EM Implosion Memos

Memo 19 : Addendum

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Additional notes

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1 Equality of electrical width of dielectric layers

Let

$$\sqrt{\frac{\epsilon_{r_{\mathrm{n+1}}}}{\epsilon_{r_{\mathrm{n}}}}} = \sqrt{\frac{\epsilon_{r_{\mathrm{n}}}}{\epsilon_{r_{\mathrm{n\text{-}1}}}}} = \xi. \tag{1.1}$$

Also, $\epsilon_{r_n} \propto r^{-2}$. Consider three layers with radii r_{n-1}, r_n, r_{n+1}

$$r_n\sqrt{\epsilon_{r_n}} = r_{n-1}\sqrt{\epsilon_{r_{n-1}}} \Rightarrow r_{n-1} = r_n\xi, \tag{1.2}$$

$$r_{n+1}\sqrt{\epsilon_{r_{n+1}}} = r_n\sqrt{\epsilon_{r_n}} \Rightarrow r_{n+1} = \frac{r_n}{\xi}. \tag{1.3}$$

The thickness of the shells (r_{n-1}, r_n) and (r_n, r_{n+1}) is

$$r_n - r_{n-1} = r_n - r_n\xi = r_n(1-\xi), \tag{1.4}$$

$$r_{n+1} - r_n = \frac{r_n}{\xi} - r_n = r_n\left(\frac{1-\xi}{\xi}\right). \tag{1.5}$$

The electrical thickness of the layer (r_{n-1}, r_n) is $(r_n - r_{n-1})\sqrt{\epsilon_{r_n}}$. Similarly, the electrical thickness of the layer (r_{n+1}, r_n) is $(r_{n+1} - r_n)\sqrt{\epsilon_{r_{n+1}}}$. Therefore,

$$(r_n - r_{n-1})\sqrt{\epsilon_{r_n}} = r_n(1-\xi)\sqrt{\epsilon_{r_n}}, \tag{1.6}$$

$$(r_{n+1} - r_n)\sqrt{\epsilon_{r_{n+1}}} = r_n\left(\frac{1-\xi}{\xi}\right)\sqrt{\epsilon_{r_{n+1}}} = r_n(1-\xi)\sqrt{\epsilon_{r_n}}. \tag{1.7}$$

i.e., $(r_n - r_{n-1})\sqrt{\epsilon_{r_n}} = (r_{n+1} - r_n)\sqrt{\epsilon_{r_{n+1}}}$. QED.

2 Physical interpretation of ν

The pulse from the source (switch) does not have a single characteristic frequency. Therefore, it is difficult to define a wavelength for such a pulse. A single pulse, however, has a definite "spatial width" in a given medium. The spatial width is the electrical distance occupied by the pulse in the medium (for e.g. a 100 ps pulse has a width of 3 cm in air). The spatial pulse width can therefore be considered equivalent to wavelength of a wave with some characteristic frequency. ν denotes the *number* of such spatial pulse widths.

The transmission line calculations that are used to determine the r's and ϵ_r's for various layers of the focusing lens are based on plane wave approximations. Such approximations do not take into account the curvature of the dielectric (except in a very special sense described in the section below). The accuracy of the transmission line based approximate formulas can be determined by examining the transmission coefficient. One can consider a higher ν as a plane wave approximation and hence approaching the transmission line formulas. A higher ν also implies a larger lens (through the relation for r_0). An optimum ν must therefore be determined for which the transmission coefficient is close to that of the transmission line approximations and the dimensions of the focusing lens are practically acceptable. This is done through numerical simulations.

