Redesigning Resource Management in Wireless Networks based on Games in Satisfaction Form

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***Abstract*—The rise in popularity of smartphones, along with the need for personalized services with different Quality of Service (QoS) requirements, has created an increased interest for energy-efficient resource management frameworks in wireless networks, where user actions and decisions are interdependent. Our focus is placed on the transformation and treatment of the uplink power control problem under the perspective of game theory in satisfaction form. The novel concept of Minimum Efficient Satisfaction Equilibrium (MESE) is introduced and its properties are investigated. Considering that each user is associated with a cost function with respect to its actions, the MESE point defines each user's transmission power that satisfies its QoS prerequisites with the lowest cost. We prove that at the MESE point the system achieves the lowest possible cumulative cost, while each user individually is penalized with the minimum cost compared to the corresponding cost of any Efficient Satisfaction Equilibrium (ESE) point. The existence, uniqueness and benefits of the MESE are studied, and a distributed low complexity algorithm based on the Best Response Dynamics that converges to the MESE point is proposed. Through modeling and simulation, the performance of the proposed novel resource management framework is evaluated, and its benefits are revealed.**

***Index Terms*—Satisfaction equilibrium, energy efficiency, game theory, power control, utility function, resource management**

I. Introduction

THE volume of the data traffic and the number of connected devices are continuously increasing, while this trend is expected to further intensify with the advent of 5G networks and the evolution of Internet of Things (IoT) [1].

Given the aforementioned setting and responding to the need for distributed solutions for resource management purposes, *Game Theory* arises as a natural choice and a powerful tool to cope with users' selfish and competitive behavior regarding the resource orchestration process within the emerging 5G networks. Accordingly various resource management problems in wireless networks have been considered in the recent literature, based on the concept of *Expected Utility Maximization* and game theory (e.g. [2]–[4]). In those approaches, the non-cooperative game theory is adopted to formulate the resource management problems and their solutions conclude to Nash equilibrium points, which are stable operational points for all the users in the network.

However, it is well-known that the Nash equilibrium points stemming from users' selfish decision-making are generally inefficient. A first step to guide the selfish users to a more efficient operating point was the introduction of pricing mechanisms, where the users are penalized with respect to their resources' consumption [5]. Still those approaches could not address the main disadvantage of the Nash equilibrium points in a holistic manner. In particular, customized heuristic pricing mechanisms are required each time to treat different resource types and networking environments. Furthermore, even when pricing is considered, each user still aims at maximizing his own perceived Quality of Service (QoS). Thus the realization of the aforementioned maximization goal does not offer a notifying difference to the experienced satisfaction.

Therefore, the unprecedented need of rethinking the resource orchestration process arises in 5G wireless networks to address the QoS demands of the significantly increasing number of users. Towards this direction, a new concept of equilibrium is introduced, *Satisfaction Equilibrium*, where the users aim to satisfy their minimum QoS prerequisites instead of targeting at QoS maximization [6], [7]. In [8] the definition of the Satisfaction Equilibrium (SE) and the general conditions for examining its existence have been discussed in detail. Also, the concept of users' effort to achieve the SE has been introduced, leading to a refinement of the SE, namely the Efficient SE (ESE). At the ESE point, all the users conclude to a resource allocation strategy, which requires the lowest effort to satisfy their minimum QoS prerequisites. In [9] and [10], the concepts of SE and ESE are applied in a simplified uplink power control problem considering interference channels in a single-cell environment. Additionally, reinforcement learning has been introduced in [11] to determine the SEs and ESEs under different conditions. However, many interesting properties that emerge when the satisfaction equilibrium framework is applied to this setting have not been revealed yet [12]. Our work aims at filling this gap, and focuses on the transformation and treatment of the uplink power control problem under the perspective of game theory in satisfaction form.

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A. Contributions and Outline

Specifically, we study in detail the satisfaction equilibrium points for QoS provisioning in wireless networks, where user actions and decisions are interdependent. This is achieved via examining the uplink power control problem for a general set of users' realistic utility functions, which are increasing with respect to the user's uplink transmission power and decreasing with respect to the intracell interference. A representative example (but not limited to this one) is the Shannon's formula. The novel concept of *Minimum Efficient Satisfaction Equilibrium (MESE)* is introduced, which is shown to be of special interest among the satisfaction equilibrium points that have already been proposed in the literature, i.e., Satisfaction Equilibrium (SE) and Efficient Satisfaction Equilibrium (ESE).

Assuming that each user is associated with a cost function of arbitrary form with respect to its actions, at the MESE point each user transmits at a power level that satisfies its QoS prerequisites with the lowest cost. It is worthwhile noting that we prove that at the MESE point, not only the system achieves the lowest possible cumulative cost, but also each user is penalized with the minimum cost compared to the corresponding cost at every other ESE point. The existence and uniqueness of the MESE point is thoroughly studied.

A distributed and low complexity Minimum Effort Best Response Dynamics algorithm is proposed, which is based on the best response dynamics behavioral rule and converges to the MESE point that is also the most energy-efficient from all the existing ESEs. A series of experiments are performed to evaluate the performance and attributes of the proposed novel resource management framework which is based on games in satisfaction form. A basic comparative study demonstrates its superiority and benefits in terms of power savings and improved network capacity, against approaches targeting energy-efficiency and/or utility maximization.

The remainder of the paper is organized as follows: Section II introduces the definitions and relevant information regarding the SE, ESE and MESE points. In Section III, the uplink power control game is formulated in its satisfaction form, the existence of the ESE points is shown, and the existence, uniqueness, and benefits of the MESE point are proven and discussed. Section IV introduces the Minimum Effort Best Response Dynamics algorithm to determine the MESE point, its convergence is shown and the corresponding complexity analysis is provided. Section V presents a detailed numerical and comparative performance evaluation, while Section VI concludes the paper.

