EM Implosion Memos

Memo 1

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Parameter Study for Prolate-Spheroidal IRA

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Abstract

This paper is a parameter study of the focal waveform produced at the second focus of a prolate-spheroidal reflector due to a TEM wave launched from the first focus.

1.1 Introduction

This is based on formulae developed in [1].

Prolate-Spheroid

A prolate-spheroidal reflector based on the two foci of an ellipse. A prolate sphere S*p* is a body of revolution with an equation for the surface.

$$\begin{aligned} &\left[\frac{\Psi}{b}\right]^2 + \left[\frac{z}{a}\right]^2 = 1 \\ &z_0 = \left[a^2 - b^2\right]^{1/2} \end{aligned} \tag{1.1}$$

The present paper is concerned with analytic calculations of the waveform at the second focus. For this purpose let us consider the geometry in Fig. 1.1 Let there be two thin perfectly conducting cone wave launchers with electrical centers lying in the *xz* plane. With respect to the negative *z* axis they are oriented at

$$\theta_1 = \theta_c \tag{1.2}$$

in the wave-launching spherical system (r_1 , θ_1 , ϕ_1). These are related to cylindrical (Ψ_1 , ϕ_1 , z_1) and Cartesian (*x*, *y*, *z*) coordinates as

$$\begin{aligned} &\Psi_1 = r_1 \sin(\theta_1) \quad , \quad z = -z_0 - r_1 \cos(\theta_1) \\ &x = \Psi_1 \cos(\phi_1) \quad , \quad y = -\Psi_1 \sin(\phi_1) \\ &\phi_1 = -\phi \quad , \quad \Psi = \Psi_1 \\ &z_1 = -z + z_0 \end{aligned} \tag{1.3}$$

At the reflector we have (subscript p)

$$\begin{aligned} &z_p = -z_0 - \Psi_p \cot(\theta_c) \\ &\left[\frac{\left[-z_0 - z_p\right]}{b}\tan(\theta_c)\right]^2 + \left[\frac{z_p}{a}\right]^2 = 1 \\ &\tan^2(\theta_c) = \left[\frac{b}{z_0 + z_p}\right]^2 \left[1 - \left[\frac{z_p}{a}\right]^2\right] \end{aligned} \tag{1.4}$$

For the present let us truncate the reflector at the $z = z_p$ plane. The portion used is S'_p, to the left. The portion of this plane inside the prolate sphere is designated S_a. It is this surface is used for integrating over the reflected TEM wave to find the fields at the second focus, $\vec{r}_0$.

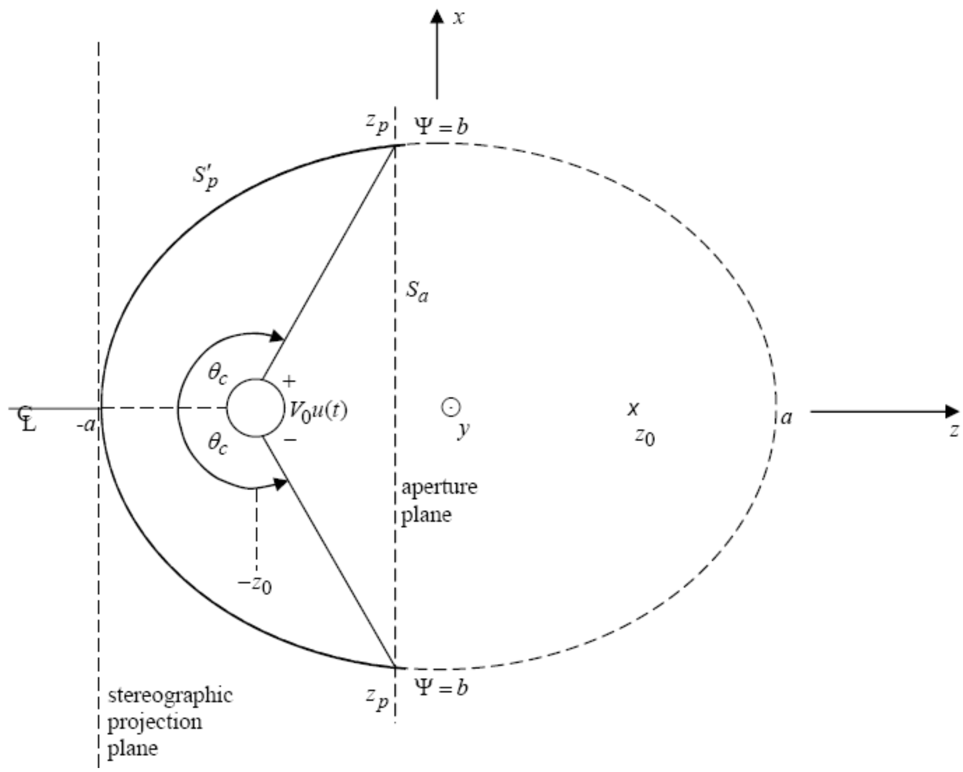


Fig. 1.1 Wave Launcher in Prolate-Spheroidal Reflector

1.2 Fields at Second Focus

Summarizing, we have

$$E_\delta = \frac{E_0}{c}[a+z_0]\left[1-\left[1+\left[\frac{\Psi_p}{z_0-z_p}\right]^2\right]^{-1/2}\right] = \frac{V_0}{\pi f_g c}\frac{a+z_0}{a-z_0}\cot\left(\frac{\theta_c}{2}\right)\left[1-\left[1+\left[\frac{\Psi_p}{z_0-z_p}\right]^2\right]^{-1/2}\right]$$

$$E_s = \frac{E_0}{2}\frac{a+z_0}{z_0-z_p}\left[1+\left[\frac{z_0-z_p}{\Psi_p}\right]^2\right]^{-1} = \frac{V_0}{2\pi f_g}\frac{1}{z_0-z_p}\frac{a+z_0}{a-z_0}\cot\left(\frac{\theta_c}{2}\right)\left[1+\left[\frac{z_0-z_p}{\Psi_p}\right]^2\right]^{-1} \qquad (1.5)$$

$$E_p = \frac{V_0}{2\pi f_g z_0}\tan\left(\frac{\theta_c}{2}\right) \;,\; E_{pa} = E_p \Delta t_p = \frac{V_0}{2\pi f_g c}\frac{a-z_0}{z_0}\tan\left(\frac{\theta_c}{2}\right) \text{ time integral or “area” of prepulse}$$

$$E_0 = \frac{V_0}{\pi f_g}\frac{1}{a-z_0}\cot\left(\frac{\theta_c}{2}\right)$$

A first observation concerns the common factor $V_0/(\pi f_g)$. For large fields one needs large voltage and low wave-launcher impedance. Note that an extra factor of $\sqrt{2}$ increase in the fields is obtained by going to a 4- arm feed in the usual sense of an IRA with arms at 45° , and a little more can be achieved by arms at about 60° from the horizontal (the *x* = 0 plane) as projected on a constant-z plane . This is just another common factor with which we can deal separately.

