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Analytical considerations for curve defining boundary of a non-uniform launching lens

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Abstract

This paper outlines analytical considerations for a curve defining the boundary of a non-uniform launching lens. A basic simulation algorithm is outlined. The use of tweaking functions are explained. A few important calculations for the times and dimensions of the launching lens setup have been provided. The lens boundary curve equation serves as a starting point for numerical simulations of the problem.

1 Introduction

The purpose of the launching lens has already been explained in detail in [1]. Within the lens we have a spherical wave centered at the switch center, while outside the lens we have an approximate spherical TEM wave centered at the first focal point of the prolate-spheroidal IRA. The launching lens essentially shifts the source so that the spherical TEM waves outside the lens appear to originate from the first focal point of the ΨRA. The launching lens also prevents dielectric breakdown when high voltages (of the order of a few kV) are applied across the switch. As derived in [2], a minimum $\epsilon_r = 25$ is required to design a uniform launching lens. Such a high dielectric constant is not only impractical but may also lead to problems of loss and dispersion of the EM wave propogating in the lens. Therefore, a non-uniform launching lens which uses a lower dielectric constant is desirable. A non-uniform launching lens typically consists of dielectric layers of varying widths. This multi-layer configuration enables one to use much lower dielectric constants to perform the same function as the unifrom lens. Henceforth "launching lens" will be used to refer to a *non-uniform* launching lens.

There are three factors one must consider when designing a launching lens:

1. Curve defining the boundary of the launching lens

2. Dielectric constants of various layers

3. Width of various layers

Manipulation of the three factors above should, in theory, enable one to obtain a spherical wave outside the lens. This paper deals with the first of the three factors above i.e. deriving an analytical form for the curve defining the boundary of the launching lens.

As mentioned in [2], two possible configurations are considered for design of the launching lens as shown in Fig. 1.1. The configuration in Fig. 1.1(a) will be referred to as the "planar" design and the configuration in Fig. 1.1(b) will be referred to as the "conical" design.

2 General simulation procedure to design a launching lens

The design of the launching lens is a numerical problem. The algorithm for a generic simulation procedure for the design of the lens is given below:

1. Determine the shape of the lens boundary curve based on suitably imposed boundary conditions.

2. Determine the dielectric constants and widths of the various layers.

3. Simulate the problem and measure time of arrival of waves at various points on a sphere outside the lens (measurements are in the near field).

4. Check if time difference between any two waves in previous step is greater than 10 ps. If not, terminate. Desired lens design has been achieved.

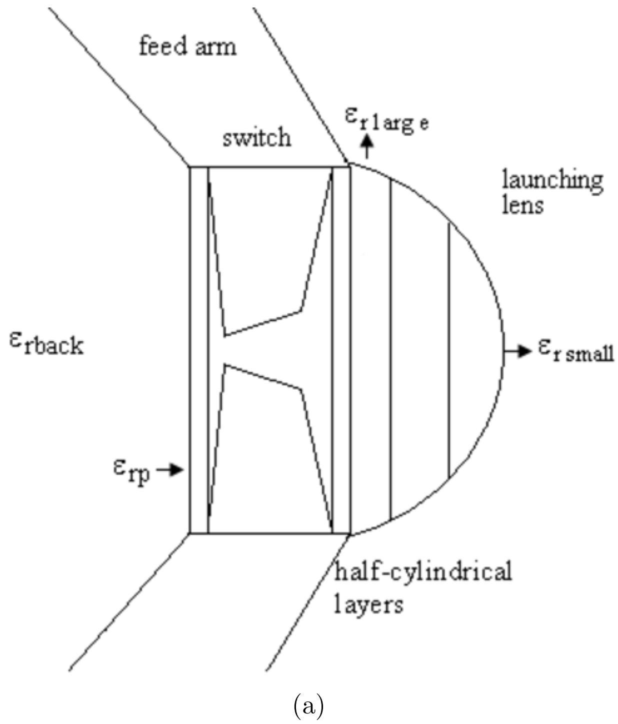

Figure 1.1: Possible configurations of non-uniform launching lens designs

5. If time difference between any two responses is greater than 10 ps, "tweak" the lens boundary curve i.e. increase width in areas to delay waves arriving too early and decrease width in areas where the waves arrive to late. Equivalently, one could also tweak the dielectric constants of the various layers. Goto Step 3.

