How to become an Influencer in Social Networks

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***Abstract*—Online Social Networks (OSNs) have become part of our everyday life, as a means of interacting with people, sharing content, attending events, etc. Influencers are professional OSN users, who have a large loyal audience and use the OSNs to market various goods or services based on brand partnerships. In this paper, we introduce a novel mechanism to enable OSN users to become influencers by strategically deciding their activity within the OSN. Initially, the concepts of social coordinates and social communities are proposed. Then, a non-cooperative game is introduced to enable the users to make optimal decisions regarding their activity in the OSN in order to become part of an influencers' social community and enjoy the benefit of additional followers. We show that the non-cooperative game is an exact potential game with at least one Nash Equilibrium and we introduce a distributed algorithm to determine its Nash Equilibrium. A detailed set of numerical results, based on real data extracted from Instagram, show the pure operation and performance of the proposed framework, as well as the impact of the social coordinates on the users' decisions. Also, a real-life case study is presented to show the applicability of the proposed framework in Instagram.**

***Index Terms*—Social Networks, Influencers, Social Activity, Game Theory.**

I. Introduction

Over the past two decades, online social networks (OSNs) have revolutionized the way humans interact, exchange information, engage in activities, and in general socialize with each other. Specifically, during the last decade, the concept of influencers has arisen in OSNs, e.g., Facebook, Instagram, and Twitter. The influencer is an individual with a large and loyal audience, characterized by high credibility in a specific marketing domain, and can persuade other users to follow its recommendations [1]. The influencers are recruited by different industrial sectors to participate in advertising campaigns for their products. Except for the profit that the influencers gain out of this process, they also achieve the privilege of social dominance among other OSN users [2]. However, *how can someone become an influencer in online social networks?* This paper tries to address this exact research question by exploiting the user's position in its social graph and introducing a novel game-theoretic approach that enables the user to perform intelligent decision-making regarding its activity in OSNs in order to become an influencer.

A. Related Work

The relationships among the users in an online social network, e.g. Facebook, Instagram, have been traditionally studied by extracting the users social graphs. The Social Network Analysis (SNA) exploits the networking and graph theory in order to extract meaningful information about the users which can be further used for several purposes, e.g., designing advertising campaigns, disaster relief planning, etc. [3]. Nowadays, there are several existing software tools that enable the OSNs researchers to extract the user's social graphs, e.g. Facebook Graph API, Tweepy and NetworkX, Gephi, and others [4]. The nodes of a social graph are the users and the edges among them are defined by their friendship/social connection. The position of the user within the social graph, i.e., the user's *social coordinates*, depends on several factors which are specifically aligned with the structure and characteristics of each OSN. For example, the social characteristics of a Facebook user in an N-dimensional social space are the number of mutual friends, the number of common groups, events, pages, the number of reactions (e.g., like, comments) to a common content (e.g., post, event) and others, with its neighbors. Thus, the user's social coordinates are defined in a relative manner with respect to other users and dynamically shape the social graph. [5].

The concept of *social communities* has recently attracted great interest from the research community, as a means of grouping users with similar behavioral characteristics. The social communities can be further exploited for targeted advertising campaigns, voting campaigns, etc. [6]. A social community prediction framework is proposed in [7] by introducing a convolutional neural network approach that exploits the social graph's local structures, topology, and content. In [8], several social network analysis metrics have been applied to an Instagram-retrieved dataset based on the Gephi tool in order to identify communities and influencers related to a specific brand hashtag. A social-influential friends discovery algorithm is proposed in [9] in order to determine the influence among the social users based on the users' semantic information and the social network's structure. The belief and opinion propagation and evolution in the same and different social communities are studied in [10] following a mean-field game-theoretic approach.

Social users tend to follow the influencers and mimic their behavior in order to ultimately increase the number of their followers, i.e., neighbors in the social graph, and potentially become influencers themselves. Thus, it is common that influencers of different levels (based on the number of their followers) tend to create a social community, in order to mutually benefit from potential additional followers and further improve their social dominance and eventual profit from targeted advertising campaigns.

B. Contributions and Outline

Apparently, even if a large part of the existing literature has studied the characteristics and the structure of social communities, the problem of how a user can become an influencer within an Online Social Network is still unexplored. Aiming to make a first step towards filling this gap, we introduce an autonomous decision-making framework based on the principles of Game Theory to enable the users to make intelligent decisions related to their activity in the OSN in order to ultimately belong in an influential social community and eventually become influencers. The main contributions of this work are summarized as follows:

1) The novel concept of *social coordinates* in OSNs is introduced in order to model the relative positions of the users within the social graph and dynamically shape it based on the user's activity in the OSN. The influential social communities are identified and the social distance of the users is defined.
2) A novel decision-making framework based on the principles of potential games is proposed to enable any user within the OSN to determine its optimal social activity in order to become an influencer. The proposed game-theoretic approach identifies the optimal user's strategies in order to move closer to the influential social communities within the social graph by strategically performing different types of activities, e.g., likes, comments, new friends, etc., within the OSN. The decision-making problem is formulated as a potential game among the users and its Nash Equilibrium, i.e., optimal activity within the OSN, is determined.
3) A detailed set of simulation-based results demonstrate the applicability of the proposed framework in a real-life and real-time implementation within an OSN. Also, a scalability analysis and comparative evaluation show its efficiency and superiority, respectively.

