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Dispersion of Water for Impulse Propagation

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Abstract

This paper calculates the dispersive loss of water when a Gaussian impulse, or a step function impulse travels over a distance. The minimum risetime t_{mr} was calculated.

1 Introduction

An antenna that uses the prolate spheroidal reflector to focus the impulse energy into a spot where a biological target could be located is under development [1]. The impulse is launched from one of the focal points as a spherical TEM wave, which is then focused at the other focal point.

One approach to achieve a better spatial resolution is to fill the system with water, so to compress the wavelength by a factor of 9 (square root of 81), for example, a 100 ps or so risetime pulse, the wavelength can be as small as 3.33 mm counting 10 GHz as the frequency. On the other hand, the size of the system can be dramatically reduced. Additional benefit includes the prevention of the breakdown due to the high voltage impulse input.

The question remains that how much the dispersive loss of water we can tolerate. The dispersion arises as the speed of the light decreases when the frequency increases in the transition stage of the real part of permitivitty (see Fig.1). The traveling speed difference at different frequency causes the broadening of the pulse.

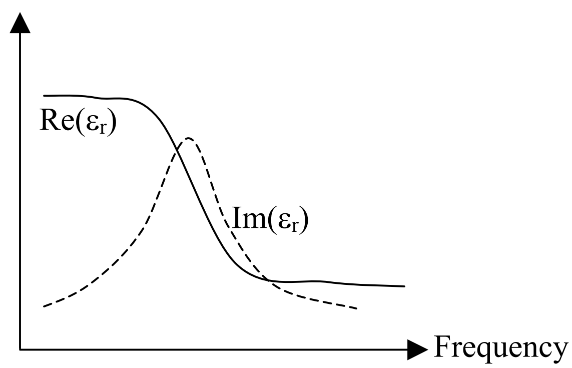


Figure 1. A schematic of dielectric permitivitty for a Debye model.

Water undergoes a dielectric dispersion centered at 25 GHz (at 37 degree Celsius), which shows up as a pronounced increase in the conductivity above 1 GHz, with a corresponding decrease in permittivity [2] Although the frequency for impulse (for example, 10 GHz for 100 ps rise) is far away from 25 GHz, the dispersive loss needs to be considered if the impulse has to travel over a distance.

2 Debye Model of Water and Impulse Propagation

The permittivity of water can be described by a Debye model, in which only one relaxation time constant is taking into account. Taking the data from[3],

$$\varepsilon_r = \frac{\sigma}{j\omega\varepsilon_0} + \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + j\omega t_0} \qquad (1)$$

where ε_s=81, ε_∞=1.8, t_0=11.5 ps, σ is the conductivity, and for deionized water, $\sigma=10^{-5}$ S/m.

Let's assume an impulse is send into a confined structure filled with deionized water, in this case, the dispersive loss is completely determined by the traveling distance. If the input pulse is a Gaussian type,

$\nu\,(t,0)=\exp[-(t-t_0)^2/\xi^2]$, (2)

t_0 gives the peak time, and ξ controls the pulse width /risetime, then in the frequency domain, the pulse can be transformed through Fourier transform, and digitized by FFT (Fast Fourier transformation). The traveling wave can be written as

$FFT[\nu\ (t,0)]\exp(-\gamma(f)\cdot z)$ (3)

Where f is the frequency and $\gamma(f)$ is the propagation factor along z direction.

$\gamma(f)=j2\pi f\sqrt{\varepsilon_r}\ /c$ (4)

c is the speed of light in the vacuum.

The distorted temporal pulse can be retrieved by an inverse Fourier transform over (3).

$\nu(t,z)=iFFT\{\ FFT[\nu\ (t,0)]\exp(-\gamma(f)\cdot z)\}$ (5)

Eq. 5 can be easily solved with Matlab program.

3 Results

3.1 Gaussian pulse

The dispersion of water was calculated for a Gaussian input impulse. Given an example, for a Gaussian input impulse with risetime (10-90%) of 140 ps traveling in a water medium, after the pulse travels 0.2 m and 0.4 m, the peak magnitude decreases to 30% and 20% respectively. In addition, the pulse width broadens (Fig. 2) and the risetime increases to 370 ps and 480 ps.

