Dynamic Spectrum Management in 5G Wireless Networks: A Real-Life Modeling Approach

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***Abstract*—In this paper a novel dynamic spectrum management scheme for 5G Non Orthogonal Multiple Access (NOMA) wireless networks is proposed, where users are offered the option to transmit via licensed and unlicensed bands. Users are enabled to determine the optimal allocation of their transmission power in each one of the bands, while the unlicensed band is treated as a Common Pool Resource (CPR) - being non-excludable and rivalrous - which may collapse due to over-exploitation. Towards providing a pragmatic modeling approach for decision making under a realistic setting of probabilistic uncertainty, while properly capturing user risk perceptions, we model the corresponding optimization problem under the principles of Prospect Theory, removing the common assumption that subjects are behaving as neutral utility maximizers, a concept that does not reflect user risk behavior peculiarities. The corresponding problem is formulated as a CPR game, while the existence and uniqueness of its Pure Nash Equilibrium point are proven, and a user centric distributed algorithm is devised that obtains the corresponding solution. Detailed evaluation results are presented, highlighting the operation and superiority of the proposed framework against conventional Expected Utility Theory based approaches, while providing useful insights about user optimal decisions under realistic behaviors.**

***Index Terms*—Licensed & Unlicensed bands, Prospect Theory, Game Theory, Resource Allocation, Common Pool Resources**

I. Introduction

The rise of the Internet of Things (IoT) era alongside the emergence of 5G next generation wireless networks have contributed to an unparalleled proliferation of the number of connected devices and the subsequent growth in data traffic [1]. Future mobile network is foreseen to reach several milestones with global mobile traffic to increase sevenfold by 2021, with 1.5 connected devices per capita and 75% of mobile data traffic being video services. This massive expansion is shedding light to the problem of spectrum shortage, with mobile users being unable to exploit bandwidth resources based on lack of infrastructure, outdated regulation policies, or limited access to data sharing. The ongoing transformation of radio cellular networks towards heterogeneity, machine-to-machine communications, and small cell deployment has shifted the interest to schemes exploiting the co-existence and use of both licensed and unlicensed spectrum bands.

The flexibility of choice and potential bundling between licensed and unlicensed spectrum sources, create a whole new vision in the field of efficient resource allocation and congestion management in modern wireless networks. The majority of relevant research works in the literature have adopted game theoretic approaches due to their distributed nature, relying mostly on Expected Utility Theory (EUT) maximization. In such a setting, users are viewed as neutral utility maximizers, ignoring decision making under conditions of uncertainty. Game theory considers rational players whose behavior remains uninfluenced by risk, an assumption which deviates from real life decisions where individuals exhibit risk seeking behavior under losses and risk averse behavior under gains. For such a user centric paradigm, Prospect Theory (PT) introduced by Kahnemann and Tversky [2] has emerged as a dominant behavioral model in formulating decision making under probabilistic uncertainty. PT succeeds in integrating user subjectivity in decisions, illustrated by an S-shaped utility function capturing user preferences tending to overweight the probability of losses and underweight that of gains.

In this work, we explore user behavioral insights and incorporate behavioral factors into modeling user decisions and devising resource allocation processes in wireless networks, by applying a PT framework. Users are enabled to exploit the degree of simultaneous use of licensed and/or unlicensed band, determining their actions based on their Quality of Service (QoS) prerequisites as well as by assessing the probability of resource failure. We consider for the first time in the literature the risks induced by over-utilization of a resource, where excessive aggregate investment and congestion may lead to complete collapse of the unlicensed band, where users may enjoy unlimited access to the available spectrum. In that respect, unlicensed band is regarded as a Common Pool Resource (CPR) offering in general higher but uncertain return or satisfaction to the users, due to the increasing risk of collapse. This is in contrast to licensed bandwidth, which is considered as a safe resource, since the Internet Service Provider (ISP) is assumed capable of regulating traffic, ensuring stability and fair share of resources among users.

A. Contributions and Outline

The main contributions and novelties of this work are summarized below:

- We introduce a holistic approach to apply PT in the treatment of resource allocation and spectrum optimization management. Specifically, in contrast to the majority of existing works, a user centric scheme allows for mobile users to shape the nature of their transmission towards the self optimization of the network, considering user heterogeneity and diverse QoS prerequisites.

