(200)                                ACH      355
          C                                                                        BIGGY      2
          C     THIS SECTION CONTAINS THE SOURCE AND FIELD ARRAYS.  THE            ACH        1
          C     SOURCES ARE UPDATED IN SAPSRC AND USED IN MARCH TO UPDATE          ACH        2
          C     THE FIELDS.  ECS STORAGE OR LARGE STORAGE MAY BE USED FOR          ACH        3
          C     VARIABLES HERE, BECAUSE OF THE LARGE ARRAY SIZE.                   ACH        4
          C                                                                        ACH        5
  000001        COMMON /PHYS3/    EXI(50,50), EZETA(50,50), EPHI( 5, 5),           ACH        6
               1                  BXI( 5, 5), BZETA( 5, 5), BPHI(50,100)           ACH        7
          C    1                  BXI( 5, 5), BZETA( 5, 5), BPHI(50,50)            ACH        8
          C                                                                        ACH        9
  000001        COMMON /DRIVR3/   XJXI(50,50), XJZETA(50,50), XJPHI( 5, 5),        ACH       10
               1                  SIGMA(50,50)                                     ACH       11
  000001        COMMON /DRIVR4/   XBJX(50), XBSIG(50), ZBJZ(50), ZBSIG(50)         ACH       12
          C                                                                        BIGGY     14
          C                                                                        ACH      356
  000001        DIMENSION    AXP(200),AXM(200),AZP(200),AZM(200),GXN(200),GZN(200) ACH      357
  000001        DIMENSION    EX2B(200),EZ2B(200),HP2B(200),SIGRC(200),QS0(200),    ACH      358
               1             QS1(200),QS2(200),EU(200),FU(200),C1S(200),EXAS(200), ACH      359
               2             D1S(200),EZBS(200),CJX(50,50),CJZ(50,50)              ACH      360
  000001        DIMENSION    EX( 50, 50),EZ( 50, 50),HP( 50, 50)                   ACH      361
  000001        DIMENSION DTODXU(200),AXMU(200),DTODZU(200),AZMU(200)              ACH      362
          C                                                                        ACH      363
  000001        EQUIVALENCE  (GXN,XNUM),(GZN,ZNUM),(EZ,EZETA),(EX,EXI),(HP,BPHI)   ACH      364
  000001        EQUIVALENCE  (CJX,XJXI),(CJZ,XJZETA)                               ACH      365
          C                                                                        ACH      366
          C     FUNDAMENTAL CONSTANTS                                              ACH      367
          C                                                                        ACH      368
  000001        DATA CL /2.997925E+08/                                             ACH      369
  000001        DATA Z0 /376.7304/                                                 ACH      370
  000001        DATA KIMP/-1/                                                      ACH      371
  000001        DATA KCXB,KCZB/1,-1/                                               ACH      372
  000001        IF(TAUED2.GT.5.E-8) KIMP=1                                         ACH      373
          C                                                                        ACH      374
          C                                                                        ACH      375
          C     A FEW FACTORS USEFUL THROUGHOUT THIS TIMESTEP.                     ACH      376
          C                                                                        ACH      377
  000005        DTAU=CL*(TAUED2-TAUED1)                                            ACH      378
  000007        Z0DTAU = Z0*DTAU                                                   ACH      379
  000011        SIG0=2./Z0DTAU                                                     ACH      380
          C                                                                        ACH      381
          C     I-DEPENDENT FACTORS.                                               ACH      382
          C     (EPSILON IS PRESENTLY STRATIFIED ONLY IN I.)                       ACH      383
          C                                                                        ACH      384
  000013        DO 19 I = 1, IM1                                                   ACH      385
  000022        DTODX = DTAU/(AAA*DXI(I))                                          ACH      386
```

```
000024          AXP(I) = (1. + DTODX)
000025          AXM(I) = (1. - DTODX)
000027          DTODXU(I)=DTAU/(AAA*(DXI(I)+DXI(I+1)))
000032          AXMU(I)=(1.-DTODXU(I))
000033          SIGRB(I) = ZODTAU/EPS(I)
000034       19 CONTINUE
          C
          C     J-DEPENDENT FACTORS
          C
000036          DO 29 J = 1, JM1
000046          DTODZ = DTAU/DZETA(J)
000047          AZP(J) = (1. + DTODZ)
000050          AZM(J) = (1. - DTODZ)
000051          DTODZU(J)=DTAU/(DZETA(J)+DZETA(J+1))
000053          AZMU(J)=(1.-DTODZU(J))
000053       29 CONTINUE
          C
          C     THIS IS THE PROCEDURE ITSELF.  HERE, ROWS
          C     OF FIELD VALUES ALONG SUCCESSIVE
          C     ELLIPSOIDS (FIXED VALUE OF *J*) ARE
          C     UPDATED VIA AN EXPLICIT OR IMPLICIT SCHEME.  J
          C     IS MARCHED OUT FROM J=1 (THE LOWER
          C     Z-AXIS) TO JM2, THE J-INDEX OF AN
          C     OUTER ELLIPSOIDAL SHELL. H-PHI=0 AT
          C     ALL POINTS ALONG THE Z-AXIS (I=1 OR J=1)
          C     AND RADIAL-FIELD BOUNDARY CONDITIONS
          C     DETERMINE E-XI AT J=1 AND E-ZETA
          C     AT I=1.
          C
          C     UPDATE SOURCES FOR THIS CYCLE.
          C
000054          CALL SOURCE(1,2)
          C
          C     BEGIN BY INITIALIZING ROW-BELOW VECTORS OF HP AND EX TO Z-AXIS
          C     VALUES.
          C
000056      101 DO 109 I=1,IM2
          C
000060          HP2B(I) = 0.
          C
          C     EX IS A RADIAL FIELD AT THE POINT THAT EACH HYPERBOLOID
          C     INTERSECTS THE LOWER Z-AXIS.
          C
000061          XKDT = XBSIG(I)*SIGRB(I)
000063          IF(XKDT .LE. 1.E-5) GO TO 107
000065          EXKDT = EXP(-XKDT)
000071          EXKDTP = (1. - EXKDT)
000072          SRA=-EXKDTP/XBSIG(I)
000076          GO TO 108
000077      107 SPHACT=(1.-3.5*XKDT)
000101          EXKDTP=XKDT*SPHACT
000102          EXKDT = (1. - EXKDTP)
000103          SRA=-SIGRB(I)*SPHACT
000106      108 EX2B(I)=EX(I,1)*EXKDT+SRA*XBJX(I)
000113      109 CONTINUE
          C
          C     NOW, PROCEED TO MARCH OUT IN I AT EACH OF THE
          C     APPROPRIATE J-VALUES.
```