2. Some Results and Definitions

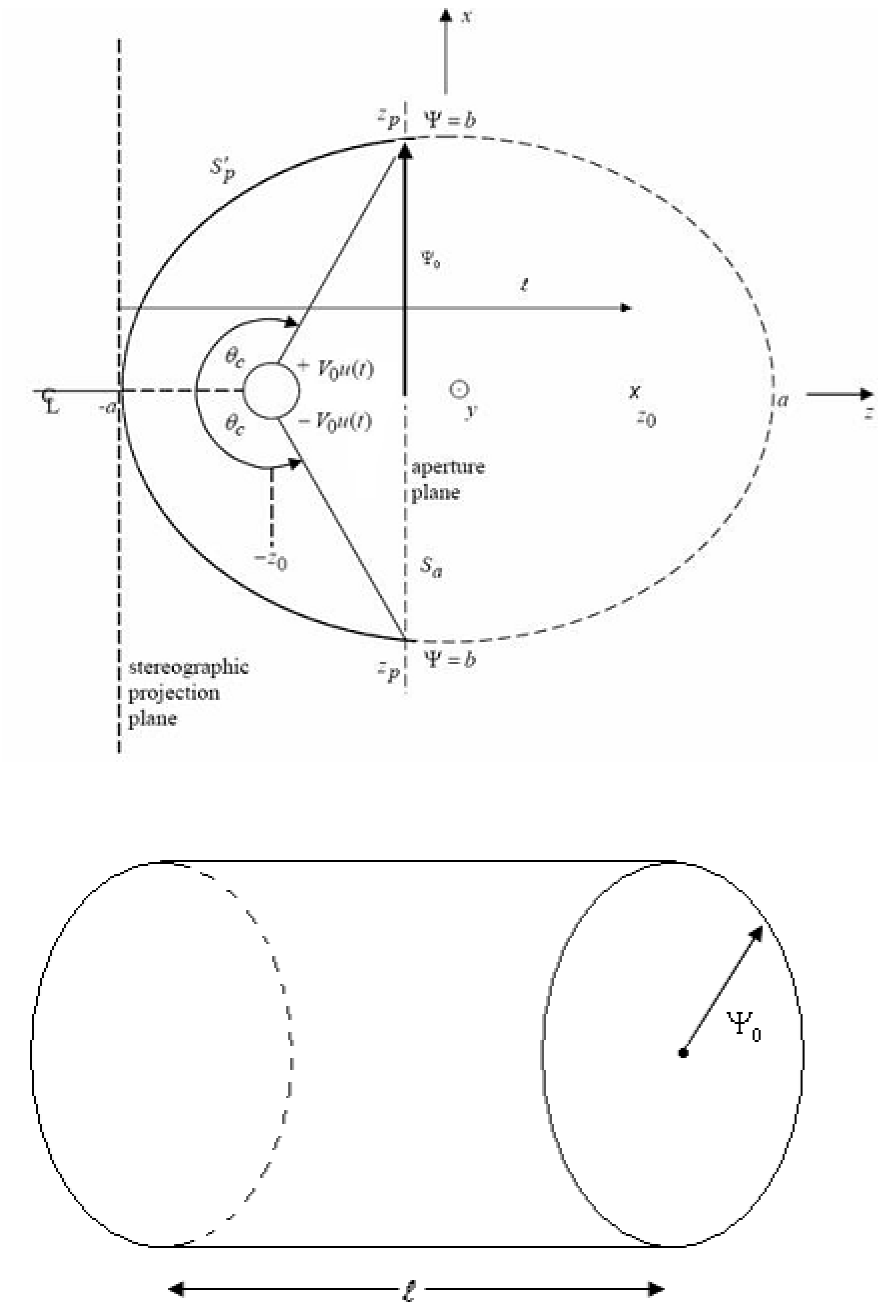


Fig. 2.1 IRA and Cylinder Geometry

In order to find an optimal design we need to compare various designs on a common basis. For this purpose let us define a volume based on a geometric shape that fill in some sense to get the maximum performance. Consider a circular cylinder as in Fig. 2.1 This has length ℓ and radius Ψ_0 .

One can see;

$$\ell = a + z_0 = a + \left[a^2 - b^2\right]^{1/2} \tag{2.1}$$

based on distance from the back of the reflector (z = -a) to the target ($z=z_0$), beyond which we will not extend S'_p. This still leaves the radius Ψ_0 which we treat via the parameter Ψ_0 / ℓ. All distances are normalized to ℓ. Note that

$$\text{for } z_p \geq 0,\ \Psi_0 = b \tag{2.2}$$
$$\text{for } z_p \leq 0,\ \Psi_0 = \Psi_p$$

2.1 Normalized Parameters

We normalized parameters with ℓ so, geometric scaled parameters are:

$$\frac{a}{\ell}, \frac{b}{\ell}, \frac{z_0}{\ell}, \frac{\Psi_0}{\ell}, \frac{\Psi_p}{\ell}, \frac{z_p}{\ell} \tag{2.3}$$

Electromagnetic scaled parameters are:

$$e_\delta = E_\delta \frac{\pi f_g c}{V_0} \quad \text{impulse} \tag{2.4}$$

$$e_p = E_p \frac{2\pi f_g \ell}{V_0} \quad \text{prepulse (step, negative)} \tag{2.5}$$

$$e_{pa} = E_p \Delta t_p \frac{2\pi f_g c}{V_0} \quad \text{prepulse integral (arae)} \tag{2.6}$$

$$e_s = E_s \frac{2\pi f_g \ell}{V_0} \quad \text{postpulse step} \tag{2.7}$$

One can see that the impulse peak is the ratio of:

$$\frac{E_\delta}{t_\delta} \tag{2.8}$$

t_δ is the “impulse width” or “rise time” of the source.