The remainder of the paper is organized as follows. Section II introduces the system model and the concepts of users' social coordinates and influential social communities. The decision-making game-theoretic problem of becoming an influencer is formulated in Section III-A and its Nash Equilibrium is determined in Section III-B. A detailed simulation-based evaluation is performed in Section IV, and Section V concludes the paper.

II. System Model

An OSN is considered, consisting of a set of social communities $C = \{1, \dots, c, \dots, |C|\}$ and a set of users $U = \{1, \dots, u, \dots, |U|\}$. Each social community consists of a number of users $|U_c|$, where U_c denotes their corresponding set. The relationships among the users within an OSN can be captured by the OSN's social graph. The nodes of the OSN social graph are the users and their position $\mathbf{X}_{u,u'} \in \mathbb{R}^N$ in the N-dimensional space is denoted as $\mathbf{X}_{u,u'} = [X^1_{u,u'}, X^2_{u,u'}, \dots, X^n_{u,u'}, \dots, X^N_{u,u'}]$. It is highlighted that a user's u position within the social graph is always relative to the interactions with other users $u', u \neq u'$. Thus, the N-dimensional space and the corresponding social coordinates $\mathbf{X}_{u,u'}$ can represent the users, u, u' interactions, e.g., the number of mutual friends among u and u', the number of common groups that the users u and u' have both joined, the number of common profiles that they follow, the number of reactions (likes, comments) a user u posts on the content uploaded by user u', the number of times a user u shared the content uploaded by user u', etc.

The influencers with common interests, e.g., politics, advertising of clothing or makeup products, tend to create social communities among each other in order to mutually exploit their large and loyal audience, thus, ultimately resulting in higher profit from advertising for them. We consider an indicative social community c of influencers with $U_c = \{1_c, \dots, u_c, \dots, |U_c|\}$, $|U_c| \geq 3$ denoting the corresponding set of influencers. The distance of the edge among two influencers u_c, ${u'}_c$ can be calculated by the Euclidean distance $d_{u_c,u'_c} = |\mathbf{X}_{u_c,{u'}_c} - \mathbf{X}_{{u'}_c,u_c}|$, where in the general case, it holds true that $\mathbf{X}_{u_c,{u'}_c} \neq \mathbf{X}_{{u'}_c,u_c}$. Towards quantifying the level of coherence of the social community, we consider the average of the distances $\bar{d}_c$ among all the influencers belonging to the examined social community c. Thus, a user u, who does not belong in the social community c and aims to mimic the influencers $|U_c|$, e.g., a teenager, a rising professional influencer, should strategically choose its activity in the OSN in order for its social graph's position $\mathbf{X}_{u,u_c}, \forall u_c \in U_c$ to become closer to the social community c. Thus, we define the error distance of a user u not belonging to the social community, from any other influencer $u_c, \forall u_c \in U_c$ belonging in the social community c, as follows:

$$\mathcal{E}(\mathbf{X}_{u,u_c}, \mathbf{X}_{u_c,u}) = [\bar{d}_c - d_{u,u_c}]^2, \forall u_c \in U_c \qquad (1)$$

where $d_{u,u_c} = |\mathbf{X}_{u,u_c} - \mathbf{X}_{u_c,u}|$ is the Euclidean distance among a user u and an influencer u_c. The goal of each user is to strategically perform its activity within the OSN in order to become part of the influencers' social community c, i.e., its position becomes closer to the influencers social community in the social graph.

III. How to become an Influencer

In this section, we formulate the optimization problem for each user in order to become an influencer. Also, in order to solve it, we follow a game-theoretic approach based on the principles of potential games and we determine the user's activity in the OSN that needs to be performed in order to become an influencer.