```
              C
000115              DO 143 J=1,JM2
              C
              C     PREPARE A FEW I-INDEPENDENT CONSTANTS USEFUL AT THIS J-VALUE.
              C
000116              AZPJ = AZP(J)
000121              AZMJ = AZM(J)
000122              ZURJ=DTODZU(J)
000123              AZMUJ=AZMU(J)
000125              IF(J.EQ.1) GO TO 115
000130              ZLRJ=DTODZU(J-1)
000131              AZMLJ=AZMU(J-1)
000133              GO TO 116
000133        115   ZLRJ=0.
000134              AZMLJ=AZMJ
000137        116   GZNJ=GZN(J)
000140              Z2J = Z2(J)
              C
              C     INITIALIZE CELL-TO-THE-RIGHT VALUES OF NEW
              C     E-ZETA AND H-PHI BEFORE PROCEEDING AT NEW J.
              C
000142              HP2R=0.
              C
              C     EZ AT THE TOP CENTER OF EACH ELLIPSOID IS FOUND FROM THE RADIAL
              C     E-FIELD BOUNDARY CONDITION ALONG THE UPPER Z-AXIS.
              C
000142              XKDT = ZBSIG(J)*SIGR0(1)
000144              IF(XKDT .LE. 1.E-5) GO TO 117
000147              EXKDT = EXP(-XKDT)
000153              EXKDTP = (1. - EXKDT)
000154              SRA=-EXKDTP/ZBSIG(J)
000160              GO TO 118
000161        117   SPHACT=(1.-0.5*XKDT)
000163              EXKDTP=XKDT*SPHACT
000164              EXKDT = (1. - EXKDTP)
000165              SRA=-SIGR0(1)*SPHACT
000167        118   EZ2R=EZ(1,J)*EXKDT+SRA*ZBJZ(J)
              C
              C     BEGINNING WITH I=1, FIND NEW
              C     E-FIELDS AT OUTER EDGES OF (I,J)-TH
              C     CELL, AND NEW H-FIELD WITHIN (I,J)-TH CELL.
              C
000175              DO 129 I=1,IM2
              C
              C     (REMAINING FACTORS NEEDED AS BASIC COEFFICIENTS IN THE (I,J)-TH
              C     RING)
              C
000177              AXFI = AXF(I)
000202              AXMI = AXM(I)
000203              XURI=DTODXU(I)
000204              AXMUI=AXMU(I)
000206              IF(I.EQ.1) GO TO 123
000211              XLRI=DTODXU(I-1)
000212              AXMLI=AXMU(I-1)
000214              GO TO 124
000214        123   XLRI=0.
000215              AXMLI=AXMI
000224        124   GOPH=-1./(2.*(Z2J-X2A2(I)))
```

```
000227          GX = GOPH*GXN(I)
000230          GZ = GOPH*GZNJ
          C
000232          XKDT = SIGMA(I,J)*SIGRO(I)
000233          IF(XKDT .LE. 1.E-5) GO TO 125
000236          EXKDT = EXP(-XKDT)
000245          EXKDTP = (1. - EXKDT)
000247          SIGR=-EXKDTP/SIGMA(I,J)
000250          GO TO 126
000252      125 SPHACT=(1.-0.5*XKDT)
000254          EXKDTP=XKDT*SPHACT
000255          EXKDT = (1. - EXKDTP)
000256          SIGR=-SIGRO(I)*SPHACT
000270      126 PHACTR=SIGB*SIGR
          C
          C     BASIC COEFFICIENTS
          C
000271          A0 = GX*AXPI
000272          A1 = GX*AXMI
000274          B0 = GZ*AZPJ
000276          B1 = GZ*AZMJ
000277          C1L=PHACTR*AZMLJ
000301          C1U=PHACTR*AZMUJ
000302          RZL=PHACTR*ZLRJ
000304          RZU=PHACTR*ZURJ
000305          D1L=PHACTR*AXMLI
000307          D1U=PHACTR*AXMUI
000310          RXL=PHACTR*XLRI
000312          RXU=PHACTR*XURI
          C
000313          HPA = HP(I,J)-A1*EZ(I,J)-A0*EZ(I+1,J)-B1*EX(I,J)
               *-B0*(EX(I,J+1)-EX2B(I))
000325          EXA = (EX(I,J+1)+EX(I,J))*EXKDT-EX2B(I)+2.*SIGR*CJX(I,J)
               *-C1U*HP(I,J)-RZU*HP(I,J+1)+RZL*HP2B(I)
000337          EZA = (EZ(I+1,J)+EZ(I,J))*EXKDT+2.*SIGR*CJZ(I,J)-D1U*HP(I,J)
               *-RXU*HP(I+1,J)
          C
          C     IF IT*S TIME TO GO IMPLICIT, TRANSFER TO THAT
          C     BRANCH OF THE PROCEDURE.
          C
000350          IF(KIMP .GT. 0) GO TO 127
          C
          C     THIS IS THE EXPLICIT BRANCH.  HERE, WE
          C     UPDATE TEMPORARY NEW-FIELD STORAGE AND
          C     RETAIN APPROPRIATE FIELD VALUES IN FIELD ARRAY
          C
000353          IF (I.GT.1) HP(I-1,J)=HP2R
000361          IF(I.EQ.IM2.AND.KCXB.EQ.1) A1=0.
000373          IF(J.EQ.JM2.AND.KCZB.EQ.1) B1=0.
000405          EZAR=EZA+RXL*HP2R
000407          HP2R = ((A0-A1)*EZ2R+HPA+A1*EZAR+B1*EXA)/
               *(1.-A1*D1L-B1*C1L)
000423          HP2B(I)=HP2R
000424          EX(I,J)=EX2B(I)
000425          EX2B(I) = EXA+C1L*HP2R
000427          IF(J.EQ.JM2.AND.KCZB.EQ.1) EX2B(I)=0.
000444          EZ(I,J) = EZ2R
000445          EZ2R = EZAR+D1L*HP2R-EZ2R
```

```
000450          IF(I.EQ.IM2.AND.KCXB.EQ.1) EZ2R=0.
000460          GO TO 129
000462   127    EXAS(I)=EXA
000463          C1S(I)=C1L
000465          IF(J.EQ.JM2.AND.KCZB.EQ.1) B1=0.
         C
000500          QDEN=1./(1.-B1*C1L)
000503          QU0=QDEN*A0
000504          QU1=QDEN*A1
000505          QU2=QDEN*(HEA+B1*EXA)
         C
000511          QS0(I)=QU0
000512          QS1(I)=QU1
000514          QS2(I)=QU2
         C
000516          IF(I.GT.1) GO TO 1281
         C
         C      BOTTOM-MOST RECURSION COEFFICIENTS
         C
000521          EU(1)=0.
000521          EU(1)=EZ2R
000523          GO TO 1282
         C
         C      REGULAR RECURSION COEFFICIENTS
         C
000523   1281   QPLS=(1.-D1L*QU1)
000526          QZRO=(1.-D1L*QU0-RXL*QL1)
000532          QMIN=-RXL*QL0
         C
000533          PUR=EZA+D1L*QU2+RXL*QL2
000543          IF(I.NE.IM2.OR.KCXB.NE.1) GO TO 1283
         C
000550          QPLS=RXUL*QU1
000551          QZRO=(1.-D0UL*QL1+RXUL*QU0)
000555          QMIN=1.-D0U*QL0
         C
000560          PUR=(EZAL+D0UL*QL2-RXUL*QU2)
         C
000567   1283   QDEN=1./(QZRO+QMIN*EU(I-1))
         C
000572          EU(I)=-QPLS*QDEN
000574          FU(I)=(PUR-QMIN*FU(I-1))*QDEN
         C
000577   1282   QL0=QU0
000600          QL1=QU1
000602          QL2=QU2
000603          IF(I.NE.IM3.OR.KCXB.NE.1) GO TO 129
         C
000616          D0UL=PHACTR*(1.+XURI)
000621          EZAL=(EZ(I+1,J)+EZ(I,J))*EXKQT+2.*SIGR*CJZ(I,J)-D0U*HF(I,J)
               *+RXL*HF(I-1,J)
000632          RXUL=RXU
         C
000634   129    CONTINUE
         C
         C      IF SCHEME IS EXPLICIT, WE JUST FINISH
         C      THE OUTER LOCATIONS.
         C
```