II. Games in Satisfaction Form

In this section, we provide some definitions and the basic notation that will be used in the rest of the paper. A game in satisfaction form is defined as [10] $\hat{G} = (K, \{A_k\}_{k\in K}, \{f_k\}_{k\in K})$, where $K = \{1, \dots |K|\}$ represents the set of players, A_k is the strategy set of player $k \in K$, $u_k(a_k, \boldsymbol{a}_{-k})$ represents player's k payoff (i.e., utility function), and $f_k(\boldsymbol{a}_{-k}) = \{a_k \in A_k : u_k(a_k, \boldsymbol{a}_{-k}) \geq u_{thr}\}$ determines the set of actions of player k that allows its satisfaction, that is its payoff to be above a threshold value u_{thr}, given the actions $\boldsymbol{a}_{-k}$ played by all the other players. A strategy profile is denoted by a vector $\boldsymbol{a} = (a_1, \dots, a_{|K|}) \in A$, $A = A_1 \times \cdots \times A_k \times \cdots \times A_{|K|}$.

Definition 1: An action profile $\boldsymbol{a}^+$ is an *SE point* for the game $\hat{G} = (K, \{A_k\}_{k\in K}, \{f_k\}_{k\in K})$ if

$$\forall k \in K, \quad a_k^+ \in f_k(\boldsymbol{a}_{-k}^+) \tag{1}$$

It should be noted that there could exist multiple strategy vectors $\boldsymbol{a}^+ = (a_1^+, \dots, a_{|K|}^+)$ satisfying player's minimum QoS prerequisites, some of which are of particular interest. A representative example is the *Efficient Satisfaction Equilibrium* (ESE) where each player of the system achieves its minimum QoS prerequisites via being simultaneously penalized with the minimum cost. To capture the notion of the players' penalty and effort associated with a given action choice, the concept of the cost function for each player is introduced. For all $k \in K$, the cost function $c_k : A_k \to [0, 1]$ satisfies the following condition: $c_k(a_k) < c_k(a_k^{'})$, $\forall (a_k, a_k^{'}) \in A_k^2$, if and only if, a_k requires a lower effort by player k than action $a_k^{'}$.

Definition 2: An action profile $\boldsymbol{a}^*$ is an *ESE point* for the game $\hat{G}$, with cost functions $\{c_k\}_{k\in K}$, if

$$\forall k \in K, \quad a_k^* \in f_k(\boldsymbol{a}_{-k}^*) \tag{2a}$$

$$\forall k \in K, \quad \forall a_k \in f_k(\boldsymbol{a}_{-k}^*), c_k(a_k) \geq c_k(a_k^*) \tag{2b}$$

Another equilibrium point of special interest is the *Minimum Efficient Satisfaction Equilibrium* (MESE). At the MESE point, all players satisfy their QoS prerequisites (Eq. 3a), with the minimum cost for themselves (Eq. 3b), as well as with the minimum total cost from the system's perspective (Eq. 3c).

Definition 3: An action profile $\boldsymbol{a}^\dagger$ is a *Minimum Efficient Satisfaction Equilibrium (MESE)* for the game $\hat{G} = (K, \{A_k\}_{k\in K}, \{f_k\}_{k\in K})$, with cost functions $\{c_k\}_{k\in K}$, and set of action profiles that are ESEs $\{E\}$ if

$$\forall k \in K, \quad a_k^\dagger \in f_k(\boldsymbol{a}_{-k}^\dagger) \tag{3a}$$

$$\forall k \in K, \quad \forall a_k \in f_k(\boldsymbol{a}_{-k}^\dagger), c_k(a_k) \geq c_k(a_k^\dagger) \tag{3b}$$

$$\forall \boldsymbol{e} \in E, \quad \sum_{k\in K} c_k(e_k) \geq \sum_{k\in K} c_k(a_k^\dagger) \tag{3c}$$

III. Rethinking Uplink Power Control

The aforementioned definitions and concepts, along with the pressing need for cost efficient (i.e., energy efficient) solutions in the era of wireless communications, motivate and support the rethinking and redefinition of the power control problem in wireless networks. Let us consider K transmitter/receiver pairs denoted by index $k \in K$. For all $k \in K$, transmitter k uses power level $p_k \in A_k$, with A_k generally defined as a compact sublattice. For each player $k \in K$, we denote p_k^{min} and p_k^{max} the minimum and maximum power levels in A_k, respectively. For every pair of devices $(i, j) \in K^2$, g_{ij} is the channel gain coefficient between transmitter i and receiver j.

For the rest of this section, we will assume and study uplink power control games in which each player has a utility function that is increasing with respect to its own transmission power and decreasing with respect to the total summation over the powers of the rest of players, as the latter quantity acts as interference to the examined player's transmission. One representative example of such utility function that satisfies the aforementioned realistic assumption, is the commonly adopted Shannon capacity which is given by:

$$u_k(p_k, \boldsymbol{p}_{-k}) = \log_2(1 + \frac{p_k g_{kk}}{\sigma_k^2 + \sum_{j \neq k} p_j g_{jk}})[\frac{bps}{Hz}] \quad (4)$$

where σ_k^2 is the noise variance at receiver k.

The considered QoS requirement for each user k is to have a channel capacity $u_k(p_k, \boldsymbol{p}_{-k})$ higher than a given threshold $u_{thr}[\frac{bps}{Hz}]$. The satisfaction correspondence of user k is subsequently expressed as:

$$\begin{aligned} f_k(\boldsymbol{p}_{-k}) &= \{p_k \in A_k \mid u_k(p_k, \boldsymbol{p}_{-k}) \geq u_{thr}\} \\ &= \{p_k \in A_k \mid p_k \geq (2^{u_{thr}} - 1)\frac{\sigma_k^2 + \sum_{j \neq k} p_j g_{jk}}{g_{kk}}\} \end{aligned} \quad (5)$$

In the above inequality, note that if a user raises its transmission power then some other users may also have to increase their transmission powers as well to get satisfied. Also, given the strategy profile of the other users $\boldsymbol{p}_{-k}$, the following statement is valid for each user k:

$$p \in f_k(\boldsymbol{p}_{-k}) \Rightarrow \forall p^* \in A_k : p^* \geq p, p^* \in f_k(\boldsymbol{p}_{-k}) \quad (6)$$

Thus, given the strategies of the other users, i.e, $\boldsymbol{p}_{-k}$, there is a transmission power p_k^{MSP} which on one hand satisfies the QoS prerequisites of the examined user k, but on the other hand playing with a lower transmission will leave the user unsatisfied. Contrary, if the user transmits with a greater power, then the user will remain satisfied. We will refer to that power p_k^{MSP} as the *Mininum Satisfying Power* (MSP) of user k given $\boldsymbol{p}_{-k}$. Note, that under the assumption we made about the monotonicities of the utility functions, the inequality (6) holds true for the rest of our analysis. In the following, we examine the existence of an ESE in the uplink power control game $\hat{G} = (K, \{A_k\}_{k \in K}, \{f_k\}_{k \in K})$ with cost functions $\{c_k\}_{k \in K}$ and payoff/utility functions $\{u_k\}_{k \in K}$ (Eq. 4).