Note that the maximum tolerable time difference between any two waves approaching the same sphere is taken to be 10 ps in the above algorithm (Step 4). The tolerable time difference is small as we are dealing with responses of the order of 100 ps.

3 Coordinate system and boundary conditions

The diagram for derivation of the lens boundary curve is shown in Fig. 3.1. The important point to note is that there are two coordinate systems, 1) source/switch as origin (r', θ') 2) focal point as origin (r, θ). Either coordinate system may be used to derive the equation of the lens curve. As will be seen, both coordinate systems yield numerically identical results, although analytically, the equations of the curve are slightly different. In our analysis and simulations we have preferred the coordinate system with the switch/source as the origin.

In general, the equation of the curve defining the lens boundary is arbitrary. Therefore we must impose suitable boundary conditions to obtain a starting point. If one considers generalized coordinates ρ, Θ (where ρ, Θ can correspond to r, θ or r', θ' in Fig. 3.1) and,

- $\Theta_{\min} \leq \Theta \leq \Theta_{\max}$,
- $\rho(\Theta_{\min}) = \rho_1$, $\rho(\Theta_{\max}) = \rho_2$.

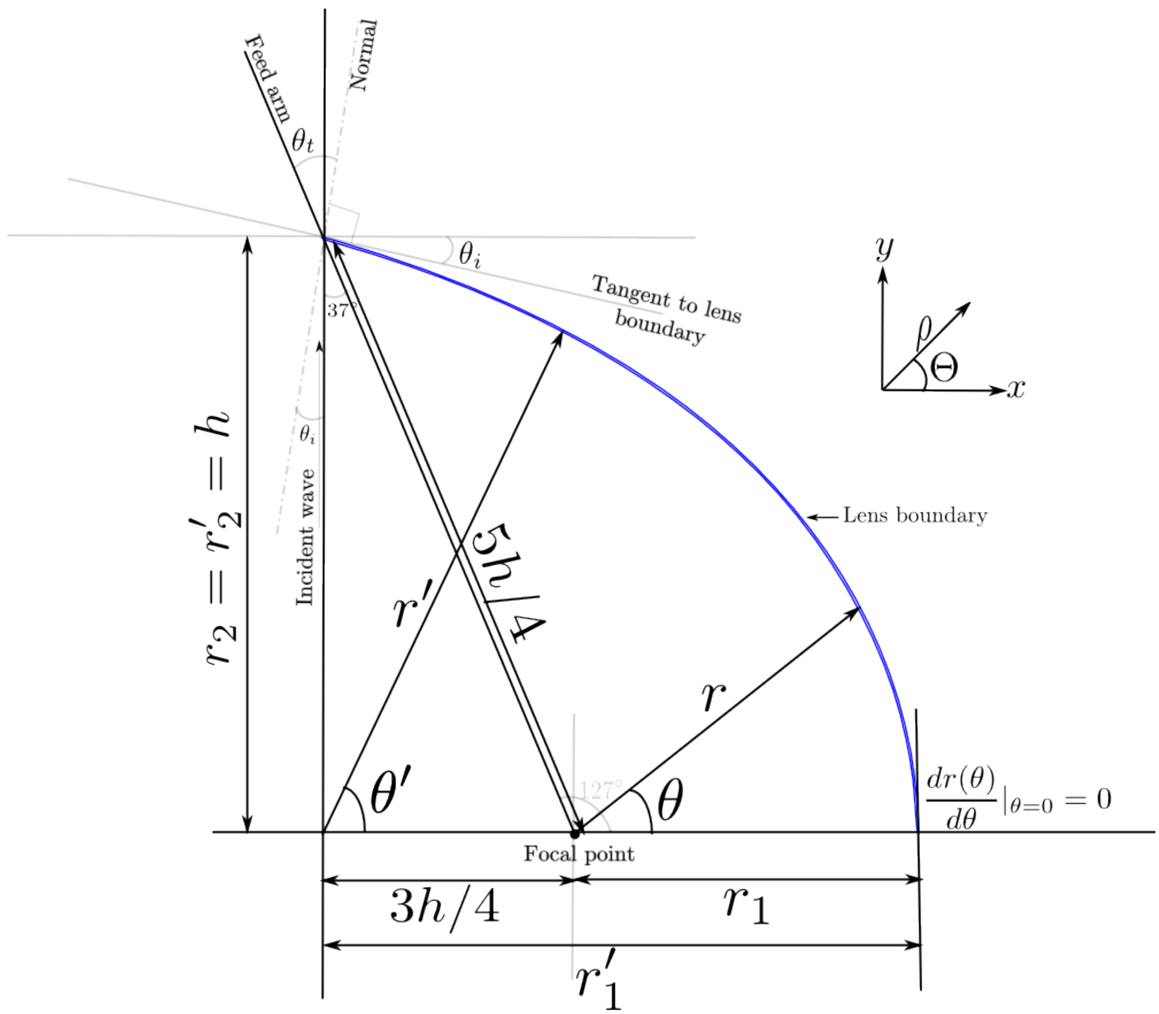

Figure 3.1: Diagram for determination of curve defining lens boundary. r, θ refers to the focal point as reference/origin while r', θ' uses the source/switch as the reference/origin.

Then we have chosen to impose the following boundary conditions

- On the function

$$\rho(\Theta)|_{\Theta=\Theta_{\min}} = \rho_1; \qquad \rho(\Theta)|_{\Theta=\Theta_{\max}} = \rho_2, \tag{3.1}$$

- On the derivatives

$$\frac{d\rho(\Theta)}{d\Theta}|_{\Theta=\Theta_{\min}} = 0; \qquad \frac{d\rho(\Theta)}{d\Theta}|_{\Theta=\Theta_{\max}} = \xi. \tag{3.2}$$

We want the wave along the lens boundary closest to the switch, to be refracted at the feed arm angle at the lens boundary-air interface. ξ represents the slope for which this condition is true. This will result in the wave being refractred toward the reflector. The angle by which the wave is refracted (and hence the slope ξ) depends on the dielectric constant of the layer closest to the switch (Snell's law). We must therefore determine the slope first.

4 Determination of slope for lens boundary at feed arm angle

The geometry of rays and lens is shown in Fig. 4.1. θ_i is the incident angle and θ_t is the transmitted angle. The 37° angle has been previously determined in [1].