```
001637        IF(KIMP.LE.0) GO TO 148
          C
          C   THIS IS THE REMAINDER OF THE IMPLICIT BRANCH. WE
          C   START WITH OUTER BOUNDARY CONDITIONS AND WORK BACK
          C   DOWN THRU I*S WITH RECURSION FORMULAS, ETC.
          C
          C   OUTER B.C. FOR NOW IS A MESS .......
          C
001643        IF(KOXB.EQ.1) GO TO 1292
001651  1291  DDU=PHACTR*(1.+XURI)
001653        EZAL=(EZ(I+1,J)+EZ(I,J))*EXKOT+2.*SIGR*CJZ(I,J)-DDU*HP(I,J)
             *+RXL*HP(I-1,J)
001664        CURLY=(DDU*QL0-1.)
001666        EZ(IM1,J)=(EZAL+FU(IM2)*CURLY+DDU*QL2)/(1.-EU(IM2)*CURLY-DDU*QL1)
          C
001676        GO TO 1293
          C
001677  1292  EZ(IM1,J)=0.
          C
001703  1293  DO 139 IC=1,IM2
001731        I=IM2-IC+1
001733        IA=I+1
          C
001734        EZ(I,J)=EU(I)*EZ(IA,J)+FU(I)
001743        IF(J.GT.1) HP(I,J-1)=HP2B(I)
001744        HP2B(I)=QS0(I)*EZ(I,J)+QS1(I)*EZ(IA,J)+QS2(I)
001752        EX(I,J)=EX2B(I)
001754        EX2B(I)=EXAS(I)+C1S(I)*HP2B(I)
          C
001760        IF(J.EQ.JM2.AND.KOZB.EQ.1) EX2B=0.
001763  139   CONTINUE
          C
          C   FIX UP LAST HP-VALUE.
          C
001767        HP(IM1,J)=0.
          C
001773        GO TO 149
          C
          C   AT THE END OF AN I-ROW, WE STORE
          C   THE LAST EZ2R AND APPLY BOUNDARY
          C   CONDITIONS TO GET BELOW-GROUND HP(IM1,J)
          C
001000  148   EZ(IM1,J) = EZ2R
001002        HP(IM2,J) = HP2R
          C
001003        HP(IM1,J)=0.
          C
001004  149   CONTINUE
          C
          C   AFTER THE JM2-TH I-ROW, STORE
          C   THE LAST EX2B-VALUES AND APPLY
          C   OUTER-ELLIPSOID BOUNDARY CONDITIONS
          C   TO H(I,JM1)
          C
001007        DO 193 I=1,IM2
          C
001024        EX(I,JM1) = EX2B(I)
001026        IF(KIMP.GT.0) HP(I,JM2)=HP2B(I)
```

```
             C                                                          ACH      677
  011030          HP(I,JM1) = HP(I,JM3-1)-3.*HP(I,JM3)+3.*HP(I,JM2)     ACH      678
             C                                                          ACH      679
  011033    199   CONTINUE                                              ACH      680
             C                                                          ACH      681
  011035          RETURN                                                ACH      682
  011036          END                                                   MRCH     203
```

SUBPROGRAM LENGTH

011400

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS

101	-	000057	107	-	000077	108	-	000107	115	-	000134
116	-	000137	117	-	000161	118	-	000170	123	-	000215
124	-	000220	125	-	000252	126	-	000262	127	-	000462
129	-	000635	139	-	000764	148	-	000774	149	-	001005
1281	-	000524	1282	-	000600	1283	-	000566	1291	-	000643
1292	-	000700	1293	-	000704						

BLOCK NAMES AND LENGTHS

PHYS1	-	[illegible]	PHYS2	-	003733	PHYS3	-	023533	DRIVR3	-	016545
DRIVR4	-	000310									

VARIABLE ASSIGNMENTS

AAA	-	000000C02	AXM	-	001430	AXMI	-	011324	AXMLI	-	011330
AXMU	-	010140	AXMUI	-	011326	AXF	-	001120	AXFI	-	011323
AZM	-	002250	AZMJ	-	011312	AZMLJ	-	011316	AZMU	-	010760
AZMUJ	-	011314	AZP	-	001740	AZPJ	-	011311	AC	-	011336
A1	-	011337	BPHI	-	011723C03	BXI	-	011641C03	BZETA	-	011672C03
B0	-	011340	B1	-	011341	CJX	-	000000C04	CJZ	-	004704C04
CL	-	011270	CURLY	-	011375	C1L	-	011342	C1S	-	006170
C1U	-	011343	DTAU	-	011275	DTODX	-	011301	DTODXU	-	007630
DTODZ	-	011303	DTODZU	-	010450	DXI	-	000001C02	DZETA	-	000311C02
D0U	-	011373	D0UL	-	011372	D1L	-	011346	D1S	-	007010
D1U	-	011347	EPHI	-	011610C03	EPS	-	003113C02	EU	-	005350
EX	-	000000C03	EXA	-	011353	EXAS	-	006500	EXI	-	000000C03
EXKDT	-	011365	EXKDIP	-	011306	EX2B	-	002560	EZ	-	004704C03
EZA	-	011354	EZAL	-	011374	EZAR	-	011355	EZBS	-	007320
EZETA	-	004704C03	EZ2B	-	003070	EZ2R	-	011322	FU	-	005660
GDPH	-	011331	GX	-	011332	GXN	-	001453C02	GZ	-	011333
GZN	-	002603C02	GZNJ	-	011317	HP	-	011723C03	HPA	-	011352
HP2B	-	003400	HP2R	-	011321	I	-	011300	IA	-	011377
IC	-	011376	IM1	-	000624C02	IM2	-	000625C02	IM3	-	000626C02
J	-	011302	JM1	-	000630C02	JM2	-	000631C02	JM3	-	000632C02
KCXB	-	011273	KCZB	-	011274	KIMP	-	011272	PHACTR	-	011335
PUR	-	011367	QDEN	-	011356	QL0	-	011366	QL1	-	011364
QL2	-	011370	QMIN	-	011365	QFLS	-	011362	QS0	-	004220
QS1	-	004530	QS2	-	005040	QU0	-	011357	QU1	-	011360
QU2	-	011361	QZRO	-	011363	RXL	-	011350	RXU	-	011351
RXUL	-	011371	RZL	-	011344	RZU	-	011345	SIG	-	003423C02
SIGMA	-	011641C04	SIGR	-	011334	SIGR0	-	003710	SIGG	-	011277
SPHACT	-	011310	SRA	-	011307	SRXNUM	-	001763C02	SRZNUM	-	002747C02
TAUED1	-	000000C01	TAUED2	-	000001C01	XBJX	-	000000C05	XBSIG	-	000062C05
XI	-	000633C02	XJPHI	-	011610C04	XJXI	-	000000C04	XJZETA	-	004704C04

APPENDIX C

```
      PROGRAM IG2D(INPUT,OUTPUT,TAPE1,FILMPL,TAPE5=INPUT,TAPE6=OUTPUT)
C
C     *** A TWO DIMENSIONAL NEAR SURFACE EMP CODE OVER AN INFINITELY
C     *** CONDUCTING GROUND
C     *** WRITTEN  27 JUNE 1973   BILL PINE
C
      REAL MU
      INTEGER PRR,PLR,WVFLG(100)
      LOGICAL REFLECT
      REAL JR,JZ
      COMMON ER1(20,20),   EZ1(20,20),   BP1(20,20)
      COMMON ER2(20,20),   EZ2(20,20),   BP2(20,20)
      COMMON  JR(20,20),    JZ(20,20),   STG(20,20)
      COMMON   R(20,20),   CON(20,20),     X(20),        Y(20)
      COMMON/TGRID/T0,DT,NT,K
      COMMON/SGRID/YMAX,DY,NY,J,X0,DX,NX,I,NG
      COMMON/WOUT/AL,BE,TP,CY,CX,YLD,EFF,EGB,HOB
      COMMON/PRNTS/PRR,NWX,NWY,NOX,NOY,IWX(10),IWY(10),IOX(10),IOY(10)
      COMMON/PLTS/PLR,NPX,NPY,IPX(10),IPY(10)
      DIMENSION A1(20),A2(20),C1(20),C2(20),E(20),FF(20),FP(20),GP(20)
      DIMENSION F(20),G(20)
      DIMENSION KOMENT(8)
      DATA CA,CB,CC,CD/17174000000000000000B,17155252525252525253B,
     $17135252525252525253B,17114210421042104210B/
      FS(X)=X*(CA+X*(-CB+X*(CC-X*CD)))
C
C     *** DATA INPUT SECTION ***
C
      READ(5,5) KOMENT
    5 FORMAT(8A10)
      READ(5,10) PLR,PRR,NOX,NOY,NWX,NWY,NPX,NPY
   10 FORMAT(8I10)
      READ(5,10) (IOX(I),I=1,NOX)
      READ(5,10)(IOY(J),J=1,NOY)
      READ(5,10)(IWX(I),I=1,NWX)
      READ(5,10) (IWY(J),J=1,NWY)
      READ(5,10) (IPX(I),I=1,NPX)
      READ(5,10)(IPY(J),J=1,NPY)
      READ(5,20) HOB,DT,NT
   20 FORMAT(2E10.0,I10)
      READ(5,20) ZMAX,DZ,NZ
      READ(5,20) R0,DR,NR
      READ(5,21) YLD,EFF,AL,BE,TP,EGB
   21 FORMAT(8E10.0)
      IF(((NZ/2)*2-NZ).EQ.0)  NZ=NZ-1
C
C     *** DEFINE THE GRID ***
C
      CA=1.0
      CB=0.5
      CC=1.0/6.0
      CD=1.0/24.0
      NX=NR
      DX=DR
      NY=NZ
```