We can find ratio of impulse to prepulse:

$$\frac{1}{E_p} \frac{E_\delta}{t_\delta} = \frac{2\ell}{ct_\delta} \frac{e_s}{e} \tag{2.9}$$

which must be large.

2.2 Calculating $\tan(\frac{\theta_c}{2})$

θ_c can be between $0 \le \theta_c \le \pi$.

By the geometric construction from Figure 2.1 we have

$$\tan(\frac{\theta_c}{2}) = \sin^{-1}(\theta_c)[1-\cos(\theta_c)] \tag{2.10}$$

$$\sin(\theta_c) = \sin(\pi-\theta_c) = \Psi_p\left[\Psi_p{}^2 + [z_p + z_o]^2\right]^{-1/2} \tag{2.11}$$

$$\cos(\theta_c) = -\cos(\pi-\theta_c) = -[z_p + z_o]\left[\Psi_p{}^2 + [z_p + z_o]^2\right]^{-1/2} \tag{2.12}$$

so

$$\begin{aligned}\tan(\frac{\theta_c}{2}) &= \frac{1}{\Psi_p}\left[\Psi_p{}^2 + [z_p + z_o]^2\right]^{1/2}\left[[z_p + z_o]\left[\Psi_p{}^2 + [z_p + z_o]^2\right]^{-1/2}\right] \\ &= \frac{1}{\Psi_p}\left[\Psi_p{}^2 + [z_p + z_o]^2\right]^{1/2}\frac{[z_p + z_o]}{\Psi_p}\end{aligned} \tag{2.13}$$

Let's try to find $\left[\Psi_p{}^2 + [z_p + z_o]^2\right]^{1/2}$ in terms of a, z_0, z_p

$$\Psi_p = b\left[1 - \left[\frac{z_p}{z_o}\right]^2\right]^{1/2} \tag{2.14}$$

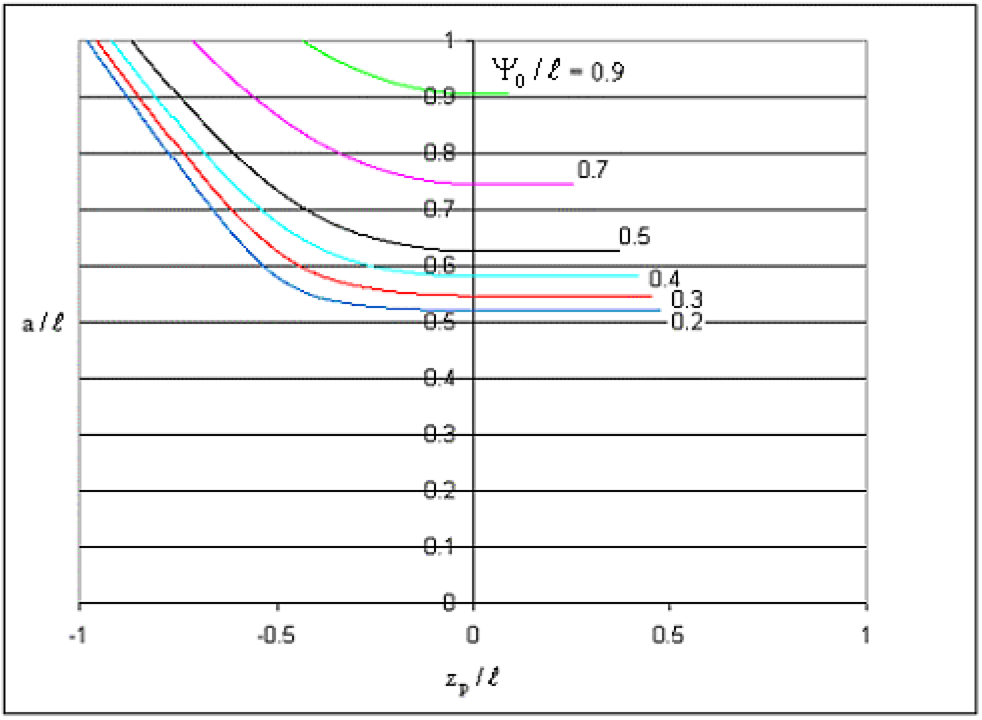
$$\begin{aligned}\left[\Psi_p{}^2 + [z_p + z_o]^2\right]^{1/2} &= \frac{1}{a}\left[b^2\left[a^2 - z_p{}^2\right] + a^2[z_p + z_0]^2\right]^{1/2} \\ &= \frac{1}{a}\left[a^4 + 2a^2 z_p z_0 + z_0{}^2 z_p{}^2\right]^{1/2} \\ &= \frac{1}{a}\left[a^2 + z_0 z_p\right]\end{aligned} \tag{2.15}$$

Substitute this in (1.21) so we can get:

$$\begin{aligned}\tan(\frac{\theta_c}{2}) &= \frac{1}{a\Psi_p}[a + z_p][a + z_0] = \frac{1}{b\left[a^2 - z_p{}^2\right]^{1/2}}[a + z_p][a + z_0] \\ &= \left[\frac{a + z_p}{a - z_p}\right]^{1/2}\frac{a + z_0}{b}\end{aligned} \tag{2.16}$$