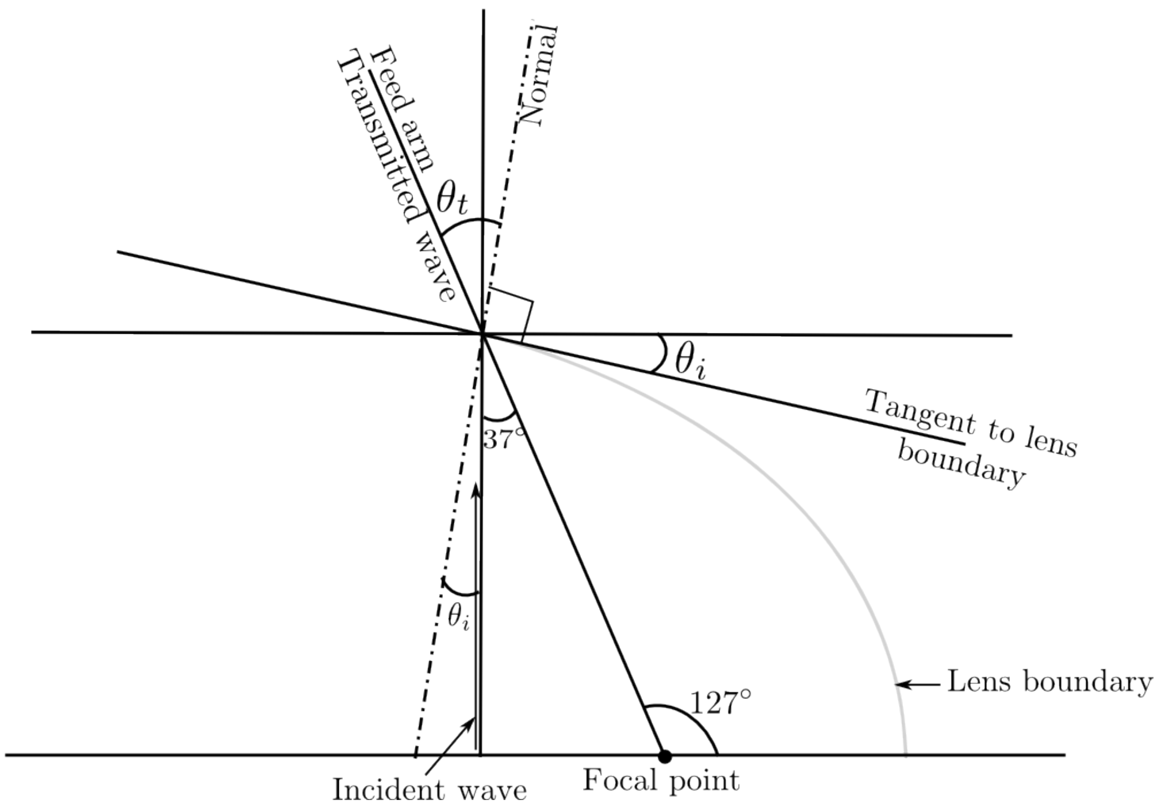

Figure 4.1: Diagram for determination of slope of lens at feed arm angle (127°)

Note from the figure that $\theta_t - \theta_i = 37° = \psi$(say). Applying Snell's law

$$\begin{aligned}
\sqrt{\epsilon_r}\sin\theta_i &= \sin\theta_t \\
\sqrt{\epsilon_r}\sin\theta_i &= \sin(\theta_i + \psi) \\
&= \sin\theta_i\cos\psi + \cos\theta_i\sin\psi \\
\Rightarrow (\sqrt{\epsilon_r} - \cos\psi)\sin\theta_i &= \cos\theta_i\sin\psi \\
\Rightarrow \cot\theta_i &= \frac{\sqrt{\epsilon_r} - \cos\psi}{\sin\psi}
\end{aligned}$$

Therefore the incident angle is given by:

$$\theta_i = \cot^{-1}\left(\frac{\sqrt{\epsilon_r} - \cos\psi}{\sin\psi}\right) \tag{4.1}$$

For $\epsilon_r = 8.903$, $\theta_i = 15.398° \approx 15.4°$. Also note that the slope ξ in polar cordinates is given by $\xi = dr/(rd\theta) = r'(\theta)/r(\theta)$.

5 Derivation of lens boundary curve

Note that the slope has been determined, the lens boundary curve can be derived as follows

1. Generalized coordinates: ρ, Θ.

   - $\Theta_{\text{min}} \leq \Theta \leq \Theta_{\text{max}}$
   - $\rho(\Theta_{\text{min}}) = \rho_1$, $\rho(\Theta_{\text{max}}) = \rho_2$

2. Determine $\rho(\Theta)$ satisfying following boundary conditions

$$\rho(\Theta)|_{\Theta=\Theta_{\text{min}}} = \rho_1; \qquad \rho(\Theta)|_{\Theta=\Theta_{\text{max}}} = \rho_2 \tag{5.1}$$

3. Impose boundary conditions on derivatives of $\rho(\Theta)$ at $\Theta = \Theta_{\text{min}}$ and $\Theta = \Theta_{\text{max}}$. If $\zeta =$*Cartesian slope* (15.4° for ϵ_r=8.903). Then

$$\zeta = \frac{d\rho}{\rho d\Theta} = \frac{\rho'(\Theta)}{\rho(\Theta)} \Rightarrow \frac{d\rho(\Theta)}{d\Theta}|_{\Theta=\Theta_{\text{max}}} = \zeta\rho(\Theta)|_{\Theta=\Theta_{\text{max}}} = \rho_2\zeta = \xi \text{ (say)} \tag{5.2}$$

   The conditions on the derivatives are

$$\frac{d\rho(\Theta)}{d\Theta}|_{\Theta=\Theta_{\text{min}}} = 0; \qquad \frac{d\rho(\Theta)}{d\Theta}|_{\Theta=\Theta_{\text{max}}} = \xi = \rho_2\zeta \tag{5.3}$$