A. Problem Formulation

The goal of each user u is to become a member of the social community c that has identified of its interest, e.g., a makeup-related social community. Thus, its goal is to achieve a distance from each influencer d_{u,u_c}, $\forall u_c \in U_c$ as closely as possible to the social community's c average distance $\bar{d}_c$. However, it is highlighted that the status/shape of the social graph dynamically changes based on all the users' activity in the OSN. Thus, the goal of each user u that aims at joining the social community is to minimize its error distance from each influencer belonging to the social community c and from the users $u' \neq u, \forall u' \in U$ who strategically try to achieve the same goal, as ultimately all of them will eventually belong in the same social community. For practical purposes, we can identify the set users U, as the ones that have a maximum distance from the centroid of the social community c. The corresponding distributed optimization problem is formulated as follows for each user u:

$$\min_{\substack{\{\mathbf{X}_{u,j}\}\forall j\in U_c\cup U\\ j\neq u}} \sum_{\substack{\forall j\in U_c\cup U\\ j\neq u}} \mathcal{E}(\mathbf{X}_{u,j}, \mathbf{X}_{j,u}) \tag{2}$$

By examining the overall set of users U from a system's oriented perspective, the common goal of all users is to minimize their overall error distance and the corresponding optimization problem is formulated as follows:

$$\min_{\substack{\{\mathbf{X}_{u,j}\}\forall j\in U_c\cup U\\ j\neq u}} \sum_{\forall u\in U} \sum_{\substack{\forall j\in U_c\cup U\\ j\neq u}} \mathcal{E}(\mathbf{X}_{u,j}, \mathbf{X}_{j,u}) \tag{3}$$

B. Problem Solution

Towards addressing the optimization problems (2) and (3) and determining each user's optimal strategy that should follow in terms of its activity in the OSN in order to become an influencer and belong in the social community c, we formulate the non-cooperative game $G = [U, \{S_u\}_{\forall u\in U}, \{E_u\}_{\forall u\in U}]$ among the users belonging in the set of users U. The set $S_u, \forall u \in U$ denotes the user's u strategy set, where $s_u = \mathbf{X}_{u,u_c}, \forall u_c \in U_c$ denotes the user's u strategy, and $E_u(s_u) = \sum_{j\in U_c\cup U} \mathcal{E}(\mathbf{X}_{u,j}, \mathbf{X}_{j,u})$ represents the user's payoff function, where $\mathcal{E}(\mathbf{X}_{u,j}, \mathbf{X}_{j,u}) = 0$, for $j = u$.

In order to determine a stable solution for all the examined users $|U|$, our goal is to determine a Nash Equilibrium for the non-cooperative game G.

Definition 1: **(Nash Equilibrium - NE)** A strategy vector $\mathbf{s}^* = (s_1^*, \dots, s_u^*, \dots, s_{|U|}^*)$ is a Nash Equilibrium for the non-cooperative game G, iff $E_u(s_u^*, \mathbf{s}_{-u}^*) \leq E_u(s_u', \mathbf{s}_{-u}^*), \forall s_u' \in S_u, \forall u \in U$, where $\mathbf{s}_{-u}^* = [s_1^*, \dots, s_{u-1}^*, s_{u+1}^*, \dots, s_{|U|}^*]$.

In the rest of the analysis in this section, we prove that the non-cooperative game G is an exact potential game and admits at least one Nash Equilibrium.

Definition 2: **(Exact Potential Game):** A non-cooperative game $G = [U, \{S_u\}_{\forall u\in U}, \{E_u\}_{\forall u\in U}]$ with payoff function E_u and potential function $\Phi(s_u, \mathbf{s}_{-u})$ is an exact potential game, if it holds true that $\Phi(s_u, \mathbf{s}_{-u}) - \Phi(s_u', \mathbf{s}_{-u}) = E_u(s_u, \mathbf{s}_{-u}) - E_u(s_u', \mathbf{s}_{-u})$, for all the strategies $\forall s_u' \in S_u$, for all the users $\forall u \in U$.

Theorem 1: The non-cooperative game $G = [U, \{S_u\}_{\forall u\in U}, \{E_u\}_{\forall u\in U}]$ is an exact potential game with potential function $\Phi(s_u, \mathbf{s}_{-u}), \forall u \in U$, which is given as follows:

$$\Phi(s_u, \mathbf{s}_{-u}) = \frac{E(s_u, \mathbf{s}_{-u})}{2} \tag{4}$$

where $E(s_u, \mathbf{s}_{-u}) = \sum_{\forall u\in U} \sum_{\forall j\in U_c\cup U} \mathcal{E}(\mathbf{X}_{u,j}, \mathbf{X}_{j,u})$.