A. Existence of ESE and MESE

To prove the existence of at least one ESE point in the uplink power control game $\hat{G}$ in our setting we first mention the Tarski and Knaster's fixed point theorem [13].

Theorem 1 (Tarski and Knaster's fixed point theorem): Let $\mathcal{L}$ be a complete lattice and let $f : \mathcal{L} \to \mathcal{L}$ be an order-preserving function. Then, the set of fixed points of f in $\mathcal{L}$ is also a complete lattice.

Let A be the set of the strategy space of the game $\hat{G}$ as defined above. Let us also define the lattice $\mathcal{L} = \langle A, \preceq \rangle$, where $\preceq$ is the component-wise less or equal. Note that $\mathcal{L}$ is a complete lattice as all of its subsets have both a supremum and an infimum. The next step is to construct an appropriate function $g : \mathcal{L} \to \mathcal{L}$. For that purpose, we will use the notation $BR_k(\boldsymbol{p}_{-k})$ as the best response function of a user k, while the strategies of the rest of the users are $\boldsymbol{p}_{-k}$. That is, the transmission power $p_k \in A_k : p_k = \arg\min_{p_k \in f_k(\boldsymbol{p}_{-k})} c(p_k)$. Let us define the function $g : \mathcal{L} \to \mathcal{L}$ as follows:

$$g(\boldsymbol{p}) = (BR_1(\boldsymbol{p}_{-1}), \ldots, BR_{|K|}(\boldsymbol{p}_{-|K|})) \quad \forall \boldsymbol{p} \in A$$

Note that if $f_k(\cdot) \neq \emptyset$ for every user k, then $BR_k(\boldsymbol{p}_{-k}) \in A_k$, $\forall \boldsymbol{p}_{-k} \in A_{-k}$, $\forall k \in K$. Following those definitions we conclude to the following proposition.

Proposition 1: If an uplink power control game in satisfaction form $\hat{G}$ with cost function $\{c_k\}_{k \in K}$ and utility function $\{u_k\}_{k \in K}$ (Eq. 4), has the f_k functions for every user k non empty for every input then it possesses at least one ESE.

Proof: The proof comes from the Theorem 1. As mentioned, $\mathcal{L}$ is a complete lattice. We can also note that $\forall \boldsymbol{p}, \boldsymbol{p}' \in A : \boldsymbol{p} \preceq \boldsymbol{p}'$ it holds that:

$$\begin{aligned} &(BR_1(\boldsymbol{p}_{-1}), \ldots, BR_{|K|}(\boldsymbol{p}_{-|K|})) \preceq \\ &(BR_1(\boldsymbol{p}'_{-1}), \ldots, BR_{|K|}(\boldsymbol{p}'_{-|K|})) \end{aligned}$$

Or equivalently $g(\boldsymbol{p}) \preceq g(\boldsymbol{p}')$. That means that for every user k, when the rest of the users have played $\boldsymbol{p}_{-k}$ and then they increase their powers, k's best response will be a greater or equal transmission power than it was. The latter holds true because of the monotonicity we assumed on the utility functions, and thus either k's best response will still satisfy user's k QoS prerequisites or user k should increase its transmission power in order to be satisfied, thus, inevitably playing an action (transmission power) that is related to a greater cost than before. In that fashion, we also proved that g is an order-preserving function. Following the previous analysis, Tarski-Kraskel's theorem ensures the existence of a fixed point of function g. That is, $\exists \boldsymbol{p} \in A$:

$$\begin{aligned} &\boldsymbol{p} = g(\boldsymbol{p}) \Leftrightarrow \\ &(p_1, \ldots, p_{|K|}) = (BR_1(\boldsymbol{p}_{-1}), \ldots, BR_{|K|}(\boldsymbol{p}_{-|K|}) \end{aligned}$$

That would mean that for the strategy profile $\boldsymbol{p}$, every user has played its best response strategy given the strategies of the rest of the users. So, $\boldsymbol{p}$ is an ESE for the game $\hat{G}$. ∎

Given the proof of the existence of at least on ESE, we can easily conclude to the existence of at least one MESE as well.

B. Uniqueness and Benefits of MESE

Below, we provide some propositions that hold true in the considered uplink power control game, to show the main benefits of the MESE point and the specific conditions under which it is unique. We assume that the f_k functions are non empty for every input and every player k, where this assumption ensures the possession of at least one ESE (Proposition 1).

Proposition 2: In an uplink power control game $\hat{G}$ as mentioned above, if an action profile $\boldsymbol{p}^+$ is an SE of the game and it holds that $\forall k \in K, \forall p_k \in A_k : p_k \geq p_k^+, c_k(p_k) \geq c_k(p_k^+)$ there exists one action profile $\boldsymbol{p}^*$ that is an ESE in which it holds that $c_k(p_k^+) \geq c_k(p_k^*), \forall k \in K$.

Proof: For the proof we exclude the powers $p_d : p_d > p_k^+, \forall k \in K$. Thus, the modified strategy space is denoted by A_k', and the corresponding game is $\hat{G}'$. In the game $\hat{G}'$, we know that the strategy p_k^+ will satisfy the user k, $\forall k \in K$, regardless the strategies of the rest of the users (Eq. 6). Applying proposition 1 that proves the existence of an action profile $\boldsymbol{p}^*$ that is an ESE for $\hat{G}'$, we have that

$$\forall k \in K, \forall p_k \in A_k' : p_k \in f_k(\boldsymbol{p}_{-k}^*), \quad c_k(p_k) \geq c_k(p_k^*) \quad (7)$$

Because by default p_k^+ is the maximum transmission power of the set A'_k of the k^{th} user in $\hat{G}'$, it means that $p_k^+ \geq p_k^*$ and consequently $c_k(p_k^+) \geq c_k(p_k^*)$ based on Eq. 7. So, the above statement combined with our assumption regarding the monotonicity of the utility function, enables us to conclude to the following statement regarding the initial game $\hat{G}$:

$$\forall k \in K, \quad \forall p \in A_k : p \in f_k(\boldsymbol{p}^*_{-k}), \quad c_k(p) \geq c_k(p_k^*)$$

Due to the above statement and given that $\boldsymbol{p}^*$ is an SE in $\hat{G}$, we conclude that $\boldsymbol{p}^*$ is also an ESE in $\hat{G}$. Thus, we have also proven implicitly that $\sum_{k\in K} c_k(p_k^+) \geq \sum_{k\in K} c_k(p_k^*)$. ■

Assuming the above setting, the following holds true.