```
      DY=DZ
      YMAX=ZMAX
      X0=R0
      JGND=NY/2 + 1
      DO 30 I=1,NX
   30 X(I)=R0 + FLOAT(I-1)*DR
      DY=2.0*YMAX/FLOAT(NY-1)
      K=NY
      DO 40  J=1,NY
      Y(K)=YMAX-FLOAT(J-1)*DY
   40 K=K-1
      DO 50  I=1,NX
      F(I)=X(I)
   50 FP(I)=1.0
      DO 60  J=1,NY
      G(J)=Y(J)
   60 GP(J)=1.0
C
C     *** DEFINE THE CONSTANTS ***
C
      C=2.998E+8
      PI=3.14159265
      EPS=8.854E-12
      MU=4.0E-7*PI
      EMU=1.0/SQRT(EPS*MU)
      NXM1=NX-1
      NXM2=NX-2
      NYM1=NY-1
      NYM2=NY-2
      NG=NY/2 + 1
      EPI=1.0/(2.0*EPS)
      CALL SETP
      TS=R(1,NYM1)/C + 0.5*DZ/C + DT
      IF(T0.LT.TS)  T0=TS
C
C     *** SUMMARY OF RUN DATA PRINTED
C
      CALL DATE(DAY)
      CALL TIME(HMS)
      CALL RDS(KOMENT,DAY,HMS)
C
C     *** ADVANCE THE FIELDS IN TIME ***
      DO 100  K=1,NT
      T=T0 + FLOAT(K)*DT
      TMP=T-0.5*DT
      RWF=C*TMP
      CALL SIGMA(TMP)
      CALL CURENT(TMP)
      DO 110 I=1,NX
      DO 110 J=1,NY
      ER1(I,J)=ER2(I,J)
      BP1(I,J)=BP2(I,J)
      EZ1(I,J)=EZ2(I,J)
  110 CONTINUE
C
```

```
C      *** STEP THE FIFLDS IN RANGE ***
C
       DO 200  I=2,NXM1
       IM1=I-1
       IP1=I+1
       AA2=FMU*(FP(IM1)/DX+0.5/F(I-1))

       AA3=-FP(I)*DT/(2.0*DX)
       DO 201  JJ=JGND,NY
       J=NY-JJ+JGND
       JIM=NY-J +1
       IF(RWF.GT.R(I,JIM))  GO TO 202
  201  CONTINUE
       REFLECT=.FALSE.
       DO 203  J=JGND,NY
       IF(RWF.GT.R(I,J))  GO TO 204
  203  CONTINUF
       GO TO 511
  204  J1DMN=J
       J1DMX=NY
       GO TO 205
  202  J1DMN=J+1
       J1DMX=NY-1
       J2DMX=J
       J2DMN=NY-J+1
       REFLECT=.TRUE.
       IF((J2DMX-J2DMN).LE.0.0)  REFLECT=.FALSE.
  205  CONTINUF
C
C      *** ADVANCE THE FIELDS OVER ALTITIDE TO GET IMPLICIT COEFFIEICENTS
C
       E(J2DMN-1)=0.
       FF(J2DMN-1)=0.
       BP2(I,J2DMX+1)=0.
       DO 250  J=J1DMN,J1DMX
       ERB4=SQRT(ER1(I,J)**2 + EZ1(I,J)**2)
       CJR=- SQRT(JR(I,J)**2+JZ(I,J)**2)
       ARG=+SIG(I,J)*DT/EPS
       IF(ARG.EQ.0.0) GO TO 250
       EARG=EXP(-ARG)
       OEARG=1.0-EARG
       IF(ARG.LT.1.0E-5) OEARG=FS(ARG)
       ERN=ERB4*EARG-CJR*OEARG/SIG(I,J)
       ER2(I,J)=ERN*X(I)/R(I,J)
       EZ2(I,J)=-ERN*(HOB-Y(J))/R(I,J)
  250  CONTINUE
       IF(.NOT.REFLECT)  GO TO 200
       DO 300  J=J2DMN,J2DMX
       JIM=NY-JWF+1
       JM1=J-1
       JP1=J+1
       GAM1=(SIG(I,J)+SIG(I,JM1))*EPI
       TH1=DT*GAM1
       ET1=EXP(-TH1)
       IF(TH1.GT.1.0E-4)  GO TO 310
```

```
      OET1=FS(TH1)
      GO TO 320
  310 OET1=1.0-ET1
  320 GAM2=(SIG(IM1,J)+SIG(I,J))*EPI
      TH2=GAM2*DT
      ET2=EXP(-TH2)
      IF(TH2.GT.1.0E-4)    GO TO 330
      OET2=FS(TH2)
      GO TO 340
  330 OET2=1.0-ET2
  340 GAM3=(SIG(I,J)+SIG(I,JP1))*EPI
      TH3=GAM3*DT
      ET3=EXP(-TH3)
      IF(TH3.GT.1.0E-4)    GO TO 350
      OET3=FS(TH3)
      GO TO 360
  350  OET3=1.0-ET3
  360 A1(J)=OET1*GP(JM1)*EMU/(2.0*GAM1*DY)
      A2(J)=OET2*AA2/GAM2
      A3=AA3
      A4=GP(J)*DT/(2.0*DY)
      A5=1.0-A3*A2(J)
      AL1=OET3*GP(J)*EMU/(2.0*GAM3*DY)
      C1(J)=ER1(I,JM1)*ET1 + OET1*(-JR(I,JM1)/EPS-
     $(GP(JM1)*EMU/(2.0*DY))*(BP1(I,J)-BP1(I,JM1)))/GAM1
      C2(J)=-EZ2(IM1,J)+(EZ1(IM1,J)+EZ1(I,J))*ET2+OET2*(-2.0*JZ(IM1,J)
     $/EPS-(FP(IM1)*EMU/DX)        *(BP1(I,J)-BP1(IM1,J)-BP2(IM1,J))-
     $(EMU/(2.0*F(IM1))*(BP2(IM1,J)+BP1(IM1,J)+BP1(I,J)))))/GAM2
      C3=BP1(I,J)+(DT*FR(I)/(2.0*DX))*(EZ1(IP1,J)-EZ1(I,J)-EZ2(IM1,J))-
     $(GP(J)*DT/(2.0*DY))*(ER1(I,J)-ER1(I,JM1))
      C4=C3-A3*C2(J)
      CL1=ER1(I,J)*ET3+OET3*(-JR(I,J)/EPS-
     $(GP(J)*EMU/(2.0*DY))*(BP1(I,JP1)-BP1(I,J)))/GAM3
      B1=A4*AL1
      B2=A5+A1(J)*A4-AL1*A4
      B3=A1(J)*A4
      B4=C4+A4*(C1(J)-CL1)
      E(J)=-B1/(B2+B3*E(J-1))
      FF(J)=(B4-B3*FF(JM1))/(B2+B3*E(JM1))
  300 CONTINUE
C
C     *** NOW ADVANCE OVER ALTITUDE TO GET THE FIELDS ***
      DO 400   JJ=J2DMN,J2DMX
      J=J2DMX-JJ+J2DMN
      JP1=J+1
      BP2(I,J)=E(J)*BP2(I,JP1)-FF(J)
      EZ2(I,J)=C2(J)-A2(J)*BP2(I,J)
  400 CONTINUE
      DO 410   J=J2DMN,J2DMX
      JM1=J-1
      ER2(I,J)=C1(J)-A1(J)*(BP2(I,J)-BP2(I,JM1))
  410 CONTINUE
  200 CONTINUE
C
C     *** SET THE FIELDS AT THE OUTER BOUNDRY ***
```