4. **Four** boundary conditions have been imposed. Therefore, we need at least four undetermined constants. Consider the simplest form of $\rho(\Theta)$

$$\rho(\Theta) = a_0 + a_1\Theta + a_2\Theta^2 + a_3\Theta^3 \tag{5.4}$$

5. Solving the above equation for imposed boundary conditions we obtain the constants $a_0 - a_3$ as

$$a_0 = \frac{\rho_1(\Theta_{\text{max}} - 3\Theta_{\text{min}})\Theta_{\text{max}}^2 + \Theta_{\text{min}}^2(\rho_2(3\Theta_{\text{max}} - \Theta_{\text{min}}) + \xi\Theta_{\text{max}}(\Theta_{\text{min}} - \Theta_{\text{max}}))}{(\Theta_{\text{max}} - \Theta_{\text{min}})^3} \tag{5.5}$$

$$a_1 = \frac{\Theta_{\text{min}}\left(-6\rho_1\Theta_{\text{max}} + 6\rho_2\Theta_{\text{max}} + \xi\left(-2\Theta_{\text{max}}^2 + \Theta_{\text{min}}\Theta_{\text{max}} + \Theta_{\text{min}}^2\right)\right)}{(\Theta_{\text{min}} - \Theta_{\text{max}})^3} \tag{5.6}$$

$$a_2 = \frac{-3\rho_1(\Theta_{\text{max}} + \Theta_{\text{min}}) + 3\rho_2(\Theta_{\text{max}} + \Theta_{\text{min}}) - \xi(\Theta_{\text{max}} - \Theta_{\text{min}})(\Theta_{\text{max}} + 2\Theta_{\text{min}})}{(\Theta_{\text{max}} - \Theta_{\text{min}})^3} \tag{5.7}$$

$$a_3 = \frac{2\rho_1 - 2\rho_2 + \xi(\Theta_{\text{max}} - \Theta_{\text{min}})}{(\Theta_{\text{max}} - \Theta_{\text{min}})^3} \tag{5.8}$$

Note : Angles assumed in subsections below are 15.4° for ϵ_r=8.903.

5.1 Source point as reference

Consider the source point as reference/origin, i.e. $\rho = r', \rho_1 = r'_1, \rho_2 = r'_2, \Theta = \theta', \Theta_{\min} = 0, \Theta_{\max} = \pi/2, \xi = -r_2 \tan(15.4°)$ (see Fig. 3.1). For $r_1 = r_2 = h = 10$ cm Fig. 5.1 shows the curve obtained.

One can also consider the simpler function:

$$\rho(\Theta) = a_0 + a_1 \sin \Theta^2 + a_2(1 - \cos \Theta) \tag{5.9}$$

The above function is "simpler" because the coefficients can be obtained by inspection (for the given boundary conditions). This function yields exactly the same curve as the polynomial function.

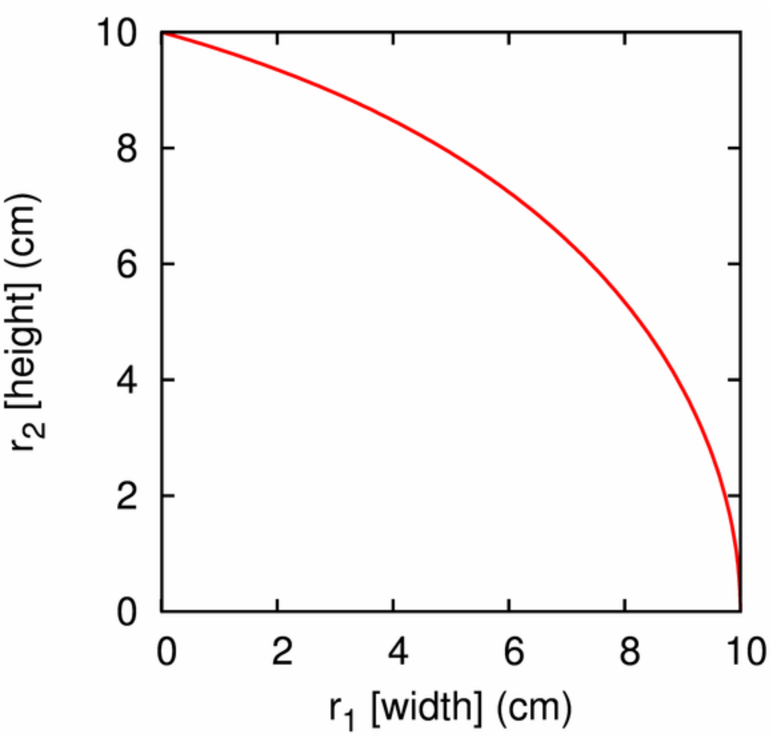

Figure 5.1: Lens boundary curve for function $r'(\theta') = a_0 + a_1\theta' + a_2\theta'^2 + a_3\theta'^3$ and/or $r'(\theta') = a_0 + a_1 \sin \theta'^2 + a_2(1 - \cos \theta')$