Proof: We determine the user's u, $\forall u \in U$ difference of payoff functions for two strategies s_u, s'_u, given that the strategies of the rest of the users $\mathbf{s}_{-u}$ remain the same.

$$E_u(s_u, \mathbf{s}_{-u}) - E_u(s_u', \mathbf{s}_{-u}) = \sum_{\forall j\in U_c\cup U} \mathcal{E}(\mathbf{X}_{u,j}, \mathbf{X}_{j,u}) - \sum_{\forall j\in U_c\cup U} \mathcal{E}(\mathbf{X}'_{u,j}, \mathbf{X}_{j,u})$$

Then, we analyze the potential function as follows.

$$\Phi(s_u, \mathbf{s}_{-u}) = \frac{1}{2} \sum_{\forall u\in U} \sum_{\forall j\in U_c\cup U} \mathcal{E}(\mathbf{X}_{u,j}, \mathbf{X}_{j,u})$$
$$= \frac{1}{2}[\sum_{j\in U_c\cup U} \mathcal{E}(\mathbf{X}_{u,j}, \mathbf{X}_{j,u}) + \sum_{\substack{\forall k\in U\\ k\neq u}} \sum_{\forall j\in U_c\cup U} \mathcal{E}(\mathbf{X}_{k,j}, \mathbf{X}_{j,k})]$$
$$= \frac{1}{2}[\sum_{j\in U_c\cup U} \mathcal{E}(\mathbf{X}_{u,j}, \mathbf{X}_{j,u}) + \sum_{\substack{\forall k\in U\\ k\neq u}} [(\sum_{\substack{\forall j\in U_c\cup U\\ j\neq u}} \mathcal{E}(\mathbf{X}_{k,j}, \mathbf{X}_{j,k})) + \mathcal{E}(\mathbf{X}_{k,u}, \mathbf{X}_{u,k})]]$$
$$= \frac{1}{2}[\sum_{j\in U_c\cup U} \mathcal{E}(\mathbf{X}_{u,j}, \mathbf{X}_{j,u}) + \sum_{\substack{\forall k\in U\\ k\neq u}} \sum_{\substack{\forall j\in U_c\cup U\\ j\neq u}} \mathcal{E}(\mathbf{X}_{k,j}, \mathbf{X}_{j,k}) + \sum_{\substack{\forall k\in U\\ k\neq u}} \mathcal{E}(\mathbf{X}_{k,u}, \mathbf{X}_{u,k})]$$

If two users k, u do not belong in the same "zone" defined by the maximum distance from the centroid of the social community c, then, the error distance is not meaningful for them, thus, $\mathcal{E}(\mathbf{X}_{k,u}, \mathbf{X}_{u,k}) = 0, \forall k, u \notin U_c \cup U$. Thus,

$$\sum_{\substack{\forall k\in U\\ k\neq u}} \mathcal{E}(\mathbf{X}_{k,u}, \mathbf{X}_{u,k}) = \sum_{\forall k\in U_c\cup U} \mathcal{E}(\mathbf{X}_{k,u}, \mathbf{X}_{u,k}) + \underbrace{\sum_{\substack{\forall k\notin U_c\cup U\\ k\neq u}} \mathcal{E}(\mathbf{X}_{k,u}, \mathbf{X}_{u,k})}_{=0} = \sum_{\forall k\in U_c\cup U} \mathcal{E}(\mathbf{X}_{k,u}, \mathbf{X}_{u,k})).$$

Thus, the potential function can be rewritten as follows.

$$\Phi(s_u, \mathbf{s}_{-u}) = \frac{1}{2}[\sum_{j\in U_c\cup U} \mathcal{E}(\mathbf{X}_{u,j}, \mathbf{X}_{j,u}) + \sum_{\substack{\forall k\in U\\ k\neq u}} \sum_{\substack{\forall j\in U_c\cup U\\ j\neq u}} \mathcal{E}(\mathbf{X}_{k,j}, \mathbf{X}_{j,k}) + \sum_{\forall k\in U_c\cup U} \mathcal{E}(\mathbf{X}_{k,u}, \mathbf{X}_{u,k})]$$
$$= \frac{1}{2}[2\sum_{j\in U_c\cup U} \mathcal{E}(\mathbf{X}_{u,j}, \mathbf{X}_{j,u}) + \sum_{\substack{\forall k\in U\\ k\neq u}} \sum_{\substack{\forall j\in U_c\cup U\\ j\neq u}} \mathcal{E}(\mathbf{X}_{k,j}, \mathbf{X}_{j,k})]$$
$$= \sum_{j\in U_c\cup U} \mathcal{E}(\mathbf{X}_{u,j}, \mathbf{X}_{j,u}) + \frac{1}{2} \sum_{\substack{\forall k\in U\\ k\neq u}} \sum_{\substack{\forall j\in U_c\cup U\\ j\neq u}} \mathcal{E}(\mathbf{X}_{k,j}, \mathbf{X}_{j,k}).$$

Then, we determine the difference of the potential function for two different strategies s_u, s_u' of user u, given the strategies of the rest of the users remain the same, as follows.