- Novel and generic enough utility functions are adopted which do not simply represent the trade off between the number of transmitted bits to the corresponding consumed power, but on the contrary reflect users' choices and priorities, a fundamental concept towards the deployment of user centric 5G networks.
- Users are not assumed to be blind utility maximizers as instructed by EUT, but their behavior modeling is specifically designed to reflect real life human decision making under uncertainty, probability weighting and risk aversion perceptions. This consideration allows the study and evaluation of user satisfaction, as expressed through his achieved utility, under more realistic and personalized assumptions.
- Dynamic transmission via both the licensed and unlicensed bands is enabled and orchestrated, empowering users to determine in real time the fraction of the spectrum that they would wish to utilize from each band under different rewards and risks.
- The optimal resource allocation problem, in terms of user optimal transmission power investment to the licensed and unlicensed band, is established as a Fragile CPR game. The corresponding non-cooperative game is solved in a distributed manner, while convergence to its unique Pure Nash Equilibrium (PNE) point is proven.

The remaining of the paper is organized as follows. In Section II a brief review of the related work in the area of spectrum management and the application areas of Prospect Theory is provided. In Section III, the system model under consideration and the proposed PT utility function formalities are presented, whereas in Section IV, the optimal resource allocation problem is formulated and analyzed. In Section V, a low complexity and distributed algorithm to determine the CPR game's solution is described. The operational characteristics and the performance benefits of the proposed framework are in depth evaluated and discussed in Section VI, through an extensive series of numerical results, including a comparative evaluation to EUT, as well as a detailed user behavioral analysis. Finally, Section VII concludes the paper.

II. Related Work

With the available spectrum capacity being at critical bottleneck, several problems referring to band co-existence and throughput optimization have been studied. In [3], Wi-Fi and LTE-U exist in the same band under a shared spectrum model, where a centralized approach delivers improvements in fair spectrum access and data throughput. In [4], the problem of direct transmission to unlicensed bands is considered. By merging band selection and data routing, an overall increase in data rates is observed, while [5] proposes a new transmission protocol for a joint utilization for licensed and unlicensed spectrum with gains in average user throughput compared to conventional Wi-Fi/LTE. [6] investigates the coexistence of unlicensed and licensed LTE, achieving superior performance in traffic balancing and data offloading.

At the same time, PT has also started gaining research attention also in the area of networked systems, spanning from smart grid networks [7], and communications systems [8], to transportation networks, [9], [10]. Although the research status of PT for resource management in wireless networks is still at a very early stage, works have already used this theory in evaluating user or operator decisions in circumstances where risk induces serious challenges to the network's performance. In particular, the authors in [11] examine system's deviation from EUT under PT leading to potential degradations in terms of throughput, delay, and pricing pressure, for a time slotted wireless random access network. Additionally, in [12] ISP profit attributes were examined, concluding in a loss aversion attitude of the providers in favor of lower gains with reduced risks, whereas in [13] users behave differently under reward and penalty cases, switching from a conservative to a more aggressive attitude depending on the loss and gain probabilities.

Our work in this paper bridges the gap between the ongoing trends in band co-existence and optimization with real life decisions that users are requested to take regarding their connectivity, given the risk of distorted access to a cellular network or to a wireless band with collapse dilemmas. In contrast to the above referenced research works, resource failure due to uncontrolled investment becomes a reality addressing pragmatic situations with regards to increasing congestion under dense deployments and spectrum scarcity.

III. System Model under a Prospect Theoretic Perspective

A. Background & System Model

Taking into account the increasing demand for higher spectral efficiency fueled by the accelerated growth of data traffic, orthogonal multiple access (OMA) techniques, despite eliminating intra cell interference by allocating each resource block to one user per time slot, are limited in terms of dynamic data control. Recently, non-orthogonal multiple access (NOMA) techniques have arisen as promising alternatives for improving bandwidth allocation and connectivity at low transmission latency and signaling cost [14]. Users are accommodated in the same resource block by having access to whole spectrum, while the induced intracell interference is eliminated by applying the Successive Interference Cancellation (SIC) technique, with each terminal decoding only the signals of other users with worse channel gains. Hence, users with decreasing channel gain conditions which affect their transmission quality, are sensing decreasing interference levels [15].