5.2 Focal point as reference

One can also consider the *focal* point as reference/origin, i.e. $\rho = r, \rho_1 = r_1, \rho_2 = r_2, \Theta = \theta, \Theta_{\min} = 0, \Theta_{\max} = 127°, \xi = -r_2 \tan(15.4° + 37°)$ (see Fig. 3.1). Equation (5.4) does *not* yield the desired smoothness at small angles. To resolve this, a higher order dependence on θ must be introduced. A fifth order functional dependence of the form $\rho(\Theta) = a_0 + a_1\Theta + a_2\Theta^2 + a_3\Theta^5$ yields a curve closer to that obtained by equation (5.4) with the source as the reference point as observed in Fig. 5.2. Note that in Fig. 5.2, $r_1 = h/4, r_2 = 5h/4$ and $h = 10$ cm for the curves with focal point as the reference. These curves have been shifted (to start from $h = 10$ cm instead of $h = 2.5$ cm) so that they can be compared to the curve obtained with the source as the reference.

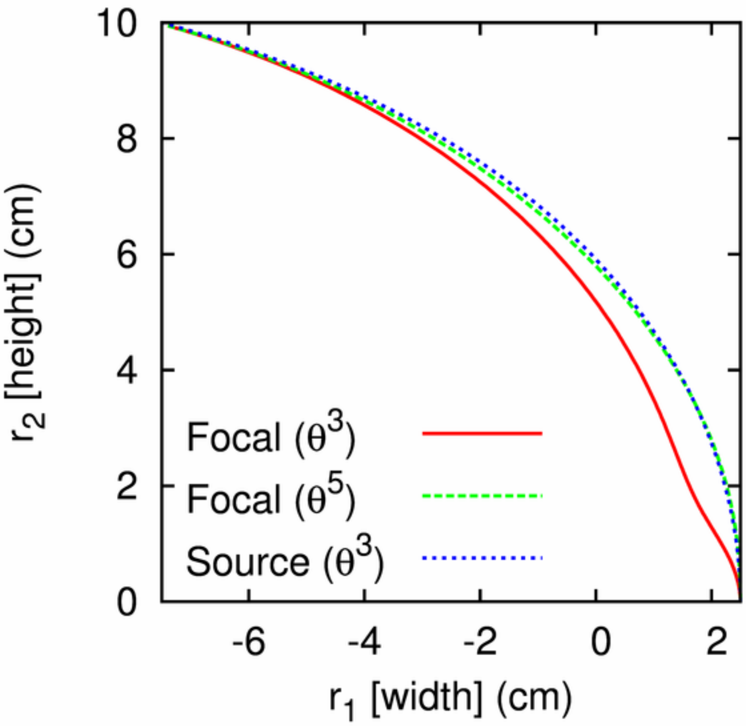

Figure 5.2: Lens boundary curves obtained by using source and focal points as reference. Focal(θ^3) is the function $r(\theta) = a_0 + a_1\theta + a_2\theta^2 + a_3\theta^3$, Focal ($\theta^5$) is the function $r(\theta) = a_0 + a_1\theta + a_2\theta^2 + a_3\theta^5$ while Source (θ^3) is $r'(\theta') = a_0 + a_1\theta' + a_2\theta'^2 + a_3\theta'^3$ using the source as the reference point.

6 Tweaking functions

A tweaking function is a function added to the lens boundary curve ($\rho(\Theta)$) such that it does not change the curve when any of the boundary conditions are applied (i.e. its value is zero for all boundary conditions). Tweaking functions may be used to change the shape of the lens boundary to compensate for the arrival times of various waves on a measurement sphere in simulations. For example, consider the tweaking function of the form $g(\theta') = \mathcal{P} * [1 - \cos(4\theta')]$. Note that in the r', θ' coordinate system $g(\theta') = g'(\theta') = 0$ at $\theta' = 0, \pi/2$. The lens boundary curve would then take the form $r'(\theta') + g(\theta')$. The shape of the lens boundary curve can be changed by manipulating the parameter $\mathcal{P}$ (too large $\mathcal{P}$ value would cause $g(\theta')$ to dominate over $r'(\theta')$). In the most general case, the tweaking functions would take the form of an infinite Fourier series.