```
C
      DO 500  J=1,NY
      ER2(NX,J)=(3.0*ER2(NXM1,J)-ER2(NXM2,J))/2.0
      BP2(NX,J)=(3.0*BP2(NXM1,J)-BP2(NXM2,J))/2.0
      EZ2(NX,J)=(3.0*EZ2(NXM1,J)-EZ2(NXM2,J))/2.0
  500 CONTINUE
C
C     *** CALL OVTPVT TO DO THE PRINTS AND PLOTS
C
  511 CONTINUE
      CALL OVTPVT(K,T)
  100 CONTINUE
      STOP10
      END
```

```
      SUBROUTINE CONST(E,RG,RE,EE)
      RG=50.8273+E*(74.5729+E*(-7.0112+E*.2939))
      IF(E.GT.1.6)  GO TO 10
      EE=E*(.0631+E*(.8108+E*(-.5918+E*.1609)))
      GO TO 20
   10 EE=E*(.3871+E*(.08945+E*(-.01037+E*.0004419)))
   20 IF(E.GT.2.1)    GO TO 30
      RE=-.003476+E*(-.03067+E*(.8050-E*.13992))
      RETURN
   30 RE=-1.0847+E*(1.4679+E*(.04584-E*.001688))
      RETURN
      END
```

```
      SUBROUTINE CURENT(TT)
      REAL JR,JZ
      COMMON ER1(20,20),    EZ1(20,20),    BP1(20,20)
      COMMON ER2(20,20),    EZ2(20,20),    BP2(20,20)
      COMMON  JR(20,20),     JZ(20,20),    SIG(20,20)
      COMMON   R(20,20),    CON(20,20),      X(20),          Y(20)
      COMMON/SGRID/YMAX,DY,NY,J,X0,DX,NX,I,NG
      REAL LAM
      COMMON/TRNS/QE,LAM,RE,EE
      COMMON/WOUT/AL,BE,TP,CY,CX,YLD,EFF,EGB,HOB
      REAL JRR
      DO 20   I=1,NX
      DO 20   J=NG,NY
      T=TT-R(I,J)/2.998E+8
      JRR=-CON(I,J)*SOURCE(T)*QE*RE
      COSINE=X(I)/R(I,J)
      SINE=(HOB-Y(J))/R(I,J)
      JR(I,J)=JRR*COSINE
      JZ(I,J)=-JRR*SINE
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE OVTPVT(ITIME,T)
      INTEGER PRR,PLR
      REAL JR,JZ
      COMMON ER1(20,20),    EZ1(20,20),    BP1(20,20)
      COMMON ER2(20,20),    EZ2(20,20),    BP2(20,20)
      COMMON  JR(20,20),     JZ(20,20),    SIG(20,20)
      COMMON   R(20,20),    CON(20,20),      X(20),          Y(20)
      COMMON/PLTS/PLR,NPX,NPY,IPX(10),IPY(10)
      COMMON/PRNTS/PRR,NWX,NWY,NOX,NOY,IWX(10),IWY(10),IOX(10),IOY(10)
      COMMON/SGRID/YMAX,DY,NY,J,X0,DX,NX,I,NG
      DIMENSION XX(8),BUF(511),LAB(6)
      EQUIVALENCE(XK,XX(1)),(T1,XX(2)),(XJR,XX(3)),(XSIG,XX(4)),(XER,XX(
     $5)),(XJZ,XX(6)),(XEZ,XX(7)),(XBP,XX(8))
      DATA LAB/2HJR,2HJZ,3HSIG,2HER,2HEZ,2HBP/
      DATA IBUF/0/
C
C     *** WRITE THE OBSERVER OUTPUT TAPE ***
C
      T1=T
      DO 10  II=1,NOX
      DO 10  IJ=1,NOY
      IK=(II-1)*NOY + IJ
      XK=IK
      I=IOX(II)
      J=IOY(IJ)
      XJR=JR(I,J)
      XSIG=SIG(I,J)
      XER=ER2(I,J)
      XJZ=JZ(I,J)
      XEZ=EZ2(I,J)
      XBP=BP2(I,J)
      DO 20  L=1,8
      BUF(IBUF+L)=XX(L)
   20 CONTINUE
      IBUF=IBUF + 8
      IF(IBUF.LT.504)   GO TO 30
      WRITE(1) BUF
      IBUF=0
   30 CONTINUE
   10 CONTINUE
C
C     *** PRINT OUT FIELD VALUES ***
C
      IF(MOD(ITIME,PRR).NE.0)  GO TO 200
      DO 100 IJ=1,NWY
      J=IWY(IJ)
      WRITE(6,120) T,Y(J)
  120 FORMAT(//*  TIME=*1PE10.3,10X,*Y=*1PE10.3,/4X,*RANGE*,9X,*JR*,11X
     $,*JZ*,10X,*SIG*,11X,*ER*,11X,*EZ*,11X,*BP*)
      DO 101  II=1,NWX
      I=IWX(II)
      TR=T-R(I,J)/2.998E+8
      WRITE(6,130) X(I),JR(I,J),JZ(I,J),SIG(I,J),ER2(I,J),EZ2(I,J),BP2(I
     $,J),TR
  101 CONTINUE
```

```
  130 FORMAT(1P8E13.4)
  100 CONTINUE
  200 CONTINUF
C
C    *** PROFILE AND RANGE PLOTS AND PRINTS ***
C
      IF(MOD(ITIME,PLR).NE.0)   GO TO 500
C     *** PROFILE PRINT ***
C
      DO 610  II=1,NPX
      I=IPX(II)
      WRITE(6,630) X(I)
  630 FORMAT(//* PROFILE AT X=* 1PE11.3,/)
      DO 610  J=1,NY
      WRITF(6,615) Y(J),JR(I,J),JZ(I,J),SIG(I,J),ER2(I,J),EZ2(I,J),BP2(I
     $,J)
  615 FORMAT(1PE11.3,1P6E20.7)
  610 CONTINUE
C
C     *** RANGE PRINT ***
      DO 620  IJ=1,NPY
      J=IPY(IJ)
      WRITE(6,640) Y(J)
  640 FORMAT(//* RANGE PRINT AT Y= *,1PE11.3,/)
      DO 620  I=1,NX
      WRITE(6,615) X(I),JR(I,J),JZ(I,J),SIG(I,J),ER2(I,J),EZ2(I,J),BP2(I
     $,J)
  620 CONTINUE
C
C     *** PROFILE PLOT ***
C
      DO 701  IJ=1,NPY
      J=IPY(IJ)
      CALL PROFILE(NX,NY,J,JR,LAB(1))
      CALL PROFILE(NX,NY,J,JZ,LAB(2))
      CALL PROFILE(NX,NY,J,SIG,LAB(3))
      CALL PROFILE(NX,NY,J,ER2,LAB(4))
      CALL PROFILE(NX,NY,J,EZ2,LAB(5))
      CALL PROFILE(NX,NY,J,BP2,LAB(6))
  701 CONTINUE
C
C     *** RANGE PLOT ***
      DO 702  II=1,NPX
      I=IPX(II)
      CALL RANGER(NX,NY,I,JR,X,LAB(1))
      CALL RANGER(NX,NY,I,JZ,X,LAB(2))
      CALL RANGER(NX,NY,I,SIG,X,LAB(3))
      CALL RANGER(NX,NY,I,ER2,X,LAB(4))
      CALL RANGER(NX,NY,I,EZ2,X,LAB(5))
      CALL RANGER(NX,NY,I,BP2,X,LAB(6))
  702 CONTINUE
  500 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PROFILE(NX,NY,J,V,Y,LAB)
      DIMENSION V(NX,NY)
      COMMON/WSPT/VB(200)
      DO 10  I=1,NY
   10 VB(I)=V(J,I)
      CALL GRAPH(3,NY,0,0,LAB,3H Y ,0,VB,Y,8.,8.,4.,0.)
      RETURN
      END
```