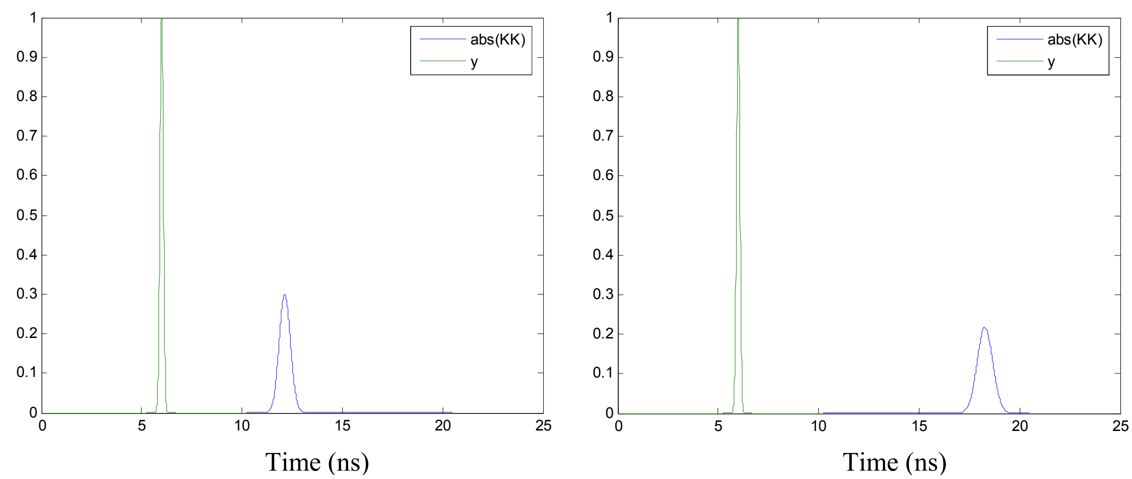


Figure 2. The impulse broadens as it travels over a distance due to the dispersive loss. The impulse is 140 ps risetime (10-90%). Left, 0.2 m, Right, 0.4 m.

One parameter, $t_{mr}=V_{max}/(dV/dt)_{max}$ is often used in the IRA (impulsed radiation antenna). Qualitativly, this can be treated as the shortest possible risetime of a pulse to reach the maximum voltage magnitude.

For a Gaussian pulse :

$$f(t)=\exp(-(t-6ns)^2/0.05ns^2)\quad (6)$$

The t_{mr} was calculated shown in Fig.3.

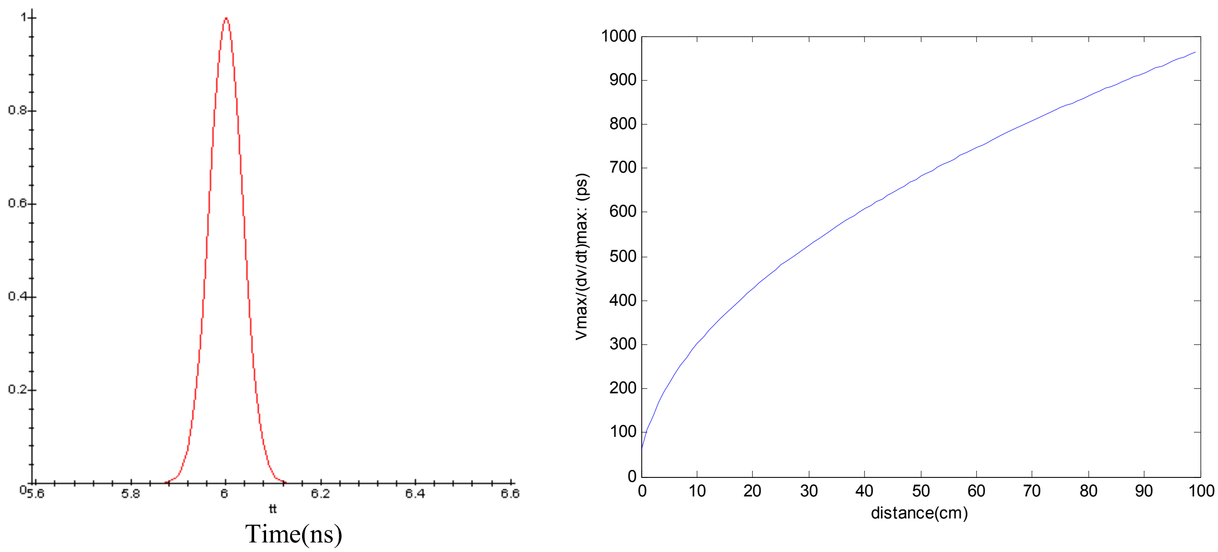


Figure 3. Gaussian impulse and its t_{mr} as impulse travels over 1 m. Left, Gaussian pulse, Right, t_{mr}.

3.2 A step function pulse

Based on the Gaussian pulse results, we can also derive the t_{mr} for a step function impulse. Noticing that a step function is the result of a integral of a Gaussian (in Fig. 4)

$$f_{step} = \int f_{gaus} dt \qquad (7)$$

For $f_{gaus} = \exp[-(t-t_0)^2/\xi^2]$,

$$f_{step} = \frac{\sqrt{\pi}}{2}\xi \operatorname{erf}[(t-t_0)/\xi] \qquad (8)$$

Also, for a step function pulse, after it travels a distance suffering a dispersive loss, the maximum amplitude of the pulse should remain the same because the frequency response of water has a low-pass property (Fig. 5).