3.Parameter Study

3.1 Geometric Parameters $\frac{a}{\ell}, \frac{b}{\ell}, \frac{z_0}{\ell}, \frac{\Psi_p}{\ell}$



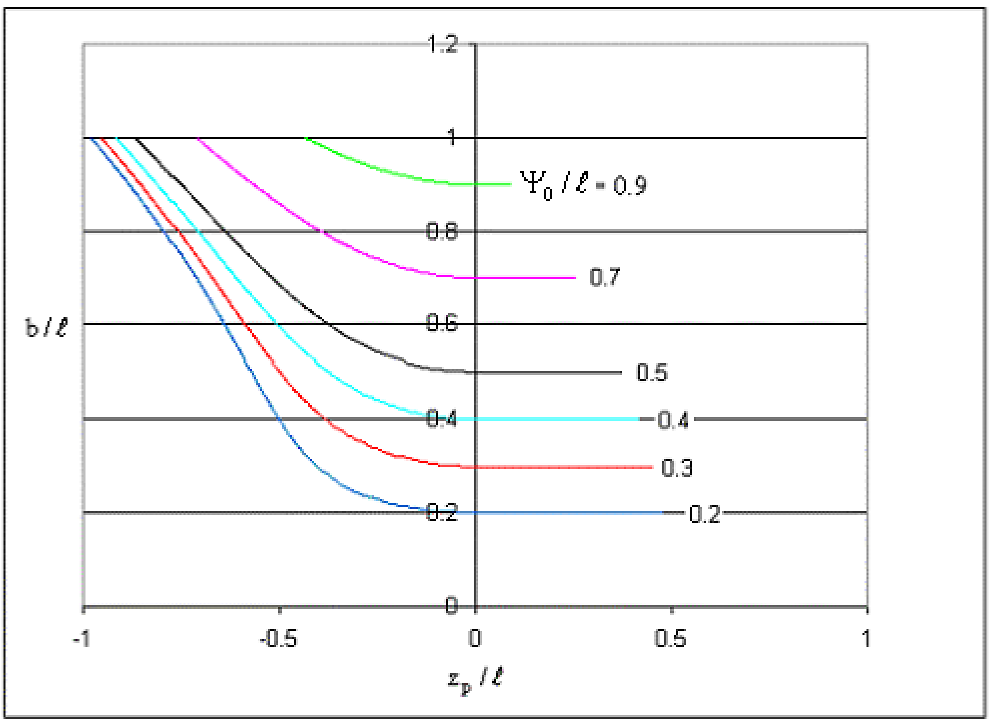


Figure 3.1 a / ℓ and b / ℓ wrt z_p / ℓ

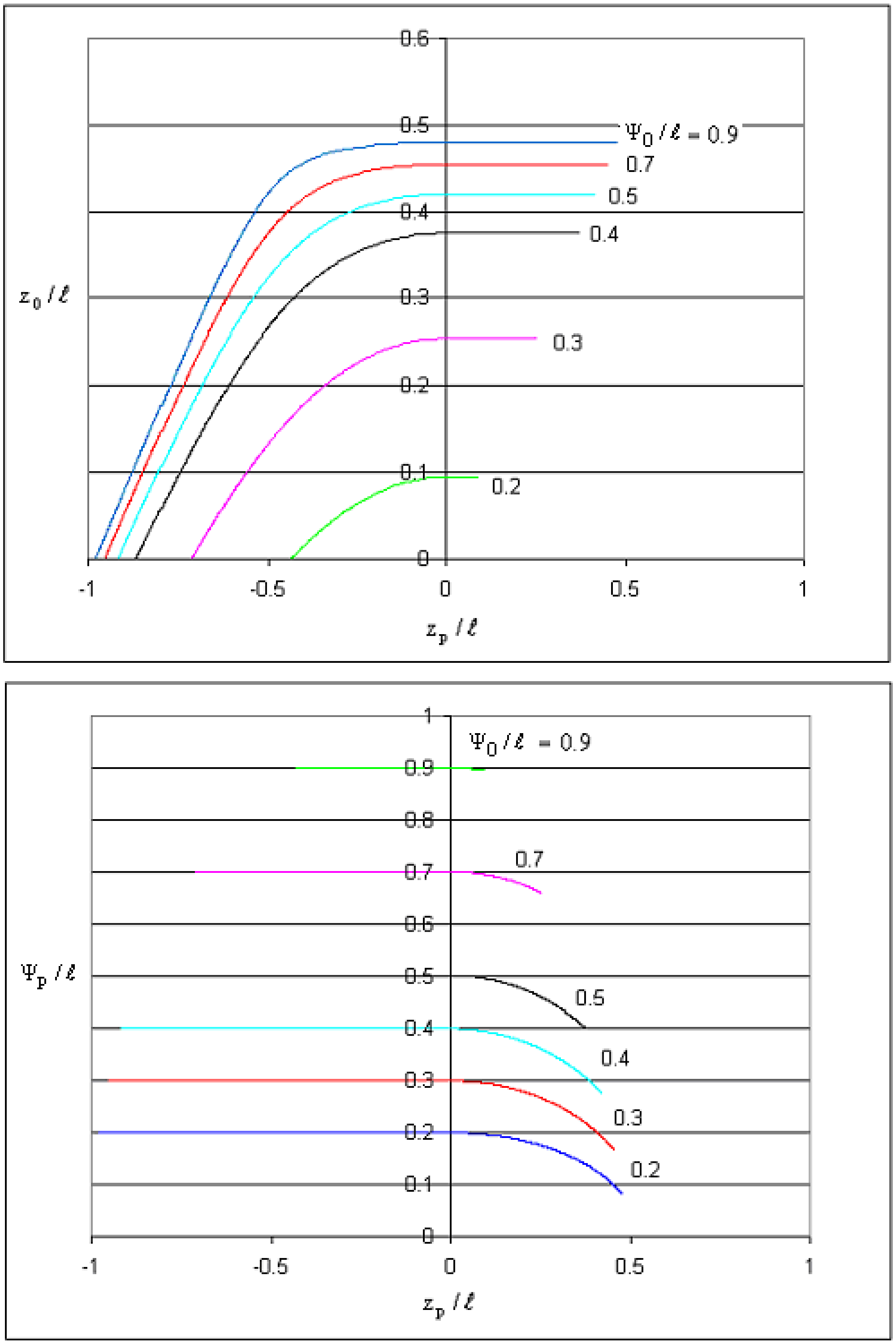
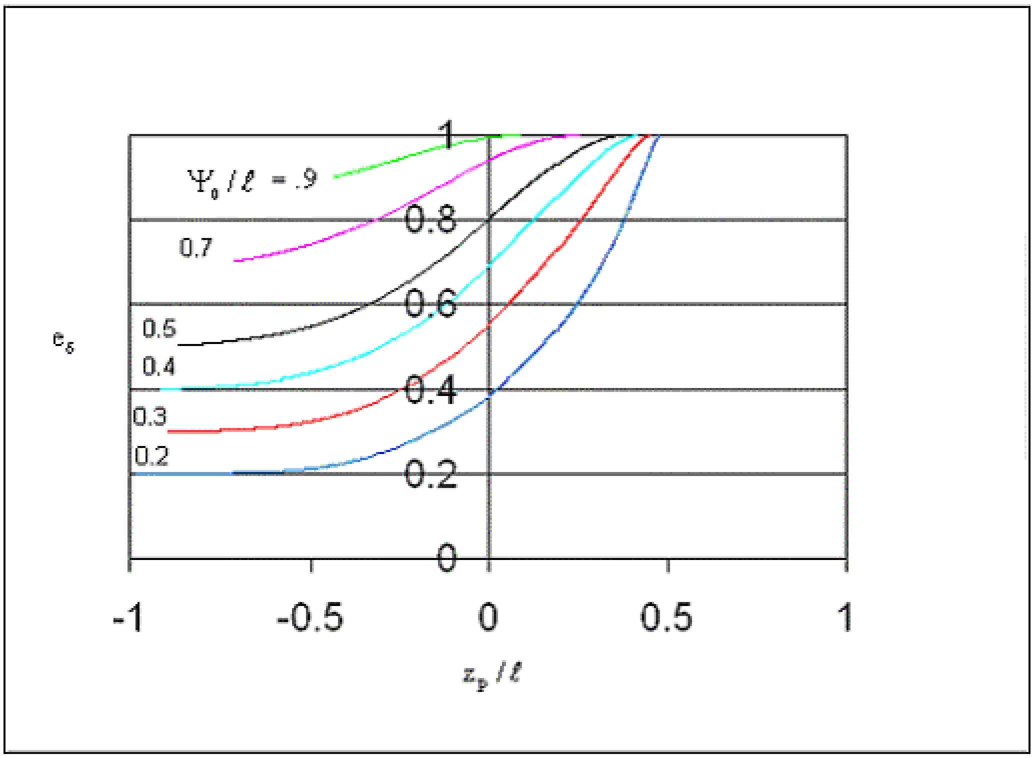