Proposition 3: In an uplink power control game $\hat{G}$ let two action profiles $\boldsymbol{p}^{*(1)}$, $\boldsymbol{p}^{(2)}$, and $\boldsymbol{p}^{*(1)}$ be an ESE. Then for each user k holds that if $c_k(p_k^{*(1)}) > c_k(p_k^{(2)})$ then $p_k^{*(1)} > p_k^{(2)}$.

Proof: This proposition is proven via the reductio ad absurdum, as follows. If $c_k(p_k^{*(1)}) > c_k(p_k^{(2)})$ and it was $p_k^{*(1)} < p_k^{(2)}$, it would mean that in the strategy profile $\boldsymbol{p}^{*(1)}$ user k would remain satisfied if it played $p_k^{(2)}$ (Eq. 6), which reduces its cost. That is a contradiction because $\boldsymbol{p}^{*(1)}$ is an ESE. In addition, $p_k^{*(1)} = p_k^{(2)}$ can't hold because $c_k(p_k^{*(1)}) \neq c_k(p_k^{(2)})$. ■

Using these propositions we prove the following statement to study the plurality of the set of the MESE points.

Proposition 4: For any two MESEs $\boldsymbol{p}^{\dagger(1)}, \boldsymbol{p}^{\dagger(2)}$ it holds that $c_k(p_k^{\dagger(1)}) = c_k(p_k^{\dagger(2)}), \quad \forall k \in K$.

Proof: Let $\{E\}$ be the set of action profiles that are ESEs. Let us now denote two MESEs of the game, $\boldsymbol{p}^{\dagger(1)}$ and $\boldsymbol{p}^{\dagger(2)}$ such that for one user k it holds that $c_k(p_k^{\dagger(1)}) \neq c_k(p_k^{\dagger(2)})$. In order for them to be MESEs the following should hold:

$$\forall \boldsymbol{p}^* \in E, \sum_{k\in K} c_k(p_k^*) \geq \sum_{k\in K} c_k(p_k^{\dagger(1)}) = \sum_{k\in K} c_k(p_k^{\dagger(2)}) \quad (8)$$

As assumed, there is one user k that $c_k(p_k^{\dagger(1)}) \neq c_k(p_k^{\dagger(2)})$ and consequently $p_k^{\dagger(1)} \neq p_k^{\dagger(2)}$. Without loss of generality,we assume that $c_k(p_k^{\dagger(1)}) < c_k(p_k^{\dagger(2)})$. Because of the fact that $\boldsymbol{p}^{\dagger(1)}$ and $\boldsymbol{p}^{\dagger(2)}$ are ESEs, from Proposition 3 we get $p_k^{\dagger(1)} < p_k^{\dagger(2)}$. Thus, the total summation over the costs of all users in $\boldsymbol{p}^{\dagger(1)}$ would be lower than the one of $\boldsymbol{p}^{\dagger(2)}$ if they do not differentiate in any other strategy. This, denotes that there should be one other user j ($j \neq k$) that $c_j(p_j^{\dagger(1)}) > c_j(p_j^{\dagger(2)})$. With the same argument it holds that $p_j^{\dagger(1)} > p_j^{\dagger(2)}$.

Let $\boldsymbol{p}^+$ be an action profile with $p_k^+ = p_k^{\dagger(1)}$ and $p_j^+ = p_j^{\dagger(2)}$. Note that $\boldsymbol{p}^+$ has lower summation over the costs of users k, j from both $\boldsymbol{p}^{\dagger(1)}$ and $\boldsymbol{p}^{\dagger(2)}$. Continuing in that fashion, $\boldsymbol{p}^+$ strategy profile picks for every user k the power that gives k the lower cost over $p_k^{\dagger(1)}$ and $p_k^{\dagger(2)}$. Because of Proposition 3, the transmission power would always be the lower of the two. If $c_k(p_k^{\dagger(1)}) = c_k(p_k^{\dagger(2)})$, let p_k^+ be the lower transmission power of the two. Note that $\boldsymbol{p}^+$ is an SE as each user k was satisfied by playing p_k^+ either at $\boldsymbol{p}^{\dagger(1)}$ or at $\boldsymbol{p}^{\dagger(2)}$ while all of the other users have played greater or equal transmission powers (Eq. 6). So, at $\boldsymbol{p}^+$ it holds that:

$$\sum_{k\in K} c_k(p_k^+) < \sum_{k\in K} c_k(p_k^{\dagger(1)}) = \sum_{k\in K} c_k(p_k^{\dagger(2)}) \quad (9)$$

Note that in order to construct $\boldsymbol{p}^+$ we chose strategies between two ESEs. Thus, because of Eq. 6 and Eq. 2b we get that $\forall k \in K, \forall p \in A_k : p \geq p_k^+, c_k(p) \geq c_k(p_k^+)$. Thus, applying proposition 2 on $\boldsymbol{p}^+$ gives us an ESE $\boldsymbol{p}^\dagger$ with

$$\sum_{k\in K} c_k(p_k^+) \geq \sum_{k\in K} c_k(p_k^\dagger) \quad (10)$$

Combining inequalities (9) and (10) we conclude:

$$\sum_{k\in K} c_k(p_k^\dagger) \leq \sum_{k\in K} c_k(p_k^+) < \sum_{k\in K} c_k(p_k^{\dagger(1)}) = \sum_{k\in K} c_k(p_k^{\dagger(2)})$$

which leads to contradiction with (8) as $\boldsymbol{p}^\dagger$ is an ESE. So, $c_k(p_k^{\dagger(1)}) = c_k(p_k^{\dagger(2)}), \forall k \in K$, which completes the proof. ■

The above proposition, shows that every MESE point gives the same cost to a given user. Consequently, if $\forall k \in K, \forall p_1, p_2 \in A_k : (p_1 \neq p_2), c_k(p_1) \neq c_k(p_2)$ (which is a common case in the uplink power control), then the MESE point is unique. The following proposition shows that each user achieves the minimum cost at a MESE point compared to the experienced cost at any ESE point.

