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Lens Design for Incoming Spherical Wave

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Abstract

In this paper a lens design procedure is discussed to obtain better focusing for a prolate-spheroidal IRA for an incoming spherical wave from the reflector.
1 Introduction

This paper is an extension of [1] and the lens design considerations are based on [2].

N layers of an increasing dielectric lens, which have the same ratio of dielectric constants between adjacent layers, are considered for a prolate-spheroidal IRA. Instead of using a half-spherical lens, a new approach is proposed for incoming spherical waves to obtain better focusing for a prolate-spheroidal IRA.

2 Design Considerations

10 layers of increasing-dielectric-constant lens are used based on the calculations in [1]. We use the same ratio of dielectric constant between subsequent layers.

\[
\varepsilon_{\text{ratio}} = \frac{\varepsilon_{r_{n+1}}}{\varepsilon_{r_n}}, \quad \left(\frac{\varepsilon_{r_{n+1}}}{\varepsilon_{r_n}}\right)^N = \varepsilon_{\text{ratio}} = \varepsilon_{r_{\max}}
\]

We use \(\varepsilon_{r_{\max}} = 81\) for the worst case scenario for biological applications. We start from free space \(\varepsilon_r = 1\) and our target dielectric is \(\varepsilon_{r_{\max}} = 81\) and \(\varepsilon_{\text{ratio}} = 1.55\) between subsequent layers.

Figure 2.1 Lens for Incoming Spherical Wave [2]
\[ \theta_{2\max} = \arctan(b / z_0) \, , \, 0 \leq \theta_2 \leq \theta_{2\max} \] (2.2) 

This represents the range of interest of angles incoming wave from the prolate-spheroidal IRA which has the dimensions as [3]. 

\[ b = \Psi_0 = 0.5 \text{ m}, \, a = 0.625 \text{ m}, \, z_0 = 0.375 \text{ m} \] (2.3) 

From (2.2) and (2.3) for the first shell \( \theta_{2\max} = 53.13^\circ \). Inside the lens the rays are changing their directions to the angle of \( \theta_1 \) with respect to the \( z' \)-axis and \( \theta_{1\max} \leq \pi / 2 \) for geometrical design purposes. \( \ell_1 \) and \( \ell_2 \) are the distances on the \( z' \)-axis, \( h \) is the height of the lens. The normalized \( \ell_1 \) and \( \ell_2 \) parameters can be defined from (5.7) in [2] as 

\[
\ell_1 = \frac{\sin(\theta_{1\max} - \theta_{2\max}) + \varepsilon_r \sin(\theta_{2\max}) - \sin(\theta_{1\max})}{(\varepsilon_r - 1)\sin(\theta_{1\max})\sin(\theta_{2\max})}^{1/2} \\
\ell_2 = \frac{\varepsilon_r[\sin(\theta_{1\max} - \theta_{2\max}) + \sin(\theta_{2\max})] - \sin(\theta_{1\max})}{(\varepsilon_r - 1)\sin(\theta_{1\max})\sin(\theta_{2\max})}^{1/2} 
\] (2.4) 

To find \( \theta_2 \) as a function of \( \theta_1 \) a quadratic equation in either \( \cos(\theta_2) \) or \( \sin(\theta_2) \) can be solved from (5.8-5.10) in [2] as 

\[
\cos(\theta_2) = \frac{AB\sin^2(\theta_1) + \cos(\theta_1) - A\varepsilon_r}{B^2 - 2AB\varepsilon_r \cos(\theta_1) + A\varepsilon_r}^{1/2} \\
A = (\ell_2 / \ell_1) - 1 \, , \, B = (\ell_2 / \ell_1) - \varepsilon_r \\
\sin(\theta_2) = \frac{A(\varepsilon_r - B\cos(\theta_1)) + B\sin(\theta_1)}{A(\varepsilon_r - B\cos(\theta_1)) + B\sin(\theta_1) - A^2 \sin^2(\theta_1)}^{1/2} - \frac{A^2 \sin^2(\theta_1)}{B^2 - 2AB\varepsilon_r \cos(\theta_1) + A\varepsilon_r}^{1/2} 
\] (2.5) 

A lens boundary curve can be defined by the coordinates of \( z' \) and \( \Psi' \) as a function of \( \theta_1 \) and \( \theta_2 \) from (5.11) and (5.12) in [2]. 

\[
\frac{z}{h} = \frac{(\ell_2 - \ell_1) / h \tan(\theta_1)}{\tan(\theta_1) - \tan(\theta_2)} \\
\frac{\Psi}{h} = \frac{z \tan(\theta_2)}{h} = \frac{(\ell_2 - \ell_1) / h \tan(\theta_1) \tan(\theta_2)}{\tan(\theta_1) - \tan(\theta_2)} 
\] (2.6) 

This approach is just for the first shell, but we can expand it to the other shells. \( \varepsilon_r = \varepsilon_{\text{ratio}} = 1.55 \) and we will have different \( \ell_1, \ell_2, \theta_{1\max}, \theta_{2\max} \) for each layer.
We can define a new coordinate system which is centered at \( z = z_0 \). We will call this system \( z' \) and it can be defined as
\[
z'/h = -(z - z_0)/h
\]  
(2.7)