Figure 3.2 z_0 / ℓ and Ψ_p / ℓ wrt z_p / ℓ

3.2 ElectromagneticParameters



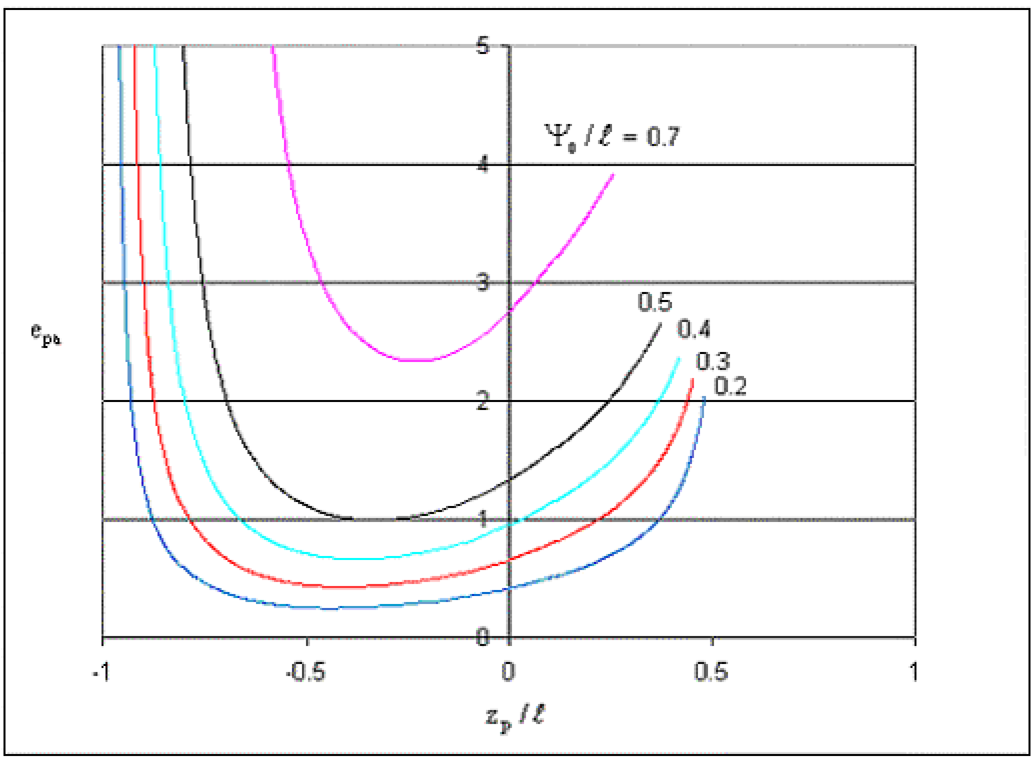