7 Calculations of various times and dimensions for launching lens setup

A few important calculations of various times and dimensions for the launching lens setup are shown with respect to Fig. 7.1. These are useful for quick calculations and diagnostics while analyzing simulation results. Although the setup shown in the figure is that for a three layer linear non-uniform case, the calculations are generic and applicable to almost any launching lens design.

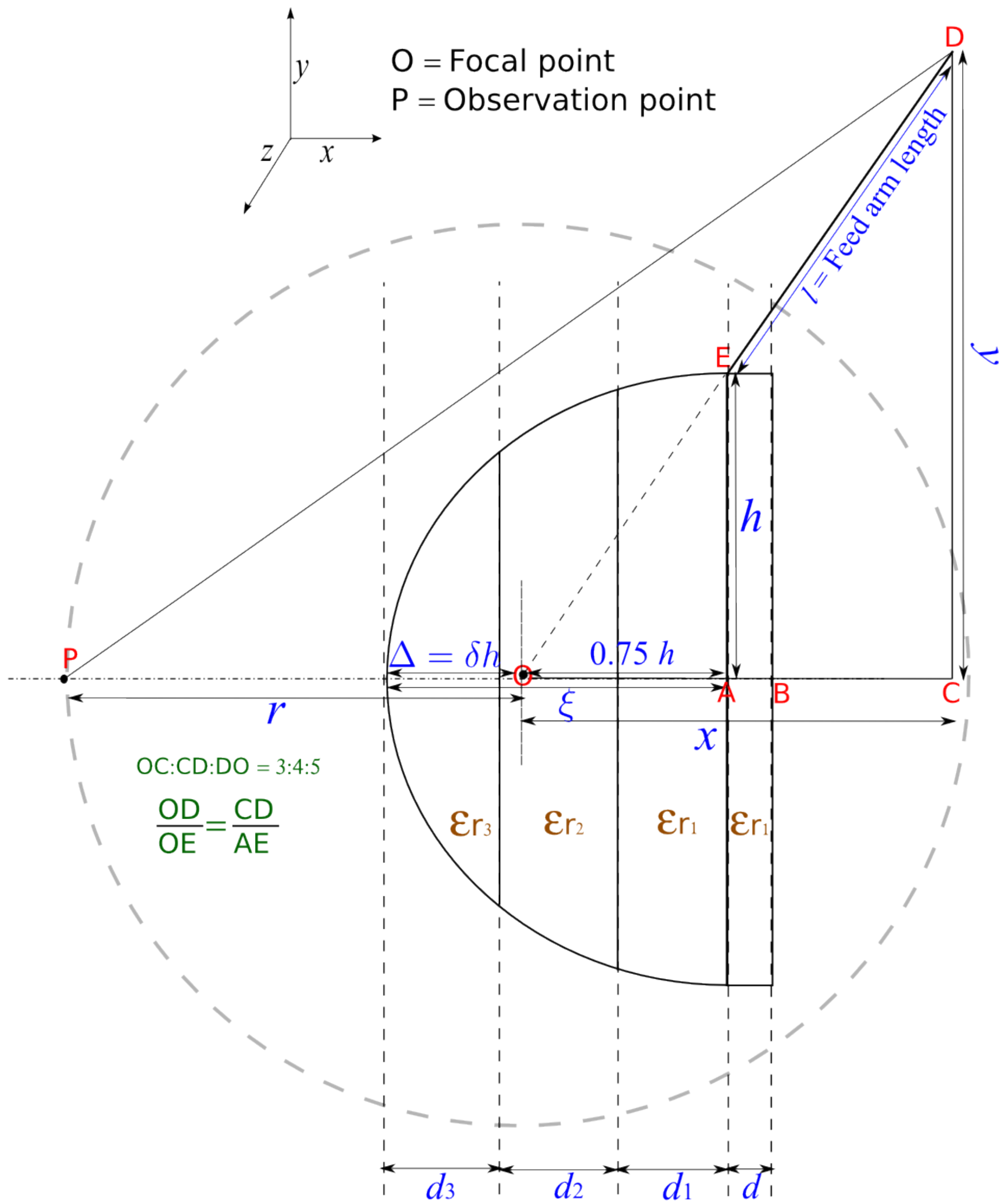

Figure 7.1: Calculations of various dimensions for launching lens

$$\begin{aligned}
h &= \text{height of lens} \\
\xi &= \text{length/width of lens} = 0.75h + \Delta = (0.75+\delta)h \\
x &= OC \\
y &= CD \\
DE &= l = \text{length of Feed arm} \\
OP &= r \\
OA &= 0.75h \\
AB &= \text{dielectric layer behind lens} = d
\end{aligned}$$