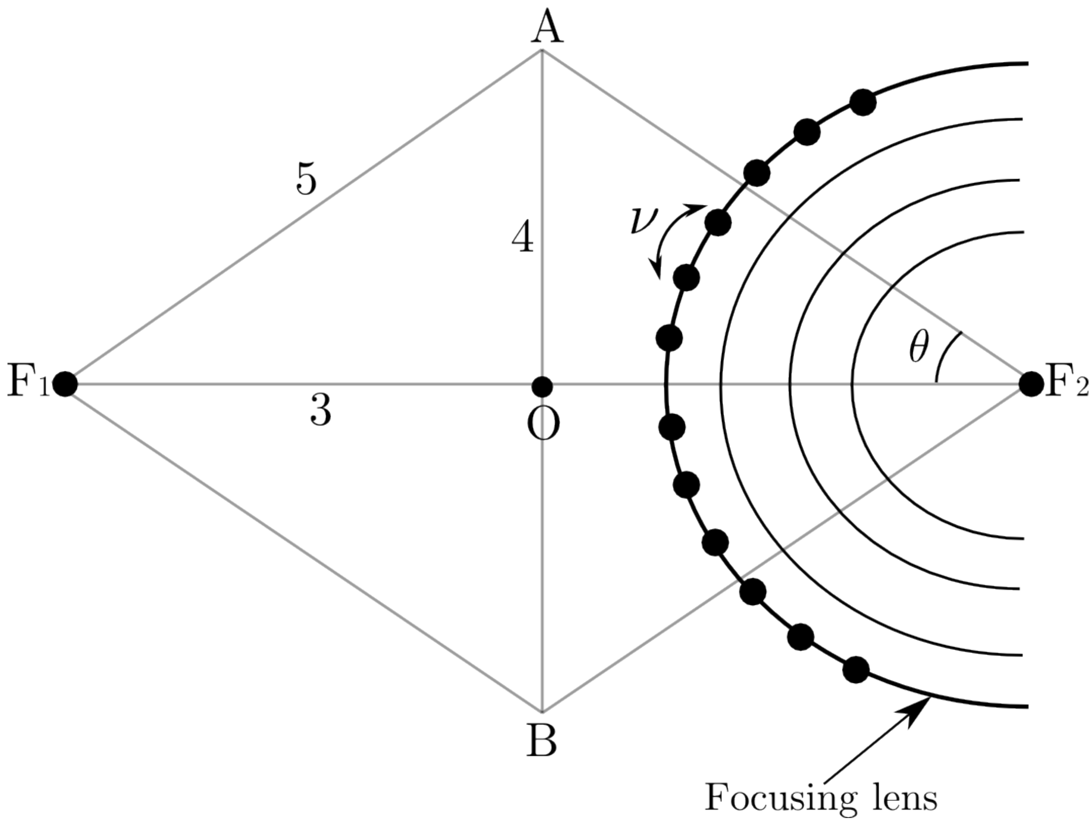

Figure 2.1: Diagram to explain physical meaning of ν. F_1 and F_2 are the first and second focal points respectively.

Curvature of dielectric

Figure 2.1 shows a diagram of the focusing lens system. F_1 and F_2 are the first and second focal points respectively. F_1A and F_1B are the feed arms. The dimensions of the right triangle F_1OA (3,4,5) are as indicated in the figure. Note that, $\triangle F_1OA \cong F_1OB \cong F_2OA \cong F_2OB$. One can therefore imagine a cone (AF_2B) of half-angle $\theta = \arctan(4/3)$ through the focusing lens. This conical region is geometrically identical to the region from which the spherical TEM wave is launched, i.e., AF_1B. Therefore, in this region (AF_2B) of the focusing lens, the sphericity is taken into account by the spheroidal wavefront of the incoming wave. ν determines the number of spatial pulse widths for a single layer as shown in Fig. 2.1. A higher ν implies a higher field amplitude at the focus, but as indicated by simulations, the electric field amplification saturates after about $\nu = 5$. The numerical simulations are necessary to take into account the curvature of the dielectric and validate the transmission line approximations used to calculate the thicknesses and dielectric constants of the various layers of the focusing lens.

3 Droop time, t_d

Consider an exponential transmission line, i.e., the transmission line impedance varies exponentially along the line. Such a transmission line can be considered as a transformer since the output voltage is some multiple of the input voltage. Ideally, an infinitely long, exponential transmission line is desirable. This is because if one considers the discrete equivalent, the reflections between Z_n and Z_{n+1} approach zero as the increment in the transmission line impedance is minimal. This implies that an input pulse suffers no loss in amplitude at the output, i.e., there is no *droop* in the pulse. This is not true if the transmission line is of finite length, for then there are bound to be reflections which will decrease the amplitude of the input pulse. The time taken for the input to droop by a given percentage is called the droop time. One must therefore determine how much droop is practically acceptable for a given length of transmission line.

The above explanation is a *over*-simplified. The impedances and responses of an exponential transmission line are in fact different for low and high frequency inputs and these must be taken into account.