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Lens Design with Constant Wavelength to Cross Section Ratio as ε_r Varies: a Log Periodic Lens

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Abstract

The basic design consideration of a variable ε_r lens with constant wavelength to cross section ratio is discussed.

1 Introduction

Basic design considerations of a dielectric lens concentrating the field on a target are discussed in [1].This paper deals with a half sphere lens design with subsequent layers. The focal point of the prolate-spheroidal IRA is $z_0 = 37.5$ cm and the other parameters are defined in [2,3]. Figure 1 presents the IRA and lens geometry.

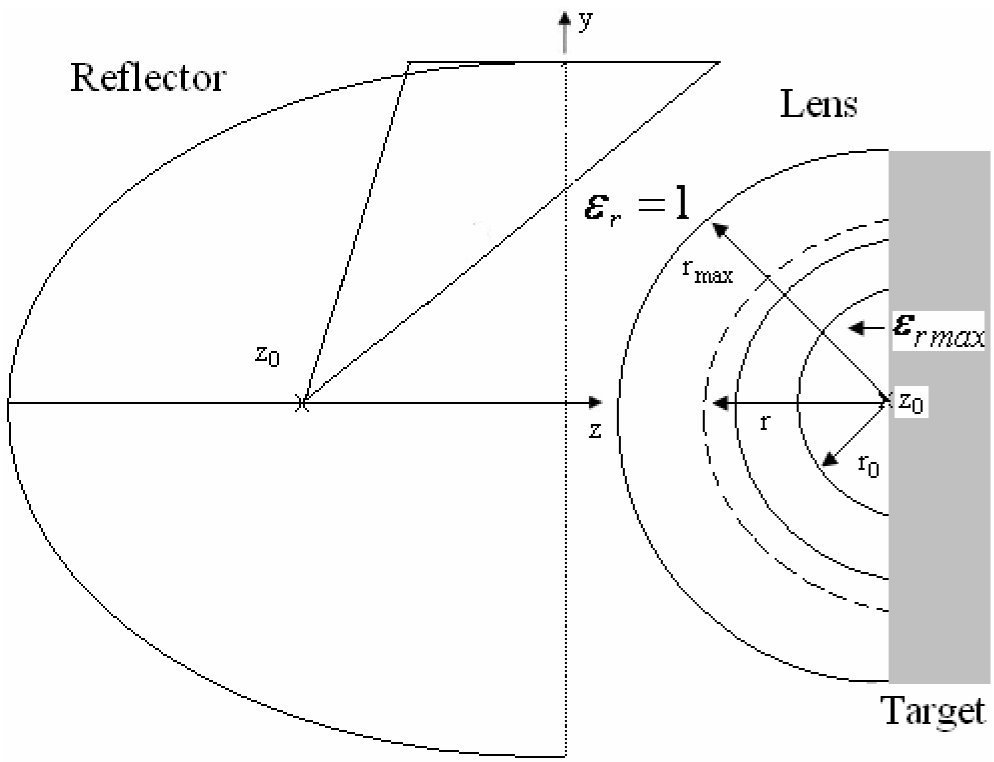

Figure 1 IRA and Lens Geometry

2. Design Considerations

We want to design a lens with constant wavelength to cross section ratio as ε_r varies. Therefore r/λ is not a function of r and

$$\lambda \propto \varepsilon_r^{-1/2} \tag{1}$$

$r\,\varepsilon_r^{-1/2}$ is not a function of r so

$$\varepsilon_r^{1/2} \propto r^{-1} \quad \text{or} \quad \varepsilon_r \propto r^{-2} \;. \tag{2}$$

One can easily define

$$\frac{r_{max}}{r_0} = \varepsilon_r^{1/2} \qquad (3)$$

Equation (3) can be written for an arbitrary point in the lens as

$$\frac{r}{r_0} = \varepsilon_r^{1/2}(r), \qquad (4)$$
$$\varepsilon_r(r) = \left(\frac{r}{r_0}\right)^2 .$$

If we choose r_0 and $\varepsilon_{r\,max}$, the rest can be determined easily.

In terms of pulse width Δt we want

$$\frac{r_o}{v} = \nu \Delta t, \quad \nu > 1, \quad \nu = 3,10,... \qquad (5)$$

ν is proportional to the number pulse widths (or number of wavelengths) in a cross section (spherical surface) and larger ν means we can think of a plane-wave approximation. v is the speed of light in the lens and it is

$$v = \frac{c}{\varepsilon_{r\,max}^{1/2}} . \qquad (6)$$

If we substitute (6) in (5), we will obtain

$$r_0 = \frac{\nu\, c\, \Delta t}{\varepsilon_{r\,max}^{1/2}} . \qquad (7)$$

3. Example

Let us assume the pulse width $\Delta t = 100\,\text{ps}$ and $c\Delta t = 3\,\text{cm}$ as in [3] and substitute this value in (7) and (3) as

$$r_0 = v[1\text{cm}] \qquad \text{for } \varepsilon_{r\max} = 9$$

$$\frac{r_{\max}}{r_0} = \varepsilon_{r\max}^{1/2} = 3 \quad \text{for } \varepsilon_{r\max} = 9 \tag{8}$$

One can note that for $\varepsilon_{r\max} = 81$, r_0 is reduced by 1/3 and r_1 (=r at $\varepsilon_r = 9$)is the same as r_0 previously.