Figure 3.3 e_δ and e_{pa} wrt z_p / ℓ

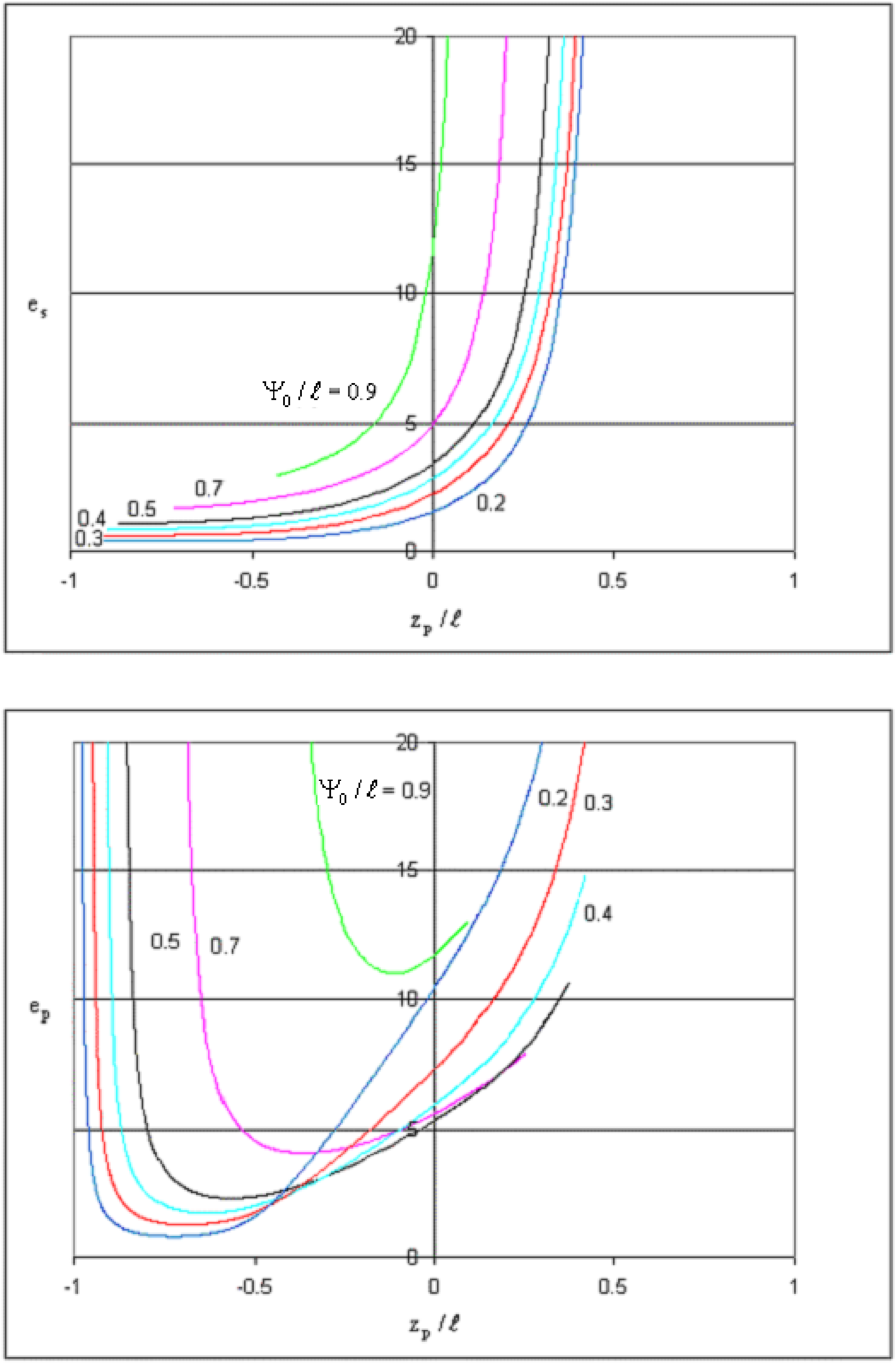
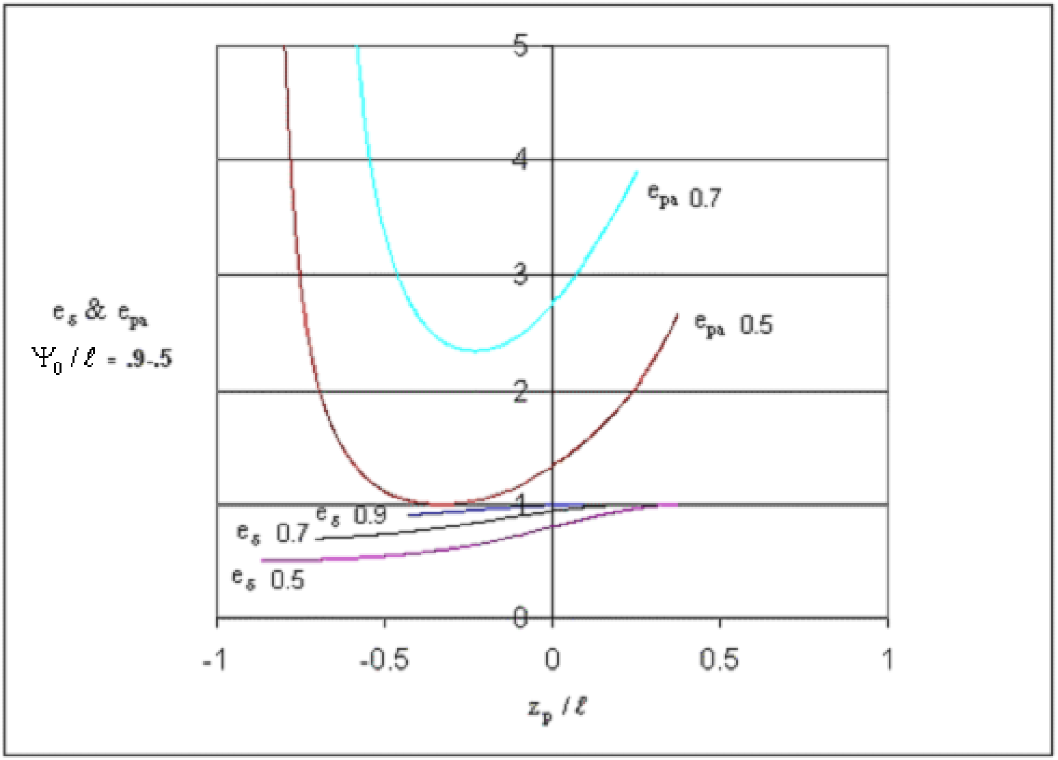