Proposition 5: In the considered uplink power control game, let $\boldsymbol{p}^\dagger$ be a MESE of the game and {E} the set of ESEs, it holds that $c_k(p_k^\dagger) \leq c_k(p_k^*), \forall k \in K, \forall \boldsymbol{p^*} \in E$.

Proof: Let us study the strategy profile $\boldsymbol{p}$ that:

$$\forall k \in K, \quad \forall \boldsymbol{p}^* \in E, \quad p_k = \arg\min_{p_k^*} c_k(p_k^*) \quad (11)$$

Thus, the strategy profile $\boldsymbol{p}$ picks for each user the power that gives the lowest cost for the user over all its strategies that belong to the set of ESEs. In case of a tie (more than one strategies that minimize the cost), it picks the lower strategy from those. Let us focus on a random user k. Let $\boldsymbol{p}^*$ be one ESE such that $p_k = p_k^*$. So, from all the ESEs of the game, $\boldsymbol{p}^*$ gives the lowest cost to user k, $c_k(p_k^*)$. Because of Eq. 11, we have: $\forall i \in K, c_i(p_i) \leq c_i(p_i^*)$. For the users i' that holds $c_{i'}(p_{i'}) < c_{i'}(p_{i'}^*)$, Proposition 3 gives $p_{i'} < p_{i'}^*$. For all users i'' that holds $c_{i''}(p_{i''}) = c_{i''}(p_{i''}^*)$ the way that we broke the ties gives us $p_{i''} \leq p_{i''}^*$. So, combining the above statements we have proven that $\forall i \in K, p_i \leq p_i^*$.

Owing to the above, user k will certainly be satisfied in strategy profile $\boldsymbol{p}$ because it was satisfied at the ESE $\boldsymbol{p}^*$ in which the other users have played greater or equal transmission powers (Eq. 6). The above analysis holds for every user k, thus every user in strategy profile $\boldsymbol{p}$ is satisfied, thus $\boldsymbol{p}$ is an SE. Note that in order to construct $\boldsymbol{p}$, we chose strategies between strategy profiles that are ESEs. Thus, based on of Eq. 6 and Eq. 2b we get that $\forall k \in K, \forall p \in A_k : (p \geq p_k), c_k(p) \geq c_k(p_k)$. Now, we can apply Proposition 2 that gives us an ESE $\boldsymbol{p}^\dagger$ that:

$$\forall k \in K \quad c_k(p_k) \geq c_k(p_k^\dagger) \quad (12a)$$

$$\sum_{k\in K} c_k(p_k) \geq \sum_{k\in K} c_k(p_k^\dagger) \quad (12b)$$

Taking into consideration Eq. 11, we can note that only the equality can hold in inequalities (12) so: $\forall k \in K \quad c_k(p_k) = c_k(p_k^\dagger)$ and $\sum_{k\in K} c_k(p_k) = \sum_{k\in K} c_k(p_k^\dagger)$. Note that we cannot find an ESE that has lower total cost than $\boldsymbol{p}$. Thus, $\boldsymbol{p}^\dagger$ is an MESE. Due to Proposition 4 every MESE assigns the same cost to a given user. That means that every MESE allocates to each user the minimum cost that it could possibly have in an ESE as exactly $\boldsymbol{p}$ does. ∎

One final observation is that if the cost function of every user is increasing with respect to its transmission power, MESE would be unique and there would not exist any strategy profile that satisfies all the users and simultaneously allocates to any user lower cost than the MESE. This is easily concluded if one applies proposition 2 on any random SE of the game.

IV. Algorithm & Convergence

In this section, we present a distributed algorithm that converges at a Minimum Efficient Satisfaction Equilibrium (MESE) of the game $\hat{G} = (K, \{A_k\}_{k\in K}, \{f_k\}_{k\in K})$. For this purpose we first introduce the *Best Response Dynamics* (BRD) in the context of a game in satisfaction form.

A. Best Response Dynamics

Best Response Dynamics is defined as the behavioral rule in which each user always chooses its uplink transmission power to be its best response (BR) as defined earlier, depending on the uplink transmission power of the rest of the users. In the context of this paper, the dynamics should not be sequential but asynchronous. As shown in [10], when the BRD start from an SE as an initial strategy profile, they converge monotonically to an ESE. The algorithm that is studied in the following section is the BRD starting with the action profile associated with the lowest effort for each user.

B. Minimum Effort Best Response Dynamics

Initially, each user k pre-processes its data with the *Preparation Phase* of the Minimum Effort Best Response Dynamics (MEBRD) algorithm. Note that after executing the preparation phase of the MEBRD algorithm, each user k has computed the vector $\mathcal{S}_k[]$. The element $\mathcal{S}_k[j]$ denotes the uplink transmission power that provides the minimum cost over the powers $P_k[j], \ldots, P_k[|A_k|-1]$. Therefore, if the Minimum Satisfying Power (MSP) is a power $p_k^{MSP} = P_k[i] \in A_k, \forall k \in K$ given $\boldsymbol{p}_{-k}$ then $BR_k(\boldsymbol{p}_{-k}) = \mathcal{S}_k[i]$. After the preparation phase, each user chooses the power that minimizes its cost function. Therefore, the starting strategy profile of the dynamics will be: $\boldsymbol{p}_{start} = (\mathcal{S}_1[0], \ldots, \mathcal{S}_{|K|}[0])$. Then, each user who is in turn to play executes the *Turn Phase* of the MEBRD algorithm. The auxiliary vectors $\mathcal{S}_k[]$ will help each user to calculate its BR in every turn with a binary search. Due to the monotonicity of the utility function and the fact that each user either does not change or increases its transmission power at each turn (as we will prove in the convergence section), user k should only do binary search from the MSP of its previous turn to its current p_k^{max}. With the binary search each user k is searching for the smallest value in $P_k[]$ that satisfies $u_k(p, \boldsymbol{p}_{-k}) \geq u_{thr}$, which is the MSP of player k given $\boldsymbol{p}_{-k}$. The algorithm stops when no one user has a new best response strategy to play.