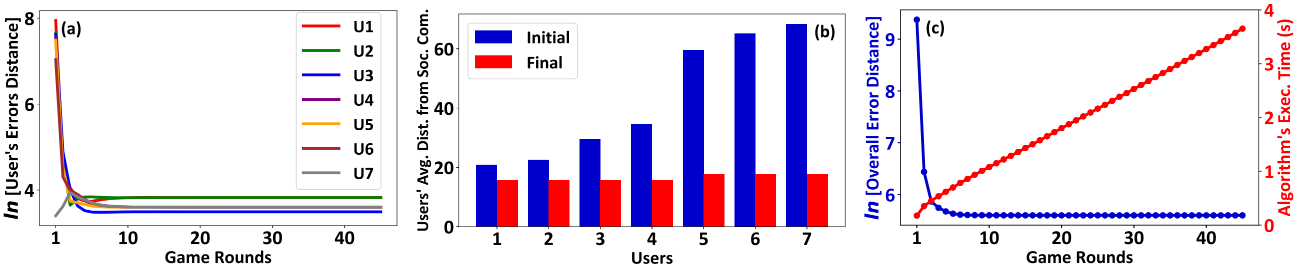

Fig. 1: Pure operation and performance.

$$
\begin{aligned}
&\Phi(s_u, \mathbf{s}_u) - \Phi(s'_u, \mathbf{s}_{-u}) = \\
&\sum_{j \in U_c \cup U} \mathcal{E}(\mathbf{X}_{u,j}, \mathbf{X}_{j,u}) + \frac{1}{2} \sum_{\substack{\forall k \in U \\ k \neq u}} \sum_{\substack{j \in U_c \cup U \\ j \neq u}} \mathcal{E}(\mathbf{X}_{k,j}, \mathbf{X}_{j,k}) \\
&- [\sum_{j \in U_c \cup U} \mathcal{E}(\mathbf{X}'_{u,j}, \mathbf{X}_{j,u}) + \frac{1}{2} \sum_{\substack{\forall k \in U \\ k \neq u}} \sum_{\substack{j \in U_c \cup U \\ j \neq u}} \mathcal{E}(\mathbf{X}_{k,j}, \mathbf{X}_{j,k}) \\
&= \sum_{j \in U_c \cup U} \mathcal{E}(\mathbf{X}_{u,j}, \mathbf{X}_{j,u}) - \sum_{j \in U_c \cup U} \mathcal{E}(\mathbf{X}'_{u,j}, \mathbf{X}_{j,u}) \\
&= E_u(s_u, \mathbf{s}_{-u}) - E_u(s'_u, \mathbf{s}_{-u})
\end{aligned}
$$

Thus, the non-cooperative game $G = [U, \{S_u\}_{\forall u \in U}, \{E_u\}_{\forall u \in U}]$ is an exact potential game and admits at least one Nash Equilibrium [11]. ■

In the following analysis, we introduce a low-complexity distributed algorithm in order to determine the Nash Equilibrium and ultimately, the optimal activity of each user within the OSN in order to become an influencer and join its targeted social community.

Algorithm 1 Social Activity Algorithm

Input: $\mathbf{X}_{u,j}$, $\forall u \in U, \forall j \in U_c \cup U$
Output: $\mathbf{s}^*$
Initialization: $ite = 0$, $Convergence = 0$, $\mathbf{s}|_{ite=0}$ based on a randomly selected initial strategy.
while $Convergence == 0$ **do**
  $ite = ite + 1$;
  A randomly selected user $u \in U_c \cup U$ determines its optimal strategy $\mathbf{s}^*_u|_{ite}$ based on Eq. (2) and determines $E_u(s^*_u|_{ite}, \mathbf{s}_{-u}|_{ite})$
  if $|E_u(s^*_u|_{ite}, \mathbf{s}^*_{-u}|_{ite}) - E_u(s^*_u|_{ite-1}, \mathbf{s}^*_{-u}|_{ite-1})| \leq \delta$, with δ a small positive value, $\forall u \in U_c \cup U$, **then**
    $Convergence = 1$
  end if
end while

IV. Numerical Results

In this section, a detailed simulation-based analysis is provided in order to capture the operational characteristics and the efficiency of the proposed model to enable users in OSNs to become influencers. We have used an open source tool, OSINTGRAM [12], to extract real data regarding the user's social activity and define their social coordinates within the social graph. Specifically, the following social coordinates have been used in the rest of the analysis: (i) number of common profiles that two users follow, ($X^1_{u,u'}$), (ii) number of common businesses that two users follow($X^2_{u,u'}$), (iii) number of times a user tags another common profile or business with another user, ($X^3_{u,u'}$), (iv) number of times a user tags simultaneously all the influencers $|U_c|$ from the social community ($X^4_{u,u'}$), and (v) the number of comments made by a user on the content of any influencer from the social community ($X^5_{u,u'}$).

