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An artificial dielectric consisting of a random array of short, thin wires embedded in a dielectric medium

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Abstract

This paper proposes an artificial dielectric consisting of a random array of short, thin wires ($\epsilon_r \to \infty$) embedded in a relatively low dielectric medium. Calculations for the average ϵ_r, RC time constant and the skin effect are outlined for such a medium.

1 Introduction

A high-ϵ_r artifical dielectric material typically consists of an ensemble of very-high-ϵ_r particles (such as barium titanate) mixed into a low-ϵ_r dielectric (with a high particle density). Even with the preferred axes of polarization (e.g. ϵ_r=3000 in some direction, say z, but relatively lower in the x and y directions) the effective ϵ_r found by averaging over 4π steradians of orientation can be quite high (e.g. 1000).

$$\epsilon_r \equiv \text{relative dielectric constant (compared to free space)} \tag{1.1}$$

2 Average ϵ_r

Since the very high ϵ_{r_2} particles are mixed into an insulation dielectric of low ϵ_{r_1}, the electric field is pushed into the low ϵ_{r_1} medium. The spacing between the particles determines how large an effective ϵ_r can be achieved. A simple model is shown in Fig. 2.1 where d_1 is the spacing between the particles in the ϵ_{r_1} medium and d_2 is the size of the ϵ_{r_2} particles. A one dimensional, series capacitance, approximation of the average ϵ_r is

$$\epsilon_{r_{\text{avg}}} \approx \left[\frac{d_1}{\epsilon_{r_1}} + \frac{d_2}{\epsilon_{r_2}}\right]^{-1} [d_1 + d_2] \tag{2.1}$$

Of course, a more detailed calculation (or experiment) can obtain a more accurate result.

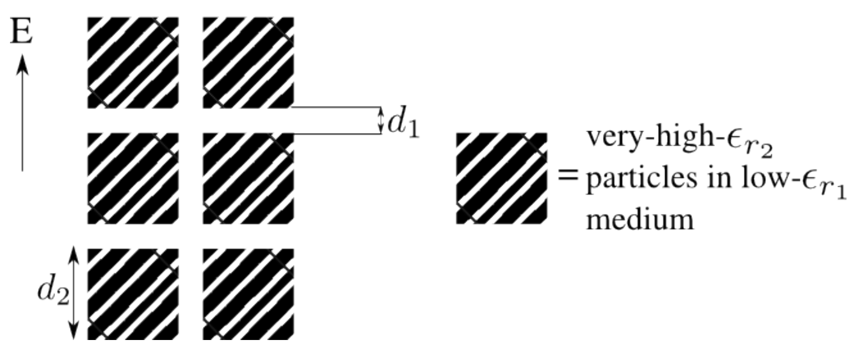


Figure 2.1: Array of very-high-ϵ_{r_2} particles in background ϵ_{r_1} dielectric

The electric field being relatively small in the particles, suppose we let $\epsilon_{r_2} \to \infty$. Then equation (2.1) becomes

$$\epsilon_{r_{\text{avg}}} \approx \epsilon_{r_1} \left[1 + \frac{d_2}{d_1}\right] \tag{2.2}$$

The relative spacing between the particles together with the background ϵ_{r_1} determine $\epsilon_{r_{\text{avg}}}$.

With little electric field in the particles, we might think of replacing these by conducting particles (e.g. copper). However, high conducting media exclude the magnetic field, lowering the effective relative permeability μ_r below 1.0.

3 Conducting spheres as very high ϵ_{r_2} particles

Let

$$\epsilon_{r_2} \gg \epsilon_{r_1}. \tag{3.1}$$

Spherical ϵ_{r_2} particles, of diameter D and number density N, can be related to the background, ϵ_{r_1}, medium as [1]

$$\epsilon_{r_2} = \epsilon_{r_1} \frac{1 + 2h}{1 - h}, \tag{3.2}$$

where

$$h = \frac{\pi D^3 N}{6} \frac{\epsilon_{r_2} - \epsilon_{r_1}}{\epsilon_{r_2} + 2\epsilon_{r_1}}. \tag{3.3}$$

From equation (3.2), for a high ϵ_{r_2}, we need h near 1. For a cube of side a (molecular cubic packing), $a^3 = 1/N$,

$$h \approx \frac{\pi}{6} \left[\frac{D}{a}\right]^2. \tag{3.4}$$

If $D/a \approx 1$ then ϵ_{r_2} is not very large and this is inconsistent with the assumption in (3.1). Not desirable!

A similar inconsistency is encountered when considering disk shaped particles.

4 Thin wires as very high ϵ_{r_2} particles

Instead of spherical or disk particles, let us consider thin wire particles as in Fig. 4.1, with

$$\frac{h}{a} \approx 10 \tag{4.1}$$

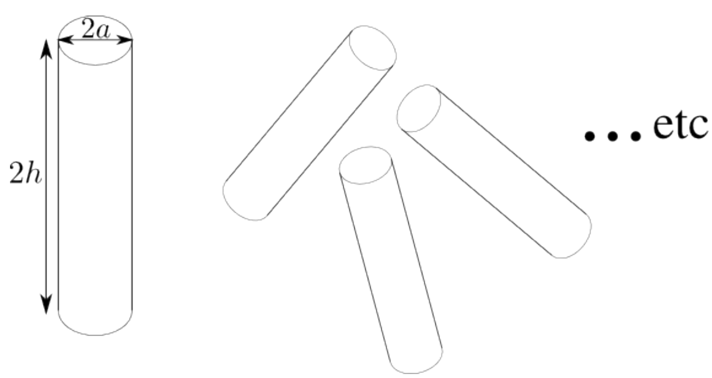


Figure 4.1: Ensemble of short, thin-wire conductors

The length of $2h$, shorts out the electric field, noting the random orientation. The magnetic field is perturbed only over the distance of the order of the radius, a.