C. Convergence

In this section we prove that the above algorithm converges, under finite number of steps, to an MESE of the considered uplink power control game. To conclude this, we prove a set of propositions as follows:

Proposition 6: The MEBRD algorithm monotonically converges to a strategy profile $\boldsymbol{p}^\dagger \in A$.

Proof: In the first turn of each user k, it examines whether it is satisfied or should increase its power in order to get satisfied. Because of the dynamics, it will not choose a lower transmission power as everyone started with the one that is associated with the lowest cost, $\mathcal{S}_k(0)$. This fact shows that in the first round, each user k is either playing again $\mathcal{S}_k(0)$ (if it is still satisfied) or increases its power.

Due to the fact that the utility functions u_k are decreasing with respect to the total summation over the powers of the users, if a set of users increases its power levels and no user does the opposite, an individual that kept its power unchanged, is now satisfied by greater power levels than before (or with exactly the same ones). Taking into account the above fact and that in the first round all users either raised their powers or held the same values, we can conclude to the following statement. In every turn all users will either keep the power levels of the previous turn (if they are still satisfied) or increase them (in order to get satisfied). It is noted that because of the assumption that $f_k()$ will be not empty for every $\boldsymbol{p}_{-k}$, user k will always have a best response. Therefore, for each user k its sequence of played strategies $\{p_k\}$ is increasing through a finite set, so, its strategy eventually converges (monotonically) to a strategy $p_k^\dagger$. ∎

Proposition 7: $p^\dagger$ is an ESE.

Proof: In this algorithm, when the turn phase of a user k is running, it chooses the transmission power that satisfies it

Algorithm 1 Preparation Phase

Sort in ascending order all powers p_k in a vector $P_k[]$;
$power \leftarrow P_k[|A_k|-1]$;
$min \leftarrow c_k(power)$;
$\mathcal{S}_k[|A_k|-1] \leftarrow power$;
for $p \leftarrow |A_k|-2,\ 0$ **do**
  if $c_k(P_k[p]) \leq min$ **then**
    $\mathcal{S}_k[p] \leftarrow P_k[p]$;
    $power \leftarrow P_k[p]$;
    $min \leftarrow c_k(P_k[p])$;
    $indexOfMin \leftarrow p$;
  else
    $\mathcal{S}_k[p] \leftarrow power$;
  end if
end for
$Msp \leftarrow P_k[0]$;
$IndexOfMsp \leftarrow 0$
play $\mathcal{S}_k(0)$;

but also has the minimum cost. When the previous power that the user selected, is not still the one mentioned above it should change strategy when its turn comes. Due to proposition 6, all users will eventually converge to a transmission power that has those two properties. Thus, because of the dynamics and its eventual convergence, $p^\dagger$ is an ESE. ∎

Proposition 8: $p^\dagger$ is a MESE.

Proof: Let $\boldsymbol{p}^* = (p_1^*, \dots, p_{|K|}^*)$ be an ESE of the game and $\boldsymbol{p}^\dagger = (p_1^\dagger, \dots, p_{|K|}^\dagger)$ be the strategy profile that our algorithm converges. Before we continue note that based on Eq. 2b, 6 it holds that

$$\forall k \in K, \quad \mathcal{S}_k(0) \leq p_k^* \tag{13}$$

Is there a possibility for one player i, to hold $p_i^\dagger > p_i^*$? Following Eq. 13 and due to the fact that in every turn a user cannot decrease its power, in order for this to happen, a user j from all the users, has played a power that exceeds its p_j^* during one of its turns. Let $\boldsymbol{p}$ be the strategy profile of the game right before this turn. Because of the above we have:

$$\forall k \in K, \quad p_k \leq p_k^* \tag{14}$$

Thus player j played a transmission power $p_j^{exc} \in A_j$ that

$$p_j \leq p_j^* < p_j^{exc} \tag{15}$$

Since the strategy profile $\boldsymbol{p}^* = (p_1^*, \dots, p_{|K|}^*)$ is an ESE, from Eq. 2b, Eq. 6, we also get that

$$\forall p \in A_j : (p \geq p_j^*), \quad c_j(p) \geq c_j(p_j^*) \tag{16}$$

Because of the fact that $\boldsymbol{p}^*$ is an SE, we get that $p_j^* \in f_j(\boldsymbol{p}_{-j}^*)$. Because of Eq. 14, and the fact that the utility functions are decreasing with respect to the total summation of the powers of the other users we obtain that:

$$p_j^* \in f_j(\boldsymbol{p}_{-j}) \tag{17}$$

Combining Eq. 15, Eq. 16, and Eq. 17 user's j best response can be neither p_j^{exc}, nor any power that is strictly greater than p_j^*. Thus, user j will not play a power that is greater than p_j^*. So the answer to the previously stated question is negative.

Algorithm 2 Turn Phase

1: **if** still satisfied **then**
2: do not change transmission power;
3: **else**
4: $[Msp, IndexOfMsp] \leftarrow BinarySearch(P_k[], Msp, |A_k|, u_k(), \boldsymbol{p}_{-k})$;{Finds new lower limit (as the vector $\boldsymbol{p_{-k}}$ has changed) using binary search in $P_k[]$ from previous Msp to P_{max} using the utility function of the player}
5: **play** $\mathcal{S}_k(IndexOfMsp)$; {When all of $P_k[indexOfMsp]$ to $P_k[|A_k - 1|](= P_{max})$) strategies satisfies you play the power that gives the lowest cost}
6: **end if**

Considering the above argument, it holds that, $\forall k \in K, p_k^\dagger \leq p_k^*$. Furthermore, since the action profiles $\boldsymbol{p}^*$ and $\boldsymbol{p}^\dagger$ are ESEs, it holds that for every user k:

$$\forall p \in A_k : p \geq p_k^\dagger, \quad c_k(p) \geq c_k(p_k^\dagger) \tag{18a}$$

$$\forall p \in A_k : p \geq p_k^*, \quad c_k(p) \geq c_k(p_k^*) \tag{18b}$$

But as we have proven for each user k it holds that $p_k^* \geq p_k^\dagger$, therefore based on Eq. 18a we conclude: $\forall k \in K, \quad c_k(p_k^\dagger) \leq c_k(p_k^*)$. Consequently, $\sum_{k\in K} c_k(p_k^\dagger) \leq \sum_{k\in K} c_k(p_k^*)$. Given that the above analysis is valid for every ESE $\boldsymbol{p}^*$, it will also hold true for one MESE. So $\boldsymbol{p}^\dagger$ is a MESE. ∎