```
      SUBROUTINE RANGER(NX,NY,I,V,X,LAB)
      DIMENSION V(NX,NY)
      COMMON/WSPT/VB(200)
      DO 10  J=1,NX
   10 VB(J)=V(J,I)
      CALL GRAPH(3,NX,0,0,3H X ,LAB,0,X,VB,8.,8.,0.,0.)
      RETURN
      END
```

```
      SUBROUTINE RDS(KOMENT,DAY,HMS)
      REAL JR,JZ
      COMMON ER1(20,20),    EZ1(20,20),    BP1(20,20)
      COMMON ER2(20,20),    EZ2(20,20),    BP2(20,20)
      COMMON  JR(20,20),     JZ(20,20),    SIG(20,20)
      COMMON   R(20,20),    CON(20,20),      X(20),         Y(20)
      COMMON/TGRID/T0,DT,NT,K
      COMMON/SGRID/YMAX,DY,NY,J,X0,DX,NX,I,NG
      REAL LAM
      COMMON/TRNS/QE,LAM,RE,EE
      COMMON/WOUT/AL,BE,TP,CY,CX,YLD,EFF,EGB,HOB
      COMMON/PRNTS/PRR,NWX,NWY,NOX,NOY,IWX(10),IWY(10),IOX(10),IOY(10)
      COMMON/PLTS/PLR,NPX,NPY,IPX(10),IPY(10)
      DIMENSION KOMENT(8),XX(6)
      WRITE(6,10)
   10 FORMAT(*1*,19X,*IG2D--A TWO DIMENSIONAL EMP CODE TO CALCULATE THE
     $FIELDS ABOVE AN INFINITELY CONDUCTING GROUND*)
      WRITE(6,20) DAY,HMS
   20 FORMAT(46X,A10,2X,A10)
      WRITE(6,30) KOMENT
   30 FORMAT(27X,8A10,//)
      WRITE(6,40) PRR,PLR
   40 FORMAT(30X,*PRINT RATIO*,63X,*PLOT RATIO*/34X,I3,71X,I3///)
      L1=NOX*NOY
      L2=NWX*NWY
      WRITE(6,41) L1,L2,NPX,NPY
   41 FORMAT(4X,*NUMBER TAPE OBSERVERS*,14X,*NUMBER PRINT OBSERVERS*,15X
     $,*NUMBER OF PROFILES*,14X,*NUMBER OF RANGE PLOTS*/13X,I3,32X,I3,32
     $X,I3,32X,I3,//)
      WRITE(6,42)
   42 FORMAT(1X,*NO.*,8X,*X*,12X,*Y*,10X,*NO.*,8X,*X*,12X,*Y*,16X,*NO.*
     $,8X,*X*,23X,*NO.*,8X,*Y*)
      KK=MAX0(L1,L2,NPX,NPY)
      I1=0
      I2=0
      J1=1
      J2=1
      DO 43  K=1,KK
      I1=I1 + 1
      I2=I2 + 1
      IF(I1.LE.NOX)  GO TO 44
      I1=1
      J1=J1 +1
   44 IF(I2.LE.NWX)  GO TO 45
      I2=1
      J2=J2 + 1
   45 I3=IOX(I1)
      I4=IWX(I2)
      J3=IOY(J1)
      J4=IWY(J2)
      IF(K.GT.10)   GO TO 777
      I5=IPX(K)
      J5=IPY(K)
  777 CONTINUE
      XX(1)=X(I3)
```

```
      XX(2)=Y(J3)
      XX(3)=X(I4)
      XX(4)=Y(J4)
      XX(5)=X(I5)
      XX(6)=Y(J5)
      IF(K.LE.L1)   GO TO 46
      XX(1)=-0.0
      XX(2)=-0.0
   46 IF(K.LE.L2)   GO TO 47
      XX(3)=-0.0
      XX(4)=-0.0
   47 IF(K.LE.NPX)   GO TO 48
      XX(5)=-0.0
   48 IF(K.LE.NPY)   GO TO 49
      XX(6)=-0.0
   49 WRITE(6,401) K,XX(1),XX(2),K,XX(3),XX(4),K,XX(5),K,XX(6)
  401 FORMAT(2(1X,I3,1P2E13.3,4X), 8X,I3,3X,1PE13.3,15X,I3,3X,E13.3)
   43 CONTINUE
      WRITE(6,60) T0,DT,NT,X0,DX,NX,YMAX,DY,NY
   60 FORMAT(4X,*TSTART*,10X,*DT*,12X,*NT*,16X,*R0*,12X,*DR*,12X,*NR*,
     $15X,*ZMAX*,11X,*DZ*,12X,*NZ*/,     2(1PE11.4,4X,1PE11.4,7X,I3,11X),
     $1PE11.4,4X,1PE11.4,7X,I3)
      WRITE(6,100)
      WRITE(6,101) YLD,EFF,AL,BE,TP,CY,EGB,LAM,EE,RE
  100 FORMAT(//58X,*SOURCE CALCULATION*,/56X,*DIRECT BEAM GAMMAS ONLY*
     $,/,6X,*YLD*,10X,*EFF*,10X,*AL*,11X,*BE*,11X,*TP*,12X,*C*,11X,*EGB*
     $,10X,*LAM*,10X,*EE*,11X,*RE*)
  101 FORMAT(1P10E13.3)
      RETURN
      END
```

```
      SUBROUTINE SETP
      REAL JR,JZ
      COMMON ER1(20,20),    EZ1(20,20),    BP1(20,20)
      COMMON ER2(20,20),    EZ2(20,20),    BP2(20,20)
      COMMON  JR(20,20),     JZ(20,20),    SIG(20,20)
      COMMON   R(20,20),    CON(20,20),      X(20),          Y(20)
      COMMON/WOUT/AL,BE,TP,CY,CX,YLD,EFF,EGB,HOB
      COMMON/SGRID/YMAX,DY,NY,J,X0,DX,NX,I,NG
      REAL LAM
      COMMON/TRNS/QE,LAM,RE,EE
      DATA PI,QE/3.141592653,1.592E-19/
      CY=1.0
      CX=CY*EXP(AL*TP)
C
C     *** FIND TOTAL AREA ***
C
      DT=TP/50
      TOTAL=0.0
      T=-0.5*DT
      DO 10  I=1,10000
      T=T + DT
      P=SOURCE(T)
      IF(P.LT.TOTAL*.0001)  GO TO 20
   10 TOTAL=TOTAL + P
      STOP 01
   20 TOTAL=TOTAL*DT
C
C
      CY=2.62E+28*YLD*EFF/(EGB*TOTAL)
      CX=CY*EXP(AL*TP)
      CALL CONST(EGB,LAM,RE,EE)
      DO 30  J=1,NY
      YY=(HOB-Y(J))**2
      DO 30  I=1,NX
      XX=X(I)*X(I)
      RR=SQRT(YY+XX)
      R(I,J)=RR
   30 CON(I,J)=EXP(-RR/LAM)/(4.0*PI*RR*RR*LAM)
      RETURN
      END
```