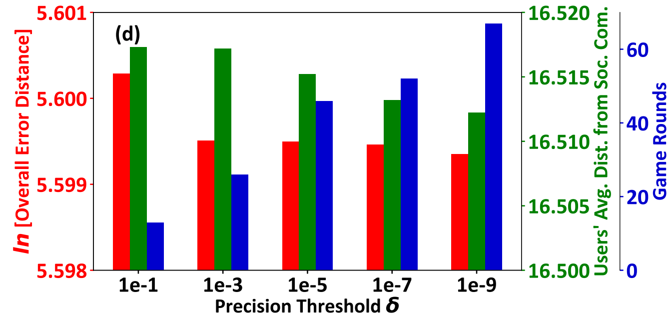

Fig. 2: Operational characteristics of the Social Activity Algorithm.

In Section IV-A, we present the pure operation and the performance of the proposed model considering the Instagram OSN as an indicative use case scenario. The impact of the different number of social coordinates is discussed in Section IV-B, and a real-life case study with real Instagram profiles is presented in Section IV-C.

A. Pure Operation and Performance

In this section, we present the pure performance and the operation of the proposed model in order to enable the users in an OSN to become influencers. Fig. 1a shows the convergence of the error distance of each examined user ($|U| = 7$) as a function of the non-cooperative game rounds. Fig. 1b presents the initial and the final average distance after the game has converged to the Nash Equilibrium, from the set of influencers ($|U_c| = 3$). Fig. 1c demonstrates the overall examined system's error distance and the execution time of the proposed algorithm as a function of the non-cooperative game's rounds. Fig. 2 illustrates the overall error distance, the number of rounds of the non-cooperative game, and the users' average distance from the influencers' social community as a function of the precision threshold δ that has been used as the convergence criterion of the proposed Social Activity Algorithm.

The results show that the users' error distance converges to low values during the execution of the non-cooperative

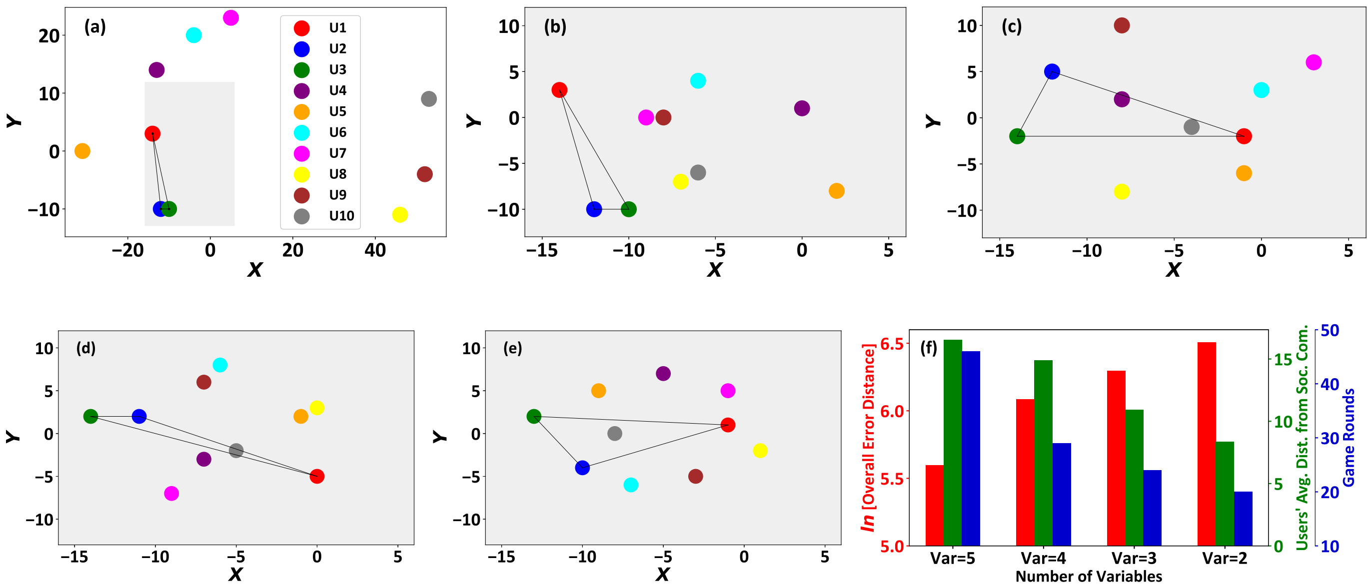

Fig. 3: Impact of the number of social coordinates on the users decisions regarding their activity in the OSN.

game (Fig. 1a), enabling the users to make optimal decisions regarding their social activity in the OSN in order to become influencers. The latter outcome is observed in Fig. 1b, where the users' final average distance from the social community approaches the corresponding value of the average distance among the influencers, i.e., $\bar{d}_c = 16.52$. Also, by studying the examined OSN from a system's perspective, we observe that the overall error distance decreases as a function of the non-cooperative game rounds (Fig. 1c), while the execution time of the proposed algorithm remains low, i.e., few seconds, enabling the users to make real-time decisions. Furthermore, the results show that as the convergence precision threshold δ decreases, the overall error distance and the users' average distance from the influencers' social community decreases, as the proposed algorithm enables the users to more accurately select their strategies. Thus, the number of rounds that are needed for the non-cooperative in order to converge increases (Fig. 2).