Figure 3.4 e_s and e_p wrt z_p / ℓ



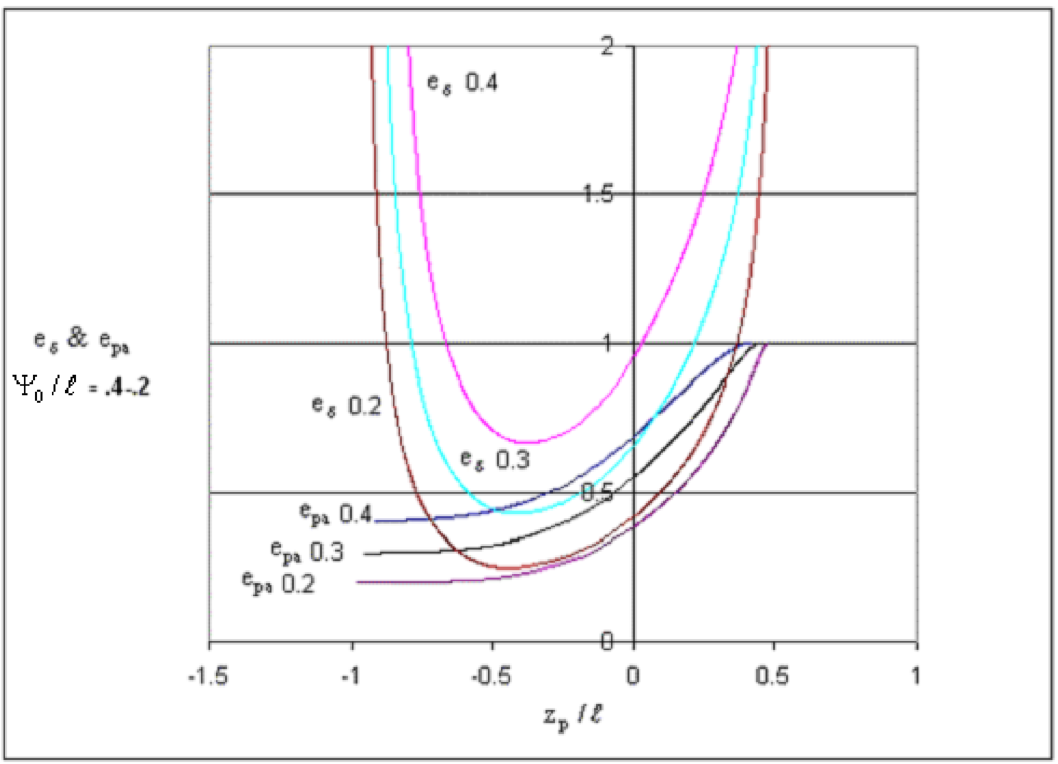


Figure 3.5 e_δ and e_{pa} wrt z_p / ℓ

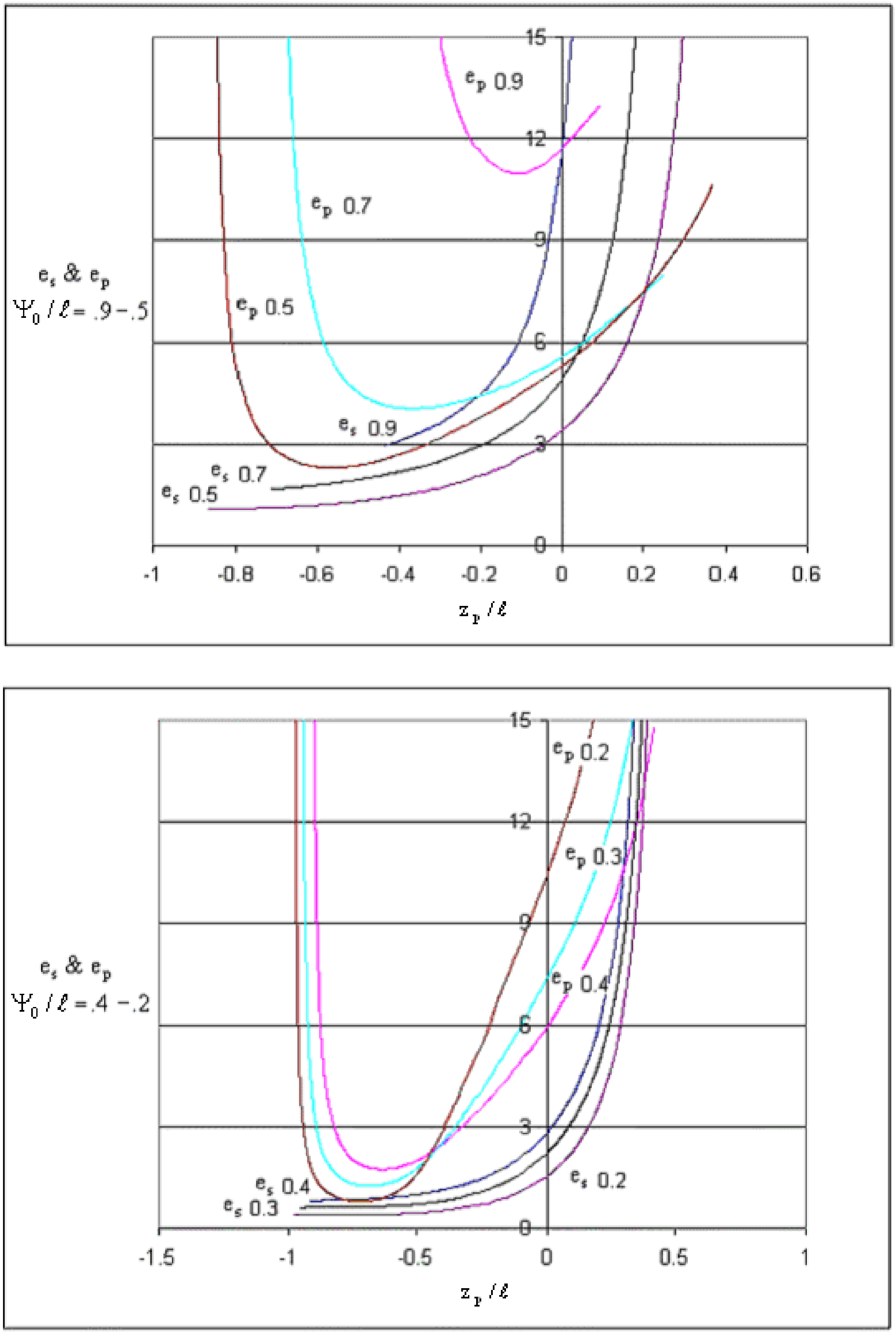


Figure 3.6 e_s and e_p wrt z_p / ℓ

4.Example Case

4.1 Feed-Point Lens

This increases (bumps up) the field after the lens.

