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Experimental Lens Design Parameters for a Prolate-Spheroidal IRA

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Abstract

The attenuations for different dielectric materials, and the use of two substitute materials instead of using just one material for a dielectric lens design of a prolate-spheroidal IRA are discussed.
1. Introduction

Early papers discussed addition of a dielectric lens before the second focal point of a prolate-spheroidal IRA [1,2]. A log-periodic lens design procedure was discussed in [3] In this paper, we discuss the attenuation (or loss tangent) of a propagating wave through a multiple layer dielectric lens. We also discuss using two substitute materials for a two stage lens layer instead of using just one material. In this case we do not have a log-periodic lens but it may be more practical for fabrication purposes.

2. Dielectric Lens Design Parameters

Figure 1. presents the lens and target geometry.

![Diagram of lens and target geometry](image)

We have a log-periodic dielectric lens which has 10 subsequent layers and the ratio of the dielectric constant between two layers is $5^{1/10} = 1.55$. 
Table 1. shows the radii and $\varepsilon_r$ values for 10 subsequent layers for a log-periodic lens. However, in our case it is hard to go to $\varepsilon_{r\max} = 81$ because of loss and dispersion. Therefore we plan to fabricate a lens with $\varepsilon_{r\max} = 9$ or $\varepsilon_{r\max} = 14$. We plan to fabricate a lens with $\varepsilon_{r\max} = 81$, if possible, in the long term.

2.1 Attenuation (or loss tangent)

One can define the complex propagation constant as

$$\gamma = [s\mu_0(\sigma + se)]^{1/2} = s[\mu_0e]^{1/2}\left[1 + \frac{\sigma}{se}\right]^{1/2},$$

where $s = j\omega$ is the complex frequency, $\mu_0$ is the permeability of the free space, $e$ is the permittivity, and $\sigma$ is the conductivity. Under the assumption of $\frac{\sigma}{se} \ll 1$, one can use binomial expansion to expand (1) as

$$\gamma = s[\mu_0e]^{1/2}\left[1 + \frac{\sigma}{2se} + ...\right]$$

In (2) the most important terms are first and second order terms

$$\gamma \cong s[\mu_0e]^{1/2} + \frac{\sigma[\mu_0e]^{1/2}}{2e} = s[\mu_0e]^{1/2} + \frac{\sigma Z_w}{2}$$

$Z_w = \sqrt{\mu_0/e}$ is the wave impedance. The wave propagation and attenuation are related with

$$e^{-\gamma d} = e^{-s[\mu_0e]^{1/2}} e^{-\sigma Z_w/2}.$$ (4)

$e^{-s[\mu_0e]^{1/2}}$ is the propagation term and the attenuation term $e^{\sigma Z_w d/2}$ can be approximately written as

$$e^{-\sigma Z_w d/2} \cong 1 - \sigma Z_w d / 2$$

for small $\sigma Z_w d / 2$, (5)

$\sigma Z_w d / 2$ is a term that can be used to determine the loss called loss factor,

loss factor $= \sigma Z_w d / 2$. (6)
If $\sigma Z_w d / 2 << 1$, then the loss is insignificant. The loss tangent can be described as

$$\tan(\delta) = \frac{\sigma}{\varepsilon \omega}. \tag{7}$$

The dielectric materials that will be used to fabricate this log-periodic lens have $\tan(\delta) < 10^{-3}$ with $\varepsilon_r = 14$ at 1 GHz. Table 1 and 2 present $\sigma Z_w d / 2$ terms for each layer, these terms are calculated using (6) and under the assumption of $\tan \delta < 10^{-3}$.

Table 1. $\sigma Z_w d / 2$ terms for each layer under the assumption of $\tan(\delta) < 10^{-3}$ for 10 layers at 1 GHz.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$r$ (meters)</th>
<th>$\varepsilon_r$</th>
<th>$\sigma Z_w d / 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>0.150</td>
<td>1.6</td>
<td>3.98E-04</td>
</tr>
<tr>
<td>2nd</td>
<td>0.120</td>
<td>2.4</td>
<td>3.73E-04</td>
</tr>
<tr>
<td>3rd</td>
<td>0.097</td>
<td>3.7</td>
<td>3.83E-04</td>
</tr>
<tr>
<td>4th</td>
<td>0.078</td>
<td>5.8</td>
<td>4.04E-04</td>
</tr>
<tr>
<td>5th</td>
<td>0.062</td>
<td>9.0</td>
<td>3.77E-04</td>
</tr>
<tr>
<td>6th</td>
<td>0.050</td>
<td>14.0</td>
<td>3.92E-04</td>
</tr>
<tr>
<td>7th</td>
<td>0.040</td>
<td>21.7</td>
<td>3.91E-04</td>
</tr>
<tr>
<td>8th</td>
<td>0.032</td>
<td>33.6</td>
<td>3.64E-04</td>
</tr>
<tr>
<td>9th</td>
<td>0.026</td>
<td>52.2</td>
<td>3.79E-04</td>
</tr>
<tr>
<td>10th</td>
<td>0.021</td>
<td>81.0</td>
<td>1.93E-03</td>
</tr>
</tbody>
</table>