B. Impact of Social Coordinates

In this section, we present the impact of the number of social coordinates that are considered in the user's decisions in order to become influencers. Fig. 3a – Fig. 3e show the social graph for $|U_c| = 3$ and $|U| = 7$, considering the initial topology (Fig. 3a), and the final topology of the Nash Equilibrium considering $5-2$ social coordinates in Fig. 3b – Fig. 3e, respectively. Also, Fig. 3f demonstrates the overall error distance, the number of game rounds, and the users' average distance from the influencers social community at the Nash Equilibrium, as a function of the number of social coordinates. It is noted that the Principal Component Analysis (PCA) has been used in (Fig. 3b – Fig. 3e) to perform the dimensionality reduction and present the social graph in the two-dimensional subspace [13].

The results show that as the number of social coordinates decreases, the users have less degrees of freedom to determine their optimal position in the social graph in order to become influencers. Thus, the social graph is dynamically moving towards a non-influential status, which is observed by the progressive movement of the social graph away from the original positions of the influencers, which is presented in the grey area of Fig. 3a. Therefore, as the number of social coordinates decreases, the overall error distance increases (Fig. 3f), as the users final positions result in a distance far away from the average distance of the influencers initial positions in the social graph. Also, the results show that as the number of social coordinates decreases, the users' average distance from the social community decreases. However, the latter phenomenon is observed because the users' social activity has dynamically dragged the influencers away from the influencers' initial influencing positions in the social graph (Fig. 3a). Thus, the outcome is non-beneficial neither for the users nor for the influencers. Moreover, we observe that as the number of social coordinates decreases, the rounds of the non-cooperative game in order to determine the Nash Equilibrium decrease (Fig. 3f), as the users' free decision variables decrease.

C. Real-life Case Study

In this section, we have performed a real-life case study, by considering three real Instagram profiles, i.e., @keithhabs, @korndiddy, @eugeneleeyang, who construct the influencers' social community, and we extracted their social graphs using the open source tool Osintgram which is an OSINT (Open Source Intelligence) tool for Instagram [12]. Table I shows the social coordinates of the influencers, considering the five social coordinates, as they have been described at the beginning of Section IV. Then, we consider three indicative users of Instagram who intend to become influencers, mimicking the three influencers' profiles that we examined, and ultimately join the influencers' social community. Table II shows the initial social coordinates of the three indicative Instagram users, while Table III presents the corresponding social coordinates of the users at the Nash Equilibrium. The latter information can be used by the user in order to identify the number of common profiles (X^1_{u,u_c}) and businesses (X^2_{u,u_c}) with the influencers that need to follow, the number of times the user needs to tag a common profile or business among the influencers (X^3_{u,u_c}), the number of times the user needs to tag the influencers (X^4_{u,u_c}), and the number of comments that need to make on the influencers' posted content (X^5_{u,u_c}). Moreover, Fig. 3a-3b graphically present the initial social coordinates of the influencers and the users, and their final positions at the Nash Equilibrium point, respectively. The results show that the strategic decision-making of the users enabled them to join the social community of the influencers.

TABLE I: Social coordinates of the influencers.

Influencer	X^1_{u,u_c}	X^2_{u,u_c}	X^3_{u,u_c}	X^4_{u,u_c}	X^5_{u,u_c}
@keithhabs	78	7	312	4	12
@korndiddy	78	7	251	4	64
@eugeneleeyang	78	7	419	4	84

TABLE II: Initial social coordinates of the users.

User	X^1_{u,u_c}	X^2_{u,u_c}	X^3_{u,u_c}	X^4_{u,u_c}	X^5_{u,u_c}
@user1	31	4	207	2	28
@user2	23	6	193	1	33
@user3	43	5	186	2	49

TABLE III: Final social coordinates of the users.