```
      SURROUTINE SIGMA(TT)
      REAL JR,JZ
      COMMON ER1(20,20),    EZ1(20,20),    BP1(20,20)
      COMMON ER2(20,20),    EZ2(20,20),    BP2(20,20)
      COMMON  JR(20,20),     JZ(20,20),    SIG(20,20)
      COMMON   R(20,20),    CON(20,20),      X(20),         Y(20)
      REAL LAM
      COMMON/TRNS/QE,LAM,RF,EE
      COMMON/SGRID/YMAX,DY,NY,J,X0,DX,NX,I,NG
      NP=NG+1
      NM=NG-1
      DO 20   I=1,NX
      JJ=NG
      DO 21   J=NP,NY
      JJ=JJ-1
      T=TT-R(I,J)/2.998E+8
      Q=CON(I,J)*SOURCE(T)*EE*1.0E+6/34.0
      SIG(I,J)=Q*0.1*1.0E-8*QE
      SIG(I,JJ)=SIG(I,J)
   21 CONTINUE
      SIG(I,NG)=0.5*(SIG(I,NP)+SIG(I,NM))
   20 CONTINUE
      RETURN
      END
```

```
      FUNCTION SOURCE(T)
      COMMON/WOUT/AL,BE,TP,CY,CX,YLD,EFF,EGB,HOB
      IF(T.LE.0.0)  GO TO 20
      IF(T.GT.4.0*TP)   GO TO 10
      SOURCE=CY*EXP(AL*T)/(1.0+EXP((AL+BE)*(T-TP)))
      RETURN
   10 SOURCE=CX*EXP(BE*(TP-T))
      RETURN
   20 SOURCE=0.0
      RETURN
      END
```

APPENDIX D

D.1

Several implicit finite difference schemes to solve the low altitude environment problem were programmed and checked out. However, none of them performed as well on the test problems as did the explicit algorithms described in Sections 2.1 and 2.2. A brief description of two of the implicit methods is given in the subsections below.

D.2

The scheme discussed here is an alternating direction implicit set of finite difference equations. For the equations implicit in the ξ direction, we combine Maxwell's equations to yield

$$\varkappa \frac{\partial E_\zeta}{\partial \tau} - Q_\tau \frac{\partial H_\varphi}{\partial \tau} = - z_o j_\zeta - z_o \sigma E_\zeta + \psi_2 \frac{\partial H_\varphi}{\partial v} \tag{D.1}$$

$$\varkappa Q_\tau G_2 \frac{\partial E_\zeta}{\partial \tau} + (G_1 - \varkappa \varkappa_m) \frac{\partial H_\varphi}{\partial \tau} = G_1 z_o (j_\xi + \sigma E_\xi) + \varkappa G_1 \psi_1 \frac{\partial E_\xi}{\partial u} + G_1 \psi_1 \frac{\partial H_\varphi}{\partial u} - \varkappa G_2 \psi_2 \frac{\partial E_\zeta}{\partial v} \tag{D.2}$$

The finite difference form of these equations chosen for the code is

$$\frac{\varkappa}{\Delta\tau}\left(E^k_{\zeta ij} - E^{k-1}_{\zeta ij}\right) - \frac{Q_\tau}{\Delta\tau}\left(H^k_{\zeta i,j} - H^{k-1}_{\zeta i,j}\right) = - z_o j^{k-1/2}_{\zeta i,j} - \frac{z_o}{2}\sigma^{k-1/2}_{i,j}\left(E^k_{\zeta ij} + E^{k-1}_{\zeta i,j}\right) + \frac{\psi_2}{2\Delta v}\left[Q_{11}\left(H^k_{\zeta i,j-1} - H^k_{\zeta i,j+1}\right) + Q_{12}\left(H^{k-1}_{\zeta i,j-1} - H^{k-1}_{\zeta i,j+1}\right)\right] \tag{D.3}$$

and

$$\frac{\varkappa Q_\tau G_2}{\Delta\tau}\left(E^{k}_{\zeta i,j} - E^{k-1}_{\zeta i,j}\right) + \frac{G_1 - \varkappa\varkappa_m}{\Delta\tau}\left(H^{k}_{\zeta i,j} - H^{k-1}_{\zeta i,j}\right) = G_1 z_o\Big(j^{k-1/2}_{\xi i,j}$$

$$+ \sigma^{k-1/2}_{i,j} E^{k-1/2}_{\xi i,j}\Big) + \frac{\varkappa G_1\psi_1}{2\Delta u}\left(E^{k-1/2}_{\xi i+1,j} - E^{k-1/2}_{\xi i-1,j}\right) + \frac{G_1\psi_1}{2\Delta u}\Big(H^{k-1/2}_{\xi i+1,j}$$

$$- H^{k-1/2}_{\xi i-1,j}\Big) - \frac{\varkappa G_2\psi_2}{2\Delta v}\left[Q_{21}\left(E^{k}_{\zeta i,j-1} - E^{k}_{\zeta i,j+1}\right) + Q_{22}\left(E^{k-1}_{\zeta i,j-1} - E^{k-1}_{\zeta i,j+1}\right)\right] \tag{D. 4}$$

Equations (D. 3) and (D. 4) can be cast in the form

$$-\mathbf{A}\vec{g}_{j+1} + \mathbf{B}_j\vec{g}_j - \mathbf{C}_j\vec{g}_{j-1} = \vec{d}_j \tag{D. 5}$$

where

$$\vec{g}_j = \begin{pmatrix} E^{k}_{\zeta i,j} \\ \\ H^{k}_{\zeta i,j} \end{pmatrix} \tag{D. 6}$$

Equation (D. 5) represents a five-diagonal matrix for the solution of the finite difference equations. It is solved in a way that is analogous to the usual three-diagonal case. We seek a solution of the form

$$\vec{g}_j = \mathbf{E}_j\vec{g}_{j+1} + \vec{f}_j \tag{D. 7}$$

By substitution into equation (D. 5), one finds

$$\mathbb{E}_j = (\mathbb{B}_j - \mathbb{C}_j \mathbb{E}_{j-1})^{-1} \mathbb{A}_j \tag{D.8}$$

$$\vec{f}_j = (\mathbb{B}_j - \mathbb{C}_j \mathbb{E}_{j-1})^{-1} (\vec{d}_j + \mathbb{C}_j \vec{f}_{j-1}) \tag{D.9}$$

The system is solved by first solving the recursion equations (D.8) and (D.9) to obtain the $\mathbb{E}_j$ and $\vec{f}_j$ terms and then obtaining the fields from equation (D.7). The boundary conditions on $\mathbb{E}_j$ and $\vec{f}_j$ are

$$\mathbb{E}_1 = 0, \qquad \vec{f}_1 = \vec{g}_1 \tag{D.10}$$

On alternate passes through the mesh, implicit equations in the ζ direction are used. In this case, Maxwell's equations are combined to yield

$$\varkappa \frac{\partial E_\xi}{\partial \tau} - \frac{\partial H_\varphi}{\partial \tau} = - z_o j_\xi - z_o \sigma E_\xi - \psi_1 \frac{\partial H_\varphi}{\partial u} \tag{D.11}$$

$$\varkappa G_1 \frac{\partial E_\xi}{\partial \tau} + (G_2 - \varkappa \varkappa_m) \frac{\partial H_\varphi}{\partial \tau} = Q_\tau G_2 z_o (j_\zeta + \sigma E_\zeta) - Q_\tau G_2 \psi_2 \frac{\partial H_\varphi}{\partial v}$$
