

EM Implosion Memos

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Calculating the Optimum Number of Layers for a Lens

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Abstract

In this paper we calculate the optimum number of layers to obtain the required field at the focal point of a prolate-spheroidal IRA based on a plane wave approximation.

## 1 Introduction

N layers of increasing dielectric lens which have the same ratio of dielectric constant consequently are considered for a prolate-spheroidal IRA that is based on [1,2].

## 2 Design Considerations

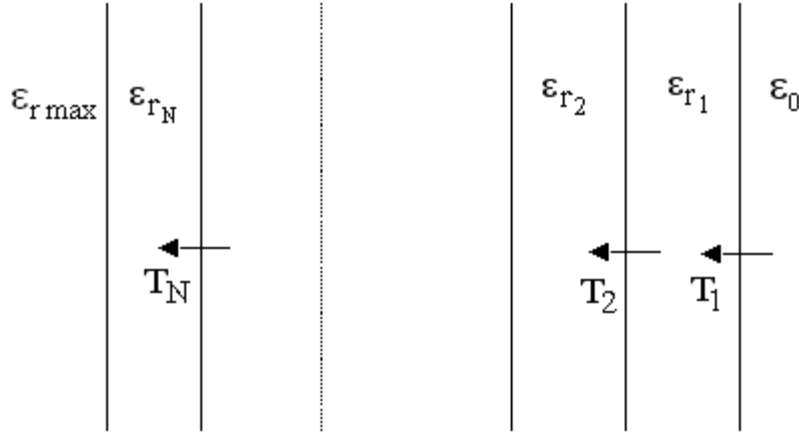


Figure.1 N layers of lens, dielectric constants and transmission coefficients

The total transmission coefficient can be defined as

$$T_{\text{total}} = \prod_{1}^{N} T_n \quad (2.1)$$

Where  $T_n$  is the transmission coefficient between nth and n+1 th layer and it can be defined as

$$T_n = \frac{2Z_{n+1}}{Z_n + Z_{n+1}} = \frac{2 \epsilon_{r_{n+1}}^{-1/2}}{\epsilon_{r_n}^{-1/2} + \epsilon_{r_{n+1}}^{-1/2}} \quad (2.2)$$

The ratio of dielectric constants between consequent layers are constrained to be the same, i.e.

$$\epsilon_{\text{ratio}} = \epsilon_{r_{n+1}} / \epsilon_{r_n} \quad (2.3)$$

For N layers

$$\left( \epsilon_{r_{n+1}} / \epsilon_{r_n} \right)^N = \epsilon_{\text{ratio}} = \epsilon_{r \text{ max}} \quad (2.4)$$

Let's substitute (2.4) in (2.2) so we have

$$T_n = \frac{2 \left( \frac{1}{N} \right)^{-1/2}}{1 + \left( \frac{1}{N} \right)^{-1/2}} \quad (2.5)$$

For N layers from (2.1)

$$T_{\text{total}} = \left( \frac{2 \left( \frac{1}{N} \right)^{-1/2}}{1 + \left( \frac{1}{N} \right)^{-1/2}} \right)^N \quad (2.6)$$

If we have a continuously increasing dielectric lens we have a total transmission coefficient as defined in (5.2) in [2]

$$T_{\text{total}} = \varepsilon_r^{\text{max}^{-1/4}} \quad (2.7)$$

### 3 Conclusion

If we have infinite number of layers, (2.6) approaches (2.7). We should decide how many layers will be acceptable to obtain the closest transmission coefficient to the continuously increasing dielectric lens case.

| $T_{total}$         | $\epsilon_{r \max}$ |              |              |              |              |              |
|---------------------|---------------------|--------------|--------------|--------------|--------------|--------------|
| <b>N</b>            | <b>16</b>           | <b>25</b>    | <b>36</b>    | <b>49</b>    | <b>64</b>    | <b>81</b>    |
| <b>2</b>            | 0.444               | 0.382        | 0.336        | 0.3009       | 0.273        | 0.25         |
| <b>3</b>            | 0.462               | 0.402        | 0.358        | 0.3237       | 0.296        | 0.274        |
| <b>4</b>            | 0.471               | 0.413        | 0.37         | 0.3362       | 0.309        | 0.287        |
| <b>5</b>            | 0.477               | 0.419        | 0.377        | 0.344        | 0.318        | 0.296        |
| <b>10</b>           | 0.488               | 0.433        | 0.392        | 0.3605       | 0.335        | 0.314        |
| <b>20</b>           | 0.494               | 0.44         | 0.4          | 0.3691       | 0.344        | 0.323        |
| <b>40</b>           | 0.497               | 0.444        | 0.404        | 0.3735       | 0.349        | 0.328        |
| <b>50</b>           | 0.498               | 0.444        | 0.405        | 0.3744       | 0.35         | 0.329        |
| <b>100</b>          | 0.499               | 0.446        | 0.407        | 0.3762       | 0.352        | 0.331        |
| $-1/4$              |                     |              |              |              |              |              |
| $\epsilon_{r \max}$ | <b>0.5</b>          | <b>0.447</b> | <b>0.408</b> | <b>0.378</b> | <b>0.354</b> | <b>0.333</b> |

Table1 Transmission coefficients for different N and  $\epsilon_{r \max}$

From Table 1 one can see that If we have 10 layers,  $N=10$ ,  $T_{total}$  approaches close to the continuously increasing dielectric lens case. So 10 layers give acceptable results.

### References

1. S. Altunc and C. E. Baum, "Extension of the Analytic Results for the Focal Waveform of a Two-Arm Prolate-Spheroidal Impulse-Radiating Antenna (IRA)", Sensor and Simulation Note 518, Nov 2006.
2. C. E. Baum, "Addition of a Lens Before the Second Focus of a Prolate-Spheroidal IRA", SSN 512, April 2006.