To calculate t_{mr} of a step function like (Eq. 8), one can first obtain $V_{max} = \int_{-\infty}^{\infty} f_{gaus} dt$ and then obtain $dV/dt|_{max} = \max(f_{gaus})$,

So

$$t_{mr} = V_{max}/dV/dt|_{max} = \int_{-\infty}^{\infty} f_{gaus} dt / \max(f_{gaus}) \qquad (9)$$

Over a distance, V_{max} remains the same because the low frequency pass property of water dispersion, but $dV/dt|_{max}$ becomes smaller.

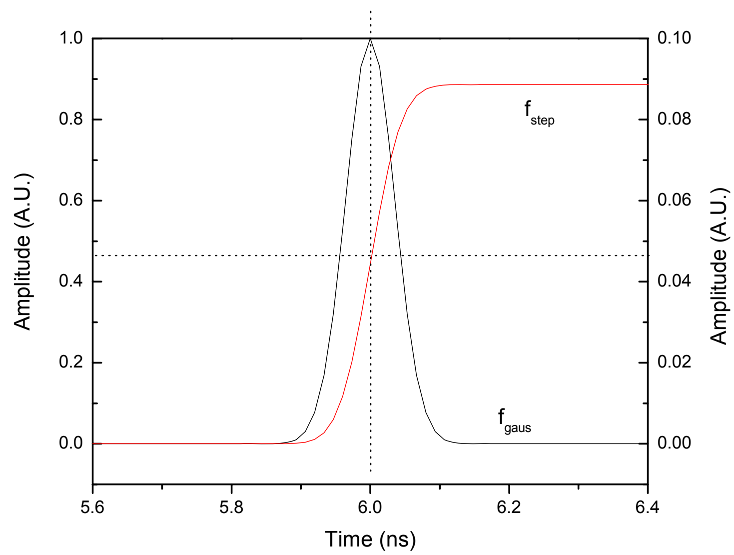


Figure 4. A step function based on the integration of a Gaussian impulse.

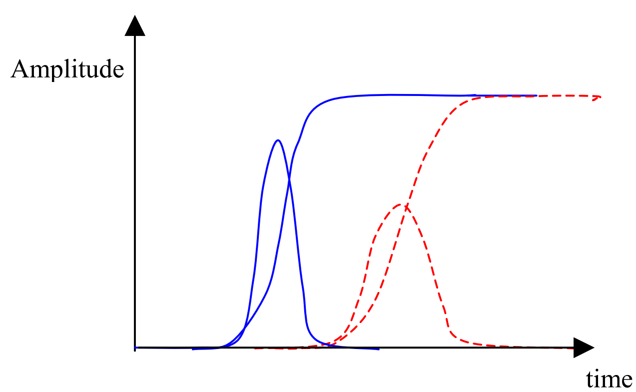


Figure 5. The dispersive loss of a Gaussian and a step pulse over a certain distance. Notice that the step function pulse always reaches its highest magnitude due to the low pass property of water. However, the peak of Gaussian pulse decreases as it travels over a distance. So the t_{mr} for a step function pulse is larger than a Gaussian pulse.

For a step function impulse,

$f_{step}=\frac{\sqrt{\pi}}{2}\xi \mathrm{erf}[(t-t_0)/\xi]$ (10)

Let's set t_0= 6 ns, ξ =0.05 ns, the pulse is shown in Fig. 4.

t_{mr} increases as the step-function impulse propagates, which means the pulse becomes slower in rise time phase.