$$+ \varkappa G_1 \psi_1 \frac{\partial E_\xi}{\partial u} - \varkappa G_2 \psi_2 \frac{\partial E_\zeta}{\partial v} \tag{D.12}$$

The finite difference equations are

$$\frac{\varkappa}{\Delta\tau}\left(E^{k}_{\xi i,j} - E^{k-1}_{\xi i,j}\right) - \frac{1}{\Delta\tau}\left(H^{k}_{\xi i,j} - H^{k-1}_{\xi i,j}\right) = - z_o j^{k-1/2}_{\xi i,j} - \frac{z_o}{2}\sigma^{k-1/2}_{i,j}$$

$$\left(E^{k}_{\xi i,j} + E^{k-1}_{\xi i,j}\right) - \frac{\psi_1}{2\Delta u}\left[Q_{11}\left(H^{k}_{\xi i+1,j} - H^{k}_{\xi i-1,j}\right) + Q_{12}\left(H^{k}_{\xi i+1,j} - H^{k-1}_{\xi i-1,j}\right)\right] \tag{D.13}$$

and

$$\frac{\varkappa G_1}{\Delta\tau}\left(E^{k}_{\xi i,j} - E^{k-1}_{\xi i,j}\right) + \frac{G_2 - \varkappa\varkappa_m}{\Delta\tau}\left(H^{k}_{\xi i,j} - H^{k-1}_{\xi i,j}\right) = Q_\tau z_o G_2\left[j^{k-1/2}_{\zeta i,j}\right.$$

$$\left. + \sigma^{k-1/2}_{i,j} E^{k-1/2}_{\zeta i,j}\right] - \frac{Q_\tau}{2\Delta v}\psi_2 G_2\left(H^{k-1/2}_{\zeta i,j-1} - H^{k-1/2}_{\zeta i,j+1}\right) + \frac{\varkappa}{2\Delta u}\psi_1 G_1$$

$$\left[Q_{21}\left(E^{k}_{\xi i+1,j} - E^{k}_{\xi i-1,j}\right) + Q_{22}\left(E^{k-1}_{\xi i+1,j} - E^{k-1}_{\xi i-1,j}\right)\right]$$

$$- \frac{\varkappa}{2\Delta v}\psi_2 G_2\left(E^{k-1/2}_{\zeta i,j-1} - E^{k-1/2}_{\zeta i,j+1}\right) \tag{D.14}$$

By making the substitution, $j \rightarrow i$, equations (D.5) through (D.10) are appropriate for obtaining the solution of the finite difference equations (D.13) and (D.14) above.

D.3

The finite difference equations discussed in this section employ second order exponential differencing. The implicit equations are tri-diagonal. The reduction from a five-diagonal system is achieved by writing the $\nabla \times \vec{E}$ term in Faraday's Law explicitly. Although the term is explicit, it is centered in time and is of second order accuracy. This is accomplished by using a difference scheme that is multilevel in the time dimension. The finite difference equations are

$$E^{k}_{\zeta i,j} = E^{k-1}_{\zeta i,j} e^{-\bar{x}} + \frac{1}{\sigma^{k}_{i,j}} \left[1 - \frac{1}{\bar{\bar{x}}}\left(1 - e^{-\bar{x}}\right)\right] \left[- j^{k}_{\zeta i,j} + \frac{Q_\tau}{z_o \Delta\tau}\left(H^{k+1/2}_{\varphi i,j} - H^{k-1/2}_{\varphi i,j}\right)\right.$$

$$\left. + \frac{\psi_2}{4 z_o \Delta v}\left(H^{k+1/2}_{\varphi i,j-1} - H^{k+1/2}_{\varphi i,j+1} + H^{k-1/2}_{\varphi i,j-1} - H^{k-1/2}_{\varphi i,j+1}\right)\right]$$

$$+ \frac{1}{\sigma^{k-1}_{i,j}} \left[\frac{1}{\bar{\bar{x}}}\left(1 - e^{-\bar{x}}\right) - e^{-\bar{x}}\right] \left[- j^{k-1}_{\zeta i,j} + \frac{Q_\tau}{z_o \Delta\tau}\left(H^{k-1/2}_{\varphi i,j} - H^{k-1/2}_{\varphi i,j}\right)\right.$$

$$\left. + \frac{\psi_2}{4 z_o \Delta v}\left(H^{k-1/2}_{\varphi i,j-1} - H^{k-1/2}_{\varphi i,j+1} + H^{k-3/2}_{\varphi i,j-1} - H^{k-3/2}_{\varphi i,j+1}\right)\right] \qquad (D.15)$$

$$E^{k}_{\xi i,j} = E^{k-1}_{\xi i,j} e^{-\bar{x}} + \frac{1}{\sigma^{k}_{i,j}} \left[1 - \frac{1}{\bar{x}}\left(1 - e^{-\bar{x}}\right)\right]\left[- j^{k}_{\xi i,j} + \frac{1}{z_o \Delta\tau}\left(H^{k+1/2}_{\varphi i,j} - H^{k-1/2}_{\varphi i,j}\right)\right.$$

$$\left. - \frac{\psi^{i}_{1}}{2 z_o \Delta u}\left(H^{k+1/2}_{\varphi i,j} - H^{k+1/2}_{\varphi i-1,j} + H^{k-1/2}_{\varphi i+1,j} - H^{k-1/2}_{\varphi i,j}\right)\right]$$

$$+ \frac{1}{\sigma^{k-1}_{i,j}}\left[\frac{1}{\bar{x}}\left(1 - e^{-\bar{x}}\right) - e^{-\bar{x}}\right]\left[- j^{k-1}_{\xi i,j} + \frac{1}{z_o \Delta\tau}\left(H^{k-1/2}_{\varphi i,j} - H^{k-3/2}_{\varphi i,j}\right)\right.$$

$$\left. - \frac{\psi^{i}_{1}}{2 z_o \Delta u}\left(H^{k-1/2}_{\varphi i,j} - H^{k-1/2}_{\varphi i-1,j} + H^{k-3/2}_{\varphi i+1,j} - H^{k-3/2}_{\varphi i,j}\right)\right] \quad (D.16)$$

and

$$\frac{Q_\tau G_2}{2\Delta\tau}\left(E^{k}_{\zeta i,j} - E^{k-2}_{\zeta i,j}\right) + \frac{G_1}{2\Delta\tau}\left(E^{k}_{\xi i,j} - E^{k-2}_{\xi i,j}\right) - \frac{\varkappa_m}{3\Delta\tau}\left(H^{k+1/2}_{\varphi i,j} - H^{k-5/2}_{\varphi i,j}\right)$$

$$= \frac{G_1 \psi^{i}_{1}}{2\Delta u}\left(E^{k-1}_{\xi i+1,j} - E^{k-1}_{\xi i-1,j}\right) - \frac{G_2 \psi_2}{2\Delta v}\left(E^{k-1}_{\zeta i,j-1} - E^{k-1}_{\zeta i,j+1}\right) \quad (D.17)$$

Equations (D.15) through (D.17) can be combined to yield the following tri-diagonal equation for the magnetic field

$$-A_j H^{k}_{\varphi i,j+1} + B_j H^{k}_{\varphi i,j} - C_j H^{k}_{\varphi i,j-1} = D_j$$

A solution of the form

$$H^{k}_{\varphi i,j} = E_j H^{k}_{\varphi i,j+1} + F_j$$

exists in which

$$E_j = \frac{A_j}{B_j + A_j E_{j-1}}$$

$$F_j = \frac{D_j - A_j F_{j-1}}{B_j + A_j E_{j-1}}$$

The appropriate boundary conditions are

$$E_1 = 0 , \qquad F_1 = H^k_{\varphi i, 1}$$

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