We consider the uplink of a NOMA wireless network consisting of a base station (BS) serving a region of radius R. Let $\aleph = \{1, ..., i, ..., N\}$ denote the set of users randomly placed within the topology, who are provided the option of transmitting in either one or simultaneously both of the available licensed and unlicensed bands. Each band corresponds to a different independent fraction of the available spectrum with the users at the beginning of the timeslot indicating the structure and nature of their transmission. For instance, users are enabled to dynamically adjust the percentage of their available transmission power allocated to the unlicensed band, potentially streamlining their operation, from fully operating within the unlicensed spectrum only, to adopting a shared scheme between the licensed and the unlicensed options.

In this work we distinguish the operation of the licensed and unlicensed bands with regards to their stability and spectral capacity. Both bands become available to users under the NOMA transmission technique, with the licensed band being regulated by the ISP where users transmit under predefined data target rates, while the unlicensed band captures a much larger portion of the overall spectrum attracting users due to the higher feasible data speeds, operating though without any regulatory framework or performance warranties. An illustrative representation of such a network structure is shown in Fig. 1. This means that an over-investment in the unlicensed spectrum by the users, would eventually lead to the collapse of this band with none of the users being able to retrieve any utility returns from their transmission. Thus, in the circumstance of band failure, a user who fully invested into the unlicensed spectrum will eventually receive zero return from this action (i.e., will eventually not transmit), whereas a user who decided to partially transmit in the licensed band will at least recover some utility surplus from this part of his operation. In this paper we adopt a scheme where all users are lured into transmitting via the unlicensed bandwidth due to the expected increased satisfaction from this resource, with some of them opting to minimize the unlicensed band collapse risk by still transmitting under the safer licensed spectrum.

B. Utility Functions Under Prospect Theoretic Perspective

We assume that individuals assess uncertain outcomes under a loss averse attitude, i.e., the utility detriment in the event of a loss is sensed as of greater magnitude in comparison to gains of equal extent given a reference point. This reference point is considered as the zero point (i.e., ground truth) of user perceived utility scale, and it is not necessarily common for all users. Additionally, the probability weighting effect is considered, implying that users tend to overweight events with small probabilities and underweight events with higher probabilities, thus instructing the formulation of a utility, where value estimations and probabilistic outcomes are integrated under a common umbrella.

Specifically, the PT utility adopted is concave in positive outcomes and convex in negative outcomes, forming an S-curved shape, indicating user risk seeking or aversion behavior as follows:

$$v_i(z_i) = \begin{cases} (z_i - z_0)^{a_i} & when\ z_i > z_0 \\ -k_i(z_0 - z_i)^{\beta_i} & otherwise \end{cases} \quad (1)$$

where z_i and z_0 denote the relative per user i outcome and the reference point suggesting the boundary among an event of gain or loss, respectively. Parameters $a_i, \beta_i \in \big(0, 1\big]$ express

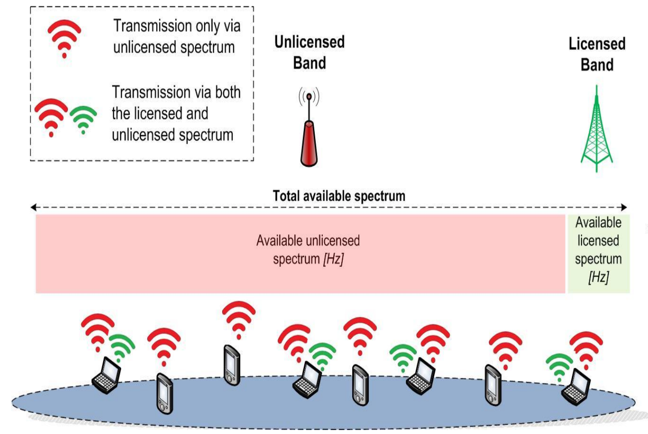

Fig. 1: Conceptual topology with licensed and unlicensed spectrum access

and quantify the sensitivity of user i with regards to a gain or a loss, while $k_i \in \big[0, \infty\big)$ reflects the impact of losses compared to gains in user's utility. In other words, a value of $k_i > 1$ implies a loss aversion attitude, where users perceive utility losses under a steeper curve compared to earns in the event of an equivalent gain. For convenience and without loss of generality, in this work we assume $a_i = \beta_i$. The above framework solidifies the alignment of PT to wireless networks, where users are called to decide how to maximize their utility under terms of probabilistic uncertainty. Licensed bandwidth is assumed to function as a safe option with lower data rate capacity, whereas the unlicensed band is regarded as a CPR available to all users offering a higher fraction of the overall spectrum, however delivering increasing risks of collapse due to high competition (if occurred). By treating unlicensed band as a CPR it means that it is non-excludable, i.e., all users have the right to access it, while simultaneously its rivalrous or subtractable, i.e., its utilization by one user reduces the degree that is exploited and utilized by another user. Subsequently, users viewed as competitive players, incorporate these estimations into their decisions directing their transmission and corresponding power through each band depending on their perceptions towards risk.