Length of feed arm:

$$\begin{aligned}
\triangle OCD \sim \triangle OEA &\Rightarrow \frac{OD}{OE} = \frac{CD}{AE} \Rightarrow OD = OE\frac{CD}{AE} \\
OE = \sqrt{OA^2 + AE^2} &= \sqrt{\left(\frac{3h}{4}\right)^2 + h^2} = \frac{5h}{4} \\
\therefore OD &= \frac{5h}{4} \times \frac{y}{h} = \frac{5y}{4}
\end{aligned}$$

$$\begin{aligned}
\therefore l = DE = \text{length of feed arm} &= OD - OE \\
&= \frac{5}{4}(y - h)
\end{aligned}$$

Important times:

- Time taken for wave to travel from source to observation point (AP) $= d_1\sqrt{\epsilon_{r_1}} + d_2\sqrt{\epsilon_{r_2}} + d_3\sqrt{\epsilon_{r_3}} + (r - \Delta)\sqrt{\epsilon_{r0}}$ (ϵ_{r0}=dielectric constant of surrounding medium, typically air).
- Time taken for wave to travel to dielectric layer behind lens and back i.e. $2AB = 2d = 2d\sqrt{\epsilon_{r_3}}$
- Time taken for wave to travel to feed arm and back i.e. $AE + ED + DE = AE + 2DE = h + 2(5/4)(y - h) = (5y - 3h)/2$
- Time taken for wave to travel from feed arm to observation point i.e. $AE + ED + DP$

$$\begin{aligned}
x^2 &= OD^2 - CD^2 = \left(\frac{5y}{4}\right)^2 - y^2 = \left(\frac{3y}{4}\right)^2 \Rightarrow x = \frac{3y}{4} \\
\therefore DP^2 &= (x + r)^2 + y^2 = \left(\frac{3y}{4} + r\right)^2 + y^2 \Rightarrow DP = \sqrt{\left(\frac{3y}{4} + r\right)^2 + y^2} \\
\Rightarrow AE + ED + DP &= h + \frac{5}{4}(y - h) + \sqrt{\left(\frac{3y}{4} + r\right)^2 + y^2} = \frac{5y - h}{4} + \sqrt{\left(\frac{3y}{4} + r\right)^2 + y^2}
\end{aligned}$$

- Round−trip time taken for wave + reflection in first dielectric layer $= 2d_1\sqrt{\epsilon_{r_1}}$
- Round−trip time taken for wave + reflection in second dielectric layer $= 2d_2\sqrt{\epsilon_{r_2}}$
- Round−trip time taken for wave + reflection in third dielectric layer $= 2d_3\sqrt{\epsilon_{r_3}}$

8 Conclusions and Future Work

An analytical equation for the curve defining the launching lens boundary has been derived by imposing suitable boundary conditions. Although the curve is generic, it serves as a good starting point for simulations. Note that the lens is a body-of-revolution (BOR) and is therefore rotationally symmetric in three dimensions. The use of tweaking functions has been explained and a few handy calculations for times and dimensions for the launching lens setup have been presented.

The next stage is to determine how the lens should be divided into various layers and what the dielectric constants of these layers should be. This would then provide us with all information needed to simulate the problem.

References

[1] Serhat Altunc, Carl E. Baum, Christos G. Christodoulou, Edl Schamiloglu, "Analytical Calculations of a Lens for Launching a Spherical TEM Wave." Sensor and Simulation Note 534, Oct. 2008.

[2] Serhat Altunc, Carl E. Baum, Christos G. Christodoulou, Edl Schamiloglu, "Switch Design for Launching a Spherical TEM Wave." Sensor and Simulation Note 536, Mar. 2009.