So the lens can be considered to have 2 parts

1) ε_r 1 to 9 as before

2) ε_r 9 to 81 is 1/3 scale of before, fitting into the space left inside the previous r_0.

One can define ε_r in steps with r_n chosen so that

$$\frac{\varepsilon_{r\,n+1}}{\varepsilon_{r\,n}} = \text{constant and} \tag{9}$$

independent of n. One can see that $\frac{r_{n+1}}{r_n}$ is also independent of n therefore we call this lens log periodic lens.

The optimum number of layers was calculated in [4] as N=10 therefore we will have

1)for ε_r 1 to 9, we will have 5 layers and the ratio $9^{1/5} = 1.55$

2)for ε_r 9 to 81, we will have 5 layers and the ratio $\left(\frac{\varepsilon_{r\,max} = 81}{9}\right)^{1/5} = 1.55$

3) for ε_r 1 to 81, we will have 10 layers and the ratio $\varepsilon_{r\,max}{}^{1/N} = 81^{1/10} = 1.55$

One can see for all cases the ratio is same.

v also depends on narrowness of the beam entering lens. Smaller θ_{max} requires larger v_{lens} and this is presented in Figure 2.

Table 1 presents r_0 values for different v values and r, ε_r values for subsequent layers where is $\varepsilon_{r\,max} = 9$.

v	2	3	4	5	10	15	20	
r_0 (meters)	0.02	0.03	0.04	0.05	0.1	0.15	0.2	
r (meters)								ε_r
5th	0.025	0.037	0.050	0.062	0.125	0.187	0.249	9.0
4th	0.031	0.047	0.062	0.078	0.155	0.233	0.310	5.8
3rd	0.039	0.058	0.077	0.097	0.193	0.290	0.387	3.7
2th	0.048	0.072	0.096	0.120	0.241	0.361	0.482	2.4
1th	0.06	0.09	0.12	0.15	0.3	0.45	0.6	1.6

Table 1: r_0 values for different values of v, r and ε_r values for subsequent layers.

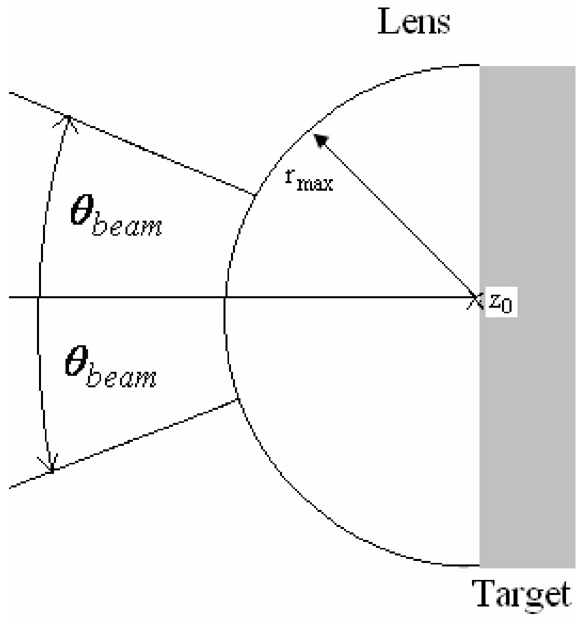

Figure 2. θ_{beam} angle of the lens

One can assume the following approximation

$$r_{max}\theta_{max} = \eta_{lens} c\Delta t \tag{10}$$

As θ_{max} decreases r_{max} must increase and larger θ_{max} requires smaller r_{max} values. Large η_{lens} means better focusing. We can say that the number of wavelengths at the entrance to the lens is maintained approximately the same as the wave propagates into the lens.

4. Conclusion

A lens with constant wavelength to cross section ratio is designed and some simple approximate results are calculated.

References

1. C. E. Baum, "Addition of a Lens Before the Second focus of a Prolate-Spheroidal IRA " Sensor and Simulation Note 512, April 2006.
2. Baum, C.E. (2007), Focal Waveform of a Prolate-Spheroidal Impulse-Radiating Antenna (IRA), Radio Sci. , Vol. 42, RS6S27, doi:1029/2006RS003556.
3. S. Altunc and C. E. Baum, "Extension of the Analytic Results for the Focal Waveform of a Two-Arm Prolate-Spheroidal Impulse-Radiating Antenna (IRA)", Sensor and Simulation Note 518, Nov 2006.
4. S. Altunc and C. E. Baum, "Lens Design for a Prolate Spheroidal IRA", Sensor and Simulation Note 525, Oct 2007.