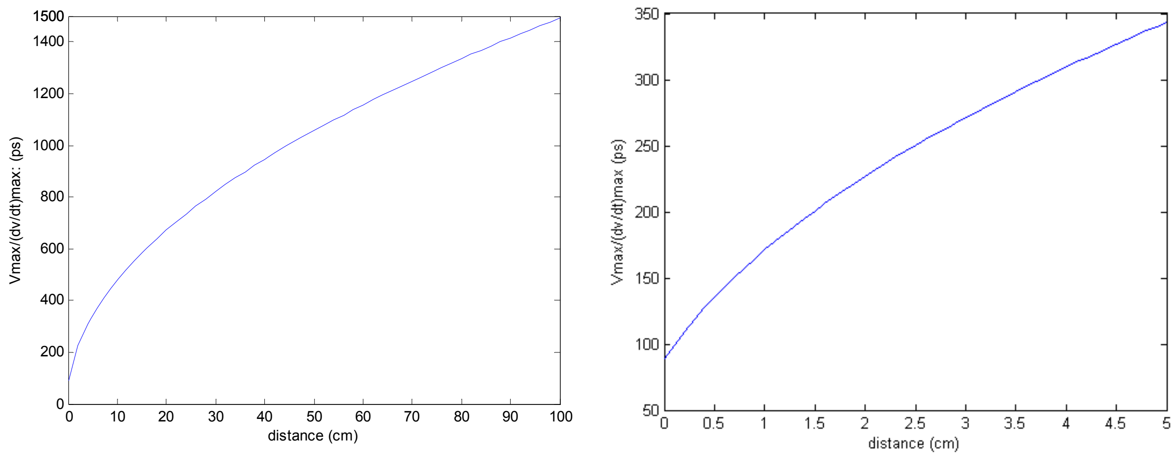
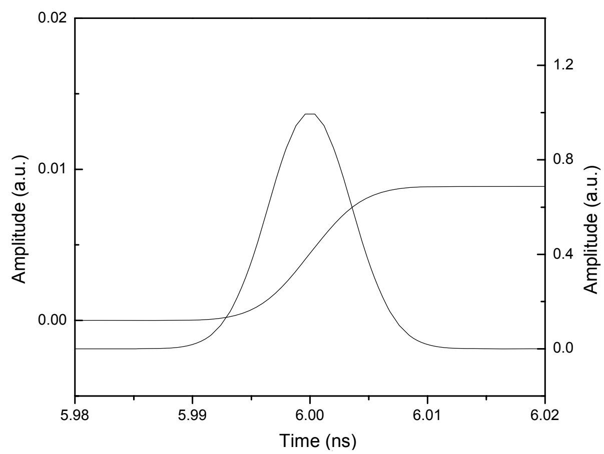


Figure 6. t_{mr} of a step function (Eq. 9) over the distance of 1 m and 5 cm.

Let's construct another pulse with Eq. (10) by setting t_0= 6 ns, ξ =0.005 ns, the pulse is shown in Fig. 7.



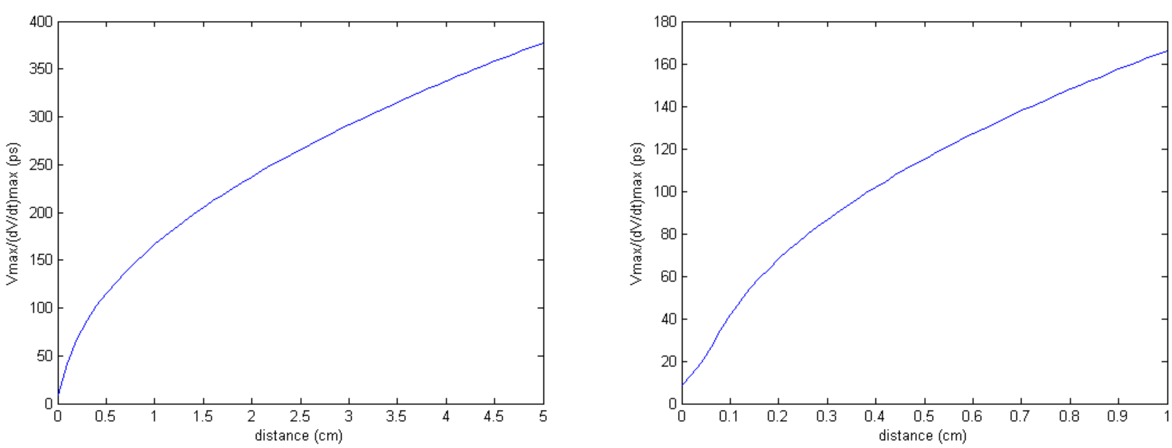


Figure 7. t_{mr} of a step function (Eq. 10) t_0= 6 ns, ξ=0.005 ns over the distance of 5 cm and 1 cm.

4 Conclusion

Whereas using water has some advantages, for example, compressing the wavelength, preventing the breakdown, and possibly enhancing the field with the lens, it is suggested to consider the dielectric dispersive loss. For a Gaussian pulse with 10-90% risetime 140 ps traveling over a distance of 0.2 m and 0.4 m, the peak magnitude decreases to 30% and 20%. The risetime (10-90%) increases to 370 ps and 480 ps respectively. For a step function pulse, t_{mr} can be doubled after propagating a few millimeters.

References

1. C. E. Baum, "Focal Waveform of a Prolate-Spheroidal IRA", Sensor and Simulation Note 509, August 2005.
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3. Measurement Note 52, "Impulse Propagation Measurement of the Dielectric Properties of Water, Dry Sand, Moist Sand and Concrete," E. G. Farr and C. A. Frost, 1997.