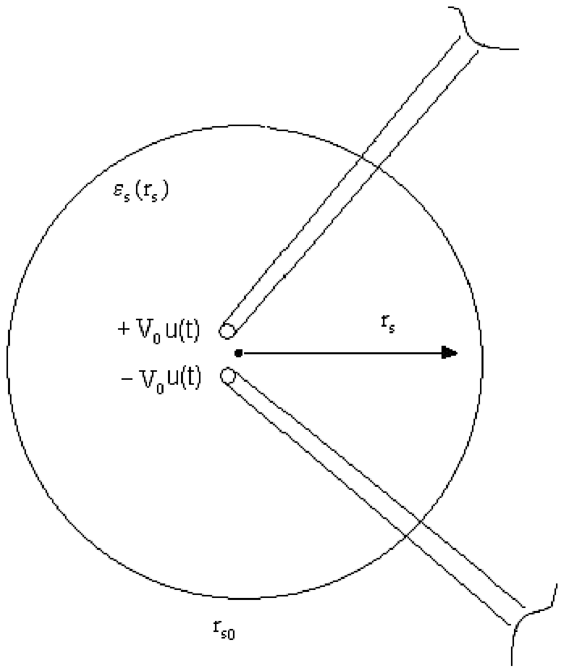


Figure 4.1 Feed-Point lens

Spherical Example:

$$\varepsilon_{rs} = \frac{\varepsilon_s}{\varepsilon_0} \quad , \quad \varepsilon_{rs}(0) \geq \varepsilon_{rs}(r_s) \geq 1 \text{ (air)} \tag{4.1}$$

Typical value

$\varepsilon_{rs} = 2.25$ (transformer oil)

Uniform transmission to air

$$T = \frac{2}{1+\varepsilon_{rs}^{-1/2}} = 1.2 \quad \text{for example} \tag{4.2}$$

Graded lens like transmission-line transformer

$$T = \varepsilon_{rs}^{1/4}(0) = 1.22 \text{ (for example not much improvement)} \tag{4.3}$$

$$\Psi_0 = \frac{b}{\ell} = 0.5 \quad , \quad \frac{z_p}{\ell} = 0 \tag{4.4}$$

$$\frac{b}{\ell} = 0.5 \quad , \quad \frac{a}{\ell} = 0.625 \quad , \quad \frac{z_0}{\ell} = 0.375$$

$$\frac{b}{a} = \frac{4}{5} \quad , \quad \frac{z_0}{a} = \frac{3}{5}$$

4.2 Geometric Parameters

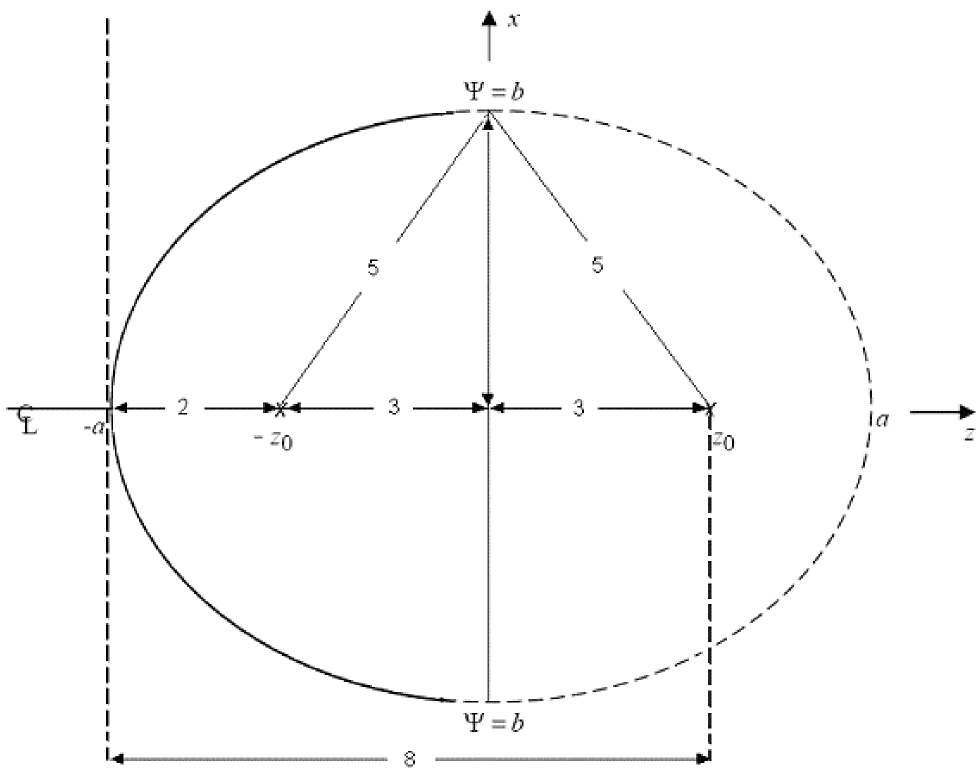


Figure 4.2 Example case parameters

4.3 Other Parameters

Source voltage (pulse rising in time t_δ);

$$V_0 = 10^5\,\mathrm{V}\ \ (2\mathrm{x}10^5 \text{ differential}) \tag{4.5}$$

4 arms at 45^0 cause increase by $\sqrt{2}$

$$\begin{aligned} &t_\delta = 100\,\mathrm{ps}\ ,\ f_g = 1.06 \\ &e_\delta = 0.8\ ,\ e_p = 5.4 \\ &E_{inc} = \frac{E_\delta}{t_\delta}\sqrt{2}\,T = \frac{V_0 e_\delta}{\pi f_g c t_\delta}\sqrt{2}\,T = 1.36\ \mathrm{MV/m} \end{aligned} \tag{4.6}$$

Ratio of impulse peak to prepulse amplitude;

$$\frac{1}{E_p}\frac{E_\delta}{t_\delta} = \frac{2\ell}{ct_\delta}\frac{e_\delta}{e_p} = 9.9\ell \quad \text{Places } \ell \text{ in range of 1 m.} \tag{4.7}$$

Spot diameter (from [1])

$$2\Delta\Psi = \frac{a}{b}ct_\delta = 3.75\,\mathrm{cm} \tag{4.8}$$

5. Concluding Remarks

This modest study has found some graphs useful for estimating the focal waveforms and focal spot size for the prolate-spheroidal IRA. Considering the sophisticated design papers which followed the introduction of the IRA concept, there is much yet to be done for the prolate-spheroidal version.

This should be adequate for designing a first experiment to demonstrate and validate the design concept.

References

1. C. E. Baum, “Focal Waveform of a Prolate-Spheroidal IRA”, Sensor and Simulation Note 509, February 2006.