For $\epsilon_{r_{\text{avg}}} \approx 9$ we have an average propagation velocity in the medium

$$v \approx \frac{c}{3} \approx 10^8 \text{ m/s}. \tag{4.2}$$

We need to propagate pulses of less than 100 ps. In such a medium the spatial pulse length is

$$l = c\Delta t \approx 1 \text{ cm}. \tag{4.3}$$

So let us consider wires with

$$\begin{aligned} 2h &\leq 1 \text{ mm}, \\ 2a &\leq 0.1 \text{ mm}. \end{aligned} \tag{4.4}$$

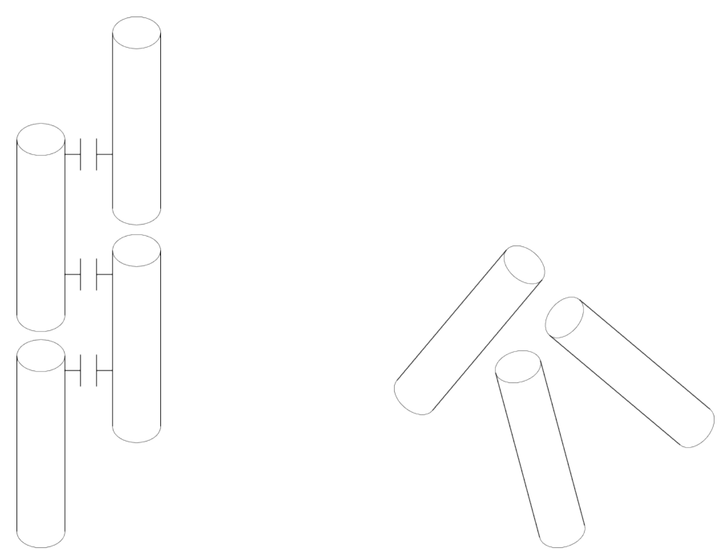
These would be resonant at

$$\begin{aligned} \lambda/2 &= 1 \text{ mm}, \\ f &= \frac{v}{\lambda} \approx 0.5 \times 10^{11} \text{ Hz} = 50 \text{ GHz}, \end{aligned} \tag{4.5}$$

which should be high enough.

Perhaps the wires should be coated with a thin layer of dielectric before mixing them in with the background dielectric. This would avoid conducting contact between the wires. Practically speaking, one might draw a long fine wire, coat it, and then chop it into small lengths. One conducting end will only relatively rarely contact another conducting end.

The overlapping of wires may result in an increased capacitance. An all aligned 3D array is shown in Fig. 4.2(a) where the spacing is of the order of a. With the random orientations shown in Fig. 4.2(b) there is still some enhancement as long as the spacing is $\ll 2h$.



(a) Aligned 3D array

(b) Random 3D array.

Figure 4.2: Aligned and random 3D arrays of thin wires.

5 RC time constant and skin effect

The rise time, t_r, is related with the capitance, C, and impedance, Z, as

$$C = \frac{t_r}{Z}, \quad Z = \frac{1}{\sqrt{\epsilon_r}} \text{ [few hundred] } \Omega. \tag{5.1}$$

One may consider a thin wire particle as a biconical antenna with a source at the center as shown in Fig. 5.1. The resistance of this antenna is given by

$$R = \frac{2}{\sigma} \frac{h}{\pi a^2}. \tag{5.2}$$

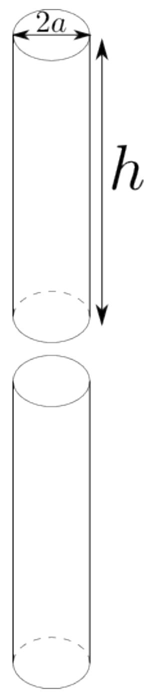


Figure 5.1: Biconical antenna approximation of very-high-ϵ_{r_2} thin wires

The RC time constant is therefore

$$RC = \frac{2}{\sigma} \frac{h}{\pi a^2} \frac{t_r}{Z} = \eta t_r, \tag{5.3}$$

where

$$\eta = \frac{2}{\sigma} \frac{h}{\pi a^2} \frac{1}{Z}, \tag{5.4}$$

and the ratio $h/a \approx 10$ or 100.

We need RC to be small compared to the times of interest, say 10 ps or less. Consider copper wires, $\sigma = 5.8 \times 10^7$ S/m. Let $h/a = 10$, $h = 1$ mm, $a = 0.1$ mm, and $Z = 100\ \Omega$. Then

$$\eta = 1.08 \times 10^{-5} \tag{5.5}$$

$$RC = 1.08 \times 10^{-5} t_r \tag{5.6}$$

Accounting for the skin depth will make the coefficient larger. At 100 GHz, $R_s = 0.83\ \Omega$, i.e.,

$$R = R_s \frac{h}{2\pi a} \approx 0.1405\ \Omega. \tag{5.7}$$

Double this to account for the two halves of a biconical antenna

$$R = 0.28\ \Omega, \tag{5.8}$$

$$C = \frac{t_r}{Z}, \tag{5.9}$$

$$t_r = \frac{h}{v} \approx \frac{0.5 \times 10^{-3}}{10^8} = 5\ \text{ps}, \tag{5.10}$$

$$RC = R\frac{t_r}{Z} \approx 0.28 \frac{5 \times 10^{-12}}{100} = 1.4 \times 10^{-14} = 14\ \text{fs};\ \ \text{excellent, low loss!} \tag{5.11}$$

So skin effect should not be a big problem.

References

[1] Henry Jasik, ed., *Antenna Engineering Handbook*. first ed., 1961. pp. 14-22.