User	X^1_{u,u_c}	X^2_{u,u_c}	X^3_{u,u_c}	X^4_{u,u_c}	X^5_{u,u_c}
@user1	40	4	331	83	106
@user2	28	92	336	1	82
@user3	140	65	332	43	97

V. Conclusion

In this paper, a novel approach, enabling the users to make optimal decisions regarding their activity in the Online Social Networks (OSNs) in order to become influencers, is introduced. The concept of social coordinates is proposed and the users' distributed decision-making, in order to become part of an influencers' social community, is formulated as a non-cooperative game. A detailed theoretical analysis is provided in order to show that the formulated non-cooperative game is a potential game and admits at least one Nash Equilibrium. Also, a distributed low-complexity algorithm is introduced in order to determine the Nash Equilibrium. A detailed set of numerical results using real data from Instagram is provided in order to show the pure performance and efficiency of the proposed framework. Part of our current and future work is to extend our study by developing an IOS and Android-based application to provide real-time feedback to the users of an OSN (initially tested for Instagram) to enable them to make real-time decisions regarding their online social activity in order to ultimately become influencers.

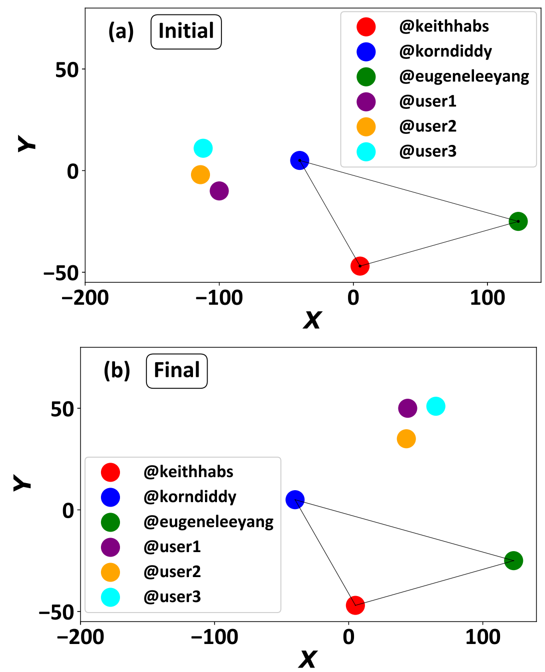

Fig. 4: Case study.

References

[1] L. Jin, Y. Chen, T. Wang, P. Hui, and A. V. Vasilakos, "Understanding user behavior in online social networks: A survey," *IEEE communications magazine*, vol. 51, no. 9, pp. 144–150, 2013.

[2] Z. Tan, F. Li, and D. Wu, "Mrainf: Multilayer relation attention based social influence prediction net with local stimulation," in *2021 IEEE Global Communications Conference (GLOBECOM)*. IEEE, 2021, pp. 01–06.

[3] S. Bhagat, G. Cormode, and S. Muthukrishnan, "Node classification in social networks," in *Social network data analytics*. Springer, 2011, pp. 115–148.

[4] S. Forouzandeh, A. Sheikhahmadi, A. R. Aghdam, and S. Xu, "New centrality measure for nodes based on user social status and behavior on facebook," *International Journal of Web Information Systems*, 2018.

[5] J. Leskovec, A. Rajaraman, and J. D. Ullman, "Mining social-network graphs," *Mining of Massive Datasets*, pp. 340–393, 2014.

[6] P. Kumar, R. Jain, S. Chaudhary, and S. Kumar, "Solving community detection in social networks: A comprehensive study," in *2021 5th International Conference on Computing Methodologies and Communication (ICCMC)*. IEEE, 2021, pp. 239–345.

[7] L. Zhang, X. Zhu, and J. Ma, "Joint connection and content embedding for link prediction in social networks," in *2021 IEEE Global Communications Conference (GLOBECOM)*. IEEE, 2021, pp. 1–6.

[8] M. Kamalzadeh and A. T. Haghighat, "Applying the approach based on several social network analysis metrics to identify influential users of a brand," in *2021 Eighth International Conference on Social Network Analysis, Management and Security (SNAMS)*, 2021, pp. 01–08.

[9] X. Pan, R. Hu, and D. Li, "Social-ifd: Personalized influential friends discovery based on semantics in lbsn," in *ICC 2020 - 2020 IEEE International Conference on Communications (ICC)*, 2020, pp. 1–6.

[10] R. A. Banez, H. Gao, L. Li, C. Yang, Z. Han, and H. V. Poor, "Belief and opinion evolution in social networks based on a multi-population mean field game approach," in *ICC 2020 - 2020 IEEE International Conference on Communications (ICC)*, 2020, pp. 1–6.

[11] D. Monderer and L. S. Shapley, "Potential games," *Games and economic behavior*, vol. 14, no. 1, pp. 124–143, 1996.

[12] N. A. Hassan and R. Hijazi, *Open Source Intelligence Methods and Tools*. Springer, 2018.

[13] H. Abdi and L. J. Williams, "Principal component analysis," *Wiley interdisciplinary reviews: computational statistics*, vol. 2, no. 4, pp. 433–459, 2010.