One corollary of the above is that since it holds that $\forall \boldsymbol{p}^* \in E, \forall k \in K, \quad p_k^\dagger \leq p_k^*$ we have also shown that from all the ESEs, $\boldsymbol{p}^\dagger$ is the most efficient one in terms of power. It is also noted that in this work we proved that MEBRD converges to the MESE point, under the assumption that for every player k, $f_k(\cdot)$ is not empty. Nevertheless, in practice this assumption is not required to hold for the proposed scheme to work, and it can be relaxed by simply adding for each player k one auxiliary (virtually maximum) transmission power, p_k^M, in its strategy space such that $\forall \boldsymbol{p}_{-k} \in A_{-k}, \quad p_k^M \in f_k(\boldsymbol{p}_{-k})$ and $c_k(p_k^M) = +\infty$. If MEBRD converges to the strategy profile $\boldsymbol{p}^\dagger = (p_1^M, \dots, p_{|K|}^M)$ then the game does not possess any ESE.

D. Complexity

In this subsection, the complexity of the algorithm is studied in the case of the users are playing sequentially in a given order. Let us concentrate on one user k in order to specify its CPU time complexity excluding the time that other users take in order to make their decisions. At first, user k should sort the array $P_k[]$ so that would be a $\mathcal{O}(|A_k| \cdot log_2(|A_k|))$ time complexity. In every cycle of turns, someone should always increase its power, or else the algorithm converged to $\boldsymbol{p}^\dagger$. The worst case is bound by the case where the game would have $\mathcal{C} = |A_1| + \dots + |A_{|K|}|$ cycles of turns. So, in $\mathcal{C} - |A_k|$ cycles, user k will find out, in constant time, that it is satisfied. On the other hand, in $|A_k|$ cycles of turns the user should do one binary search in $P_k[]$ in order to find out its next move. Therefore, for all of the cycles it will totally spend $\mathcal{O}((\mathcal{C} - |A_k|) + |A_k| \cdot log_2(|A_k|))$. Thus, the total time complexity is $\mathcal{O}((\mathcal{C} - |A_k|) + |A_k| \cdot log_2(|A_k|))$. Note, that if each user has the same cardinality in its strategy space, N, the total complexity will be $\mathcal{O}(|K| \cdot N + N \cdot log_2(N))$.

V. Performance Analysis and Evaluation

In this section, we provide some indicative numerical results to evaluate the performance of the MEBRD algorithm, while at the same time demonstrating the key benefits of the MESE point. For demonstration purposes, the utility function of each user is assumed to follow Eq. 4. The distance $d_k, \forall k \in K$ from the base station is randomly and uniformly distributed within the range of 1 to 50 *m*. The gain g_k of each user k is inversely proportional to the square of its distance d_k, i.e., $g_k = \frac{1}{d^2}$. Each user is assumed to have a finite number of power levels, while its maximum transmission power is $1Watt$.

A. Pure Operation

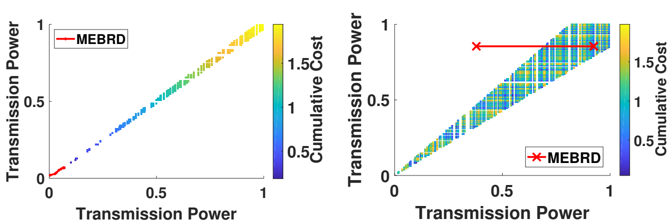
Fig. 1 presents, for a two user uplink power control game, the steps of the MEBRD to converge to an MESE point with respect to each user's transmission power, assuming either increasing cost function (Fig. 1a) or arbitrarily randomly chosen cost function (Fig. 1b). The colored region represents all the strategy profiles that are SEs and each point's color depends on the cumulative cost of the two users, where the light and dark color represents high and low cost, respectively. It is noted that in Fig. 1a each user starts with its lowest transmission power and monotonically converges to the unique MESE which is also the SE that charges each user with the lowest possible cost and power. Please also note that in Fig. 1a and Fig. 1b, the linear trend of the satisfaction region stems from the selection of the u_{thr} value, which in our case represents the utility that each user would gain if it transmitted with an intermediate power, i.e., $0.5W$, in its feasible power range. In the rest of this section we consider increasing cost function with respect to user's transmission power.

Assuming that each user is capable of achieving its QoS prerequisites, it is highly possible for the game to possess multiple ESEs. In Fig. 2, we compare the cost allocation of different ESEs (multiple curves) of an uplink power control game with four users. We confirm that the MESE achieves the lowest cumulative cost by assigning to each user the transmission power associated with the lowest effort compared to every ESE, as claimed in Proposition 5.

B. Complexity Evaluation

The time complexity of the MEBRD algorithm, as analyzed in Section IV. D, mainly depends on the number of users in the system and the number of the discrete transmission power levels that each user possesses. Fig. 3 presents the behavior of the execution time of the MEBRD algorithm with respect to the number of each user's discrete transmission power levels.

The time complexity of the MEBRD algorithm depends also on every parameter of the game, such as for instance each user's threshold u_{thr}, above which the user is satisfied. For example, if each user is satisfied by gaining very low bit rate, most of the transmission powers will satisfy the user independently of the power of the others, hence the MESE that the MEBRD algorithm converges will consist of low transmission powers and it will converge fast. Specifically, Fig. 4 presents the time needed for the MEBRD to converge to its MESE as a function of the number of users in the system,