$$\sum \sigma Z_w d / 2 = 5.44E-03$$

Table 2. $\sigma Z_w d / 2$ terms for each layer under the assumption of $\tan(\delta) < 10^{-3}$ for 6 layers at 1 GHz.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$r$ (meters)</th>
<th>$\varepsilon_r$</th>
<th>$\sigma Z_w d / 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>0.150</td>
<td>1.6</td>
<td>3.98E-04</td>
</tr>
<tr>
<td>2nd</td>
<td>0.120</td>
<td>2.4</td>
<td>3.73E-04</td>
</tr>
<tr>
<td>3rd</td>
<td>0.097</td>
<td>3.7</td>
<td>3.83E-04</td>
</tr>
<tr>
<td>4th</td>
<td>0.078</td>
<td>5.8</td>
<td>4.04E-04</td>
</tr>
<tr>
<td>5th</td>
<td>0.062</td>
<td>9.0</td>
<td>3.77E-04</td>
</tr>
<tr>
<td>6th</td>
<td>0.050</td>
<td>14.0</td>
<td>3.90E-03</td>
</tr>
</tbody>
</table>

$$\sum \sigma Z_w d / 2 = 3.90E-03$$

2.2 Using Two Substitute Materials Instead of Using Just One Material for a Lens Layer

While fabricating a log-periodic lens, one of the most important things that one should consider is how precise a material one can find that has the exact dielectric constant desired. In our case we’ll have 10% difference for each layer. Even though one finds the exact material, you should consider other parameters as loss and dispersion. Therefore, we come up with a new idea in which we use two substitute materials instead.
of using just one material for a layer of the dielectric lens. In this case we do not have a log-periodic lens but we will be flexible about the materials that will be used and we obtain better focusing because we will have a more-continuous-increasing dielectric lens. Figure 2 shows using two substitution materials instead of using just one material.

![Figure 2 Using two substitution materials instead of using just one material.](image)

The original layer has the thickness of \( d_0 \) and dielectric constant of \( \varepsilon_{\text{avg}} \), the capacitive solution can be written as

\[
d_0 = d_1 + d_2, \\
\varepsilon_1 d_1 + \varepsilon_2 d_2 = \varepsilon_{\text{avg}}. \tag{8}
\]

One can easily solve \( d_1, d_2 \) for the given \( \varepsilon_1, \varepsilon_2, \varepsilon_{\text{avg}} \). One should also provide \( \varepsilon_1 > \varepsilon \) of the previous layer and \( \varepsilon_2 < \varepsilon \) of the next layer. Therefore, from (8) we can substitute two materials for the desired single material.

If we want to obtain same transit time through the lens we can use the following formulation as

\[
t_{\text{total}} = t_1 + t_2 = \frac{d_{\text{avg}} \sqrt{\varepsilon_{\text{avg}}}}{c} = \frac{d_1 \sqrt{\varepsilon_1}}{c} + \frac{d_2 \sqrt{\varepsilon_2}}{c}, \tag{9}
\]

where \( t_{\text{total}} \) is the total propagation time through a layer, \( t_1 \) and \( t_2 \) are the propagation time through the first and second layers, and \( d_{\text{avg}} \) is the thickness of the original layer. However, in our case we’ll use the capacitive solution in (8). One can easily rewrite (9) as

\[
d_1 \sqrt{\varepsilon_1} + d_2 \sqrt{\varepsilon_2} = d_{\text{avg}} \sqrt{\varepsilon_{\text{avg}}} \tag{10}
\]
2.3 Example Case for Capacitive Solution

Let us assume that we cannot find the first layer which has its dielectric constant as $\varepsilon_{\text{avg}} = 1.6$ and the thickness $r_1 - r_2 = 3\text{ cm}$ from Table 2, but we provide two materials which have the dielectric constants as $\varepsilon_1 = 1.3$ and $\varepsilon_2 = 2$. From (8) one can easily calculate the substitute materials thicknesses as

\[
d_0 = d_1 + d_2 = 3\text{ cm}, \\
1.3d_1 + 2(3\text{ cm} - d_1) = 1.6, \\
d_1 = 1.7\text{ cm}, \ d_2 = 1.3\text{ cm}
\]

Conclusion

In this paper, we have calculated the attenuation (or loss tangent) of a propagating wave through a 10 layer dielectric lens for a given loss tangent value $\tan(\delta) < 10^{-3}$. One can see from Table 2 that the total loss is acceptable at 1 GHz. The loss factors (7) can be calculated at 10 GHz and from (6) and (7), they are just 10 times of the loss value of table 2, therefore they are in the acceptable range.

We also discuss using two substitute materials instead of using just one material for a lens layer. In this case, we do not have a log-periodic lens but it may be more practical for fabrication purposes and we will be flexible about the materials that can be used. We obtain better focusing because we shall have a more continuous increasing dielectric lens. While fabricating a log-periodic lens one should consider is how precise a material one can find that has the exact dielectric constant desired. In our case we’ll have 10% differences for each layer. Therefore, we come up with a new idea, i.e., using two materials instead of using just one material for a single layer of a dielectric lens.

References