The IRA and lens geometry is presented in Fig. 2.2 and the \( \theta_{1\text{max}} \) and \( \theta_{2\text{max}} \) can be calculated as follows

We use \( N=10 \) layers and \( \Delta \theta \) is the change in the angle as one goes from one to the next. This is constant, and it is
\[
\Delta \theta = (\theta_{1\text{max}_N} - \theta_{2\text{max}_1})/N
\]  
(2.8)

We design the lens for two different \( \theta_{1\text{max}10} \) angles.
\[ \theta_{1_{\text{max}}} = \begin{cases} 90^\circ (\pi/2) \\ 85^\circ \end{cases} \quad (2.9) \]

For the \( \pi/2 \) case \( \Delta \theta = 3.7^\circ \) and for the \( 85^\circ \) case \( \Delta \theta = 3.2^\circ \).

\( \Delta z_n \) / \( h \) is the normalized distance between each layer-beginning point on the \( z \) -axis.
\( \Delta z_n \) / \( h \) is the sum of the \( n \) distances on the \( z \) -axis.

3 Concluding Remarks

We design a lens for incoming spherical waves to obtain better focusing for a prolate-spheroidal IRA. This design is based on the same procedure as in [2]. But in this design just one layer was used. So we extended this design to \( N=10 \) layers. In this case we have different \( \ell_1/h \), \( \ell_2/h \), \( \theta_{1_{\text{max}}} \), \( \theta_{2_{\text{max}}} \), \( h_n/h \), \( z_n/h \). So we calculate these values for the first layer. Then we correct the values for the other layers.

![Graph](image)

Figure 3.1 \( \Psi/h \) vs \( z'/h \) for \( \theta_{1_{\text{max}}} = \pi/2 \)
Table 3.1 $h_n / h$, $\Delta z_n / h$, $z_n / h$, $\theta_{1\text{max}}$, $\theta_{2\text{max}}$ values for $\theta_{1\text{max10}} = \pi / 2$

<table>
<thead>
<tr>
<th>Layer</th>
<th>$h_n / h$</th>
<th>$\Delta z_n / h$</th>
<th>$z_n / h$</th>
<th>$\theta_{1\text{max}}$</th>
<th>$\theta_{2\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>0.096</td>
<td>0.000</td>
<td>0.992</td>
<td>0.927</td>
</tr>
<tr>
<td>2</td>
<td>0.9</td>
<td>0.079</td>
<td>0.096</td>
<td>1.056</td>
<td>0.992</td>
</tr>
<tr>
<td>3</td>
<td>0.8</td>
<td>0.066</td>
<td>0.175</td>
<td>1.123</td>
<td>1.056</td>
</tr>
<tr>
<td>4</td>
<td>0.7</td>
<td>0.054</td>
<td>0.241</td>
<td>1.185</td>
<td>1.120</td>
</tr>
<tr>
<td>5</td>
<td>0.6</td>
<td>0.044</td>
<td>0.295</td>
<td>1.249</td>
<td>1.185</td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
<td>0.035</td>
<td>0.339</td>
<td>1.313</td>
<td>1.249</td>
</tr>
<tr>
<td>7</td>
<td>0.4</td>
<td>0.027</td>
<td>0.374</td>
<td>1.373</td>
<td>1.313</td>
</tr>
<tr>
<td>8</td>
<td>0.3</td>
<td>0.020</td>
<td>0.401</td>
<td>1.442</td>
<td>1.378</td>
</tr>
<tr>
<td>9</td>
<td>0.2</td>
<td>0.013</td>
<td>0.421</td>
<td>1.506</td>
<td>1.442</td>
</tr>
<tr>
<td>10</td>
<td>0.1</td>
<td>0.006</td>
<td>0.434</td>
<td>1.571</td>
<td>1.506</td>
</tr>
</tbody>
</table>

First we calculate the $\Psi / h$ and $z / h$ values for the first layer then for the second layer we calculate $\Psi / h$ and $z / h$ we correct them by dividing $h_{\text{corrected}}$ value then we add the corrected values $z_n / h$ value.

Figure 3.2 $\Psi / h$ and $z / h$ for $\theta_{1\text{max10}} = 85^\circ$
Table 3.1 $\frac{h_n}{h}, \frac{\Delta z_n}{h}, \frac{z_n}{h}, \theta_{1\text{max}}, \theta_{2\text{max}}$ values for $\theta_{1\text{max}10} = 85^\circ$

As one can see from Fig 3.1 and Fig. 3.2, for $\theta_{1\text{max}10} = \frac{\pi}{2}$ case we obtain better focusing but for $\theta_{1\text{max}10} = 85^\circ$ case we can easily focus to the dielectric target regarding geometry.

We call $h$ the radius of the shell, it is a universal normalization parameter. But this calculation is not determining $h$, because it is an optical calculation (infinite frequency). To determine how large $h$ should be is a difficult problem. Clearly $h$ must be much larger than the focus pulse width, and the rise-time of the incoming wave, and it should be smaller than the radius of the reflector.

$$ct_\delta = 3\text{ cm} < h < b = 50\text{ cm}$$ (3.1)

References