(a) Increasing Cost Functions (b) Random Cost Functions

Fig. 1: Convergence of the MEBRD algorithm in a 2-user uplink power control game

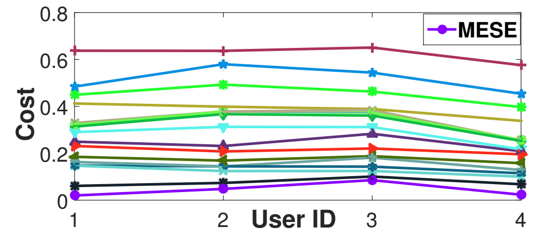


Fig. 2: Cost of various ESE points vs user ID

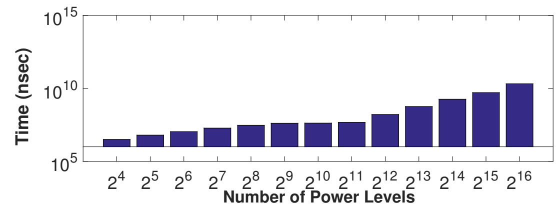


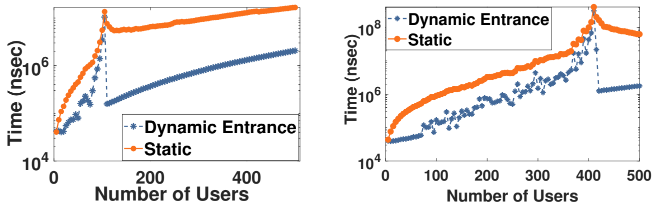
Fig. 3: Execution time of the MEBRD algorithm as a function of the number of each user's transmission power levels

assuming 150 different transmission power levels available to each user. In particular two cases are studied with respect to each user's threshold parameter u_{thr}: (a) high threshold (Fig. 4a) and (b) low threshold (Fig. 4b). In the latter case, it is easier to satisfy the requirements and more SEs are expected to exist. Please also note that the low thresholds are selected such that the game even with 400 users in the system will possess at least one equilibrium. The top curve in each subgraph (referred to as Static) represents the time needed for the MEBRD algorithm to converge. For example, when the game consists of a lower number of users (e.g., 100) each user is satisfied easily, thus experiencing a fast convergence. On the other hand, when the number of users is approaching 400, the algorithm needs more iterations to converge. After that number, the system does not possess any equilibrium points. In this case the number of required iterations decrease, since each user makes greater steps increasing its transmission power in order to meet its QoS prerequisites and eventually fail.

The second curve in the two subgraphs of Fig. 4 (referred to as Dynamic Entrance) denotes the time needed in order the MEBRD algorithm to converge, when each run of the MEBRD is not independent as before. In contrary, $five$ users every time enter the game, while the previous number of users had converged to the MESE and use this point in order to initialize their strategies in the new run. As the algorithm does not need to be in sequential turns, it will still converge to the MESE for the new number of users, while lower convergence time than before (top curve) is observed. Last it is observed that in the latter case the time needed for convergence is not necessarily strictly increasing with respect to the number of users.

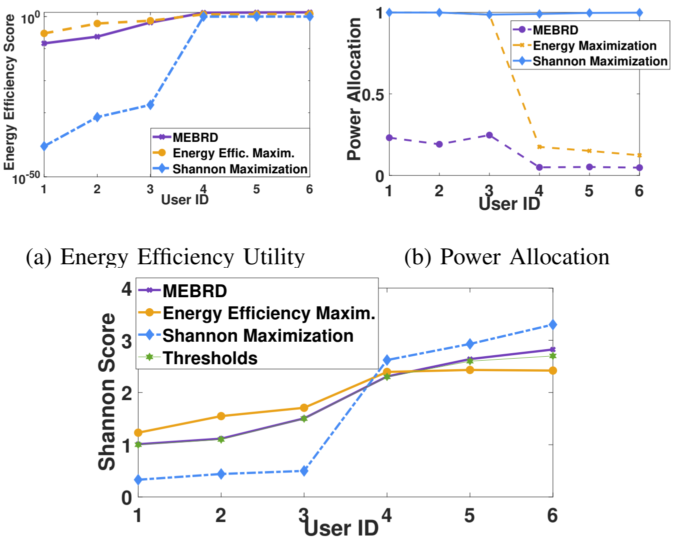
C. Comparative Results

In this subsection, we compare the MESE as a strategy profile, with the corresponding ones that an Energy Efficiency Maximization or Shannon Maximization algorithm would obtain, with respect to different performance metrics. Specifically, in this scenario, six users are considered in the system



(a) High Thresholds (b) Low Thresholds

Fig. 4: Execution time of the MEBRD algorithm as a function of the number of users in the system.



(a) Energy Efficiency Utility (b) Power Allocation

(c) Shannon Utility

Fig. 5: Comparison of strategy profiles for MEBRD, Energy-Efficiency Maximization and Shannon Maximization

that are located at decreasing distances from the base station. Therefore, users with lower ID have the highest distances from the base station, thus worse channel conditions. With reference to the energy efficiency maximization case, a utility function that represents the achievable data rate over corresponding consumed power (expressed in bits/joule) was adopted, as typically defined and used in corresponding literature [5]. In both cases of Shannon and energy efficiency utilities we set $W = 10^6 Hz$. In particular, Fig. 5a suggests that for the first 3 users (the 3 users that are the farthest from the base station) the energy efficiency maximization algorithm, as expected, scores higher in the energy efficiency metric. However, as shown in Fig. 5b this happens at the cost that each of the three users transmits with a very high transmission power compared to the MESE, hence gaining higher bit rate than their QoS prerequisites (Fig. 5c). Thus, the MESE strategy profile converges to quite low transmission powers, while assigning to each user bit rate that is close to its threshold (green line in Fig. 5c) and therefore satisfying each user's requirement. The other two strategy profiles do not properly adapt to the users' needs, since they either exceed the threshold forcing the users to transmit with high power, or leave the users unsatisfied.

VI. Concluding Remarks

In this paper, we adopted the game theory in satisfaction form, to redefine and treat the uplink power control problem in wireless networks, for a general set of users' utility functions. Given this setting, we initially defined and discussed the different equilibrium points, i.e., SE, ESE and MESE. Among those, the MESE appeared to be of high theoretical and practical interest, and therefore its properties were thoroughly examined. In particular, it is shown that at the MESE point, for any arbitrary cost function, each user transmits with a power level which satisfies its QoS prerequisites with the lowest cost not only from its own perspective, but from the whole system's perspective as well. The existence and uniqueness of the MESE point were shown and a distributed algorithm was introduced to determine the MESE point. Detailed numerical results were presented to highlight and reveal the benefits of the MESE point compared to other types of equilibrium, and the superior performance of the proposed novel resource management framework in terms of power savings and improved network capacity. As part of our current and future work, we plan to investigate and quantify the "inefficiency" introduced by the commonly adopted solutions in the literature that directly target energy efficiency maximization.

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