It should be noted that in the current work the utility functions have been specifically designed in accordance with the Shannon log-based paradigm, indicating the maximum feasible rate for successful transmission under system's physical constraints and illustrating user greedy behavior towards the maximization of their QoS satisfaction, as follows:

$$U_i(x_i) = \begin{cases} \dfrac{W_l/N_l}{P'_i} ln(1+\gamma_i) & if\ x_i = 0 \\ \dfrac{W_l/N_l}{P'_i} ln(1+\gamma_i)(1-x_i) \\ \quad + \dfrac{W_u/N}{P''_i} x_i(2 - e^{-x_\tau}) ln(1+\gamma_i) & otherwise \end{cases} \quad (2)$$

where W_l and W_u, $[Hz]$ correspond to the bandwidth segments of the licensed and unlicensed bands, N denotes the total number of users and N_l the number of users who choose to transmit not only in the unlicensed but also via the licensed band. In the following let $P_i', P_i'' \in [0, P_{max}], P_i' + P_i'' = P_{max}$ refer to the fractions of the transmission power allocated to the licensed and the unlicensed band, respectively, and γ_i is the Signal to Interference plus Noise Ratio (SINR):

$$\gamma_i = \begin{cases} \dfrac{W_l}{R_i} \dfrac{G_i P_i'}{\sigma^2 + \sum_{i>k} G_k P_k'} & (licensed\ band) \\ \dfrac{W_u}{R_i} \dfrac{G_i P_i''}{\sigma^2 + \sum_{i>k} G_k P_k''} & (unlicensed\ band) \end{cases} \tag{3}$$

where σ^2 denotes the Additive White Gaussian Noise (AWGN) at the receiver and user channel gain G_i reflects the corresponding path gain conditions.

Additionally, $x_i \in [0, 1]$ indicates the power investment of each user to the CPR and is defined as the percentage of maximum power P_{max} reserved for the transmission in the unlicensed band, thus, the transmission power to the unlicensed band is determined as: $P_i'' = x_i P_{max}$, with the remaining power, $P_i' = P_{max} - P_i''$, allocated for transmission to the licensed band. Lastly, let $x_\tau = \sum_i x_i$ denote the aggregate investment of all users in the CPR. The first branch of (2) reflects a transmission only via the licensed (safe) band since the investment to the unlicensed is zero, (i.e. $x_i = 0$), whilst the second branch depicts the utility captured from a shared transmission scheme between the two available bands. Please note that in our setting, user aggregate investment x_τ is considered to be bounded due to the physical limitations of each user. Without loss of generality we normalize aggregate investment x_τ, $x_\tau \in [0, 1]$. Thus, CPR's failure probability increases with the rise of x_τ, with the unlicensed band to collapse with certainty once $x_\tau \to 1$.

In the following, for simplicity of notation, we set:

$$\theta_i = \frac{W_l/N_l}{P_i'} ln(1 + \gamma_i)\ and\ \xi_i = \frac{W_u/N}{P_i''} ln(1 + \gamma_i) \tag{4}$$

and we define the rate of return function of the CPR as follows:

$$r(x_\tau) = 2 - e^{-x_\tau} \tag{5}$$

The rate of return indicates the gain or loss of the unlicensed band with regards to aggregate investment. For demonstration purposes, in this work we consider the rate of return from the unlicensed band to be increasing with total investment, providing higher returns than the licensed band unless the CPR fails, where zero returns are collected by the users. Nevertheless, similar analysis can be also devised by considering a decreasing rate of return function. Based on the above, we explicitly select a rate of return function increasing, concave, twice continuously differentiable, and greater than 1, $\forall x_\tau \in [0, 1]$, and assuming that all users decide to invest to the CPR, i.e., $x_i > 0$, by subtracting θ_i from (2) we obtain:

$$U_i(x_i) = \begin{cases} 0 & if\ x_i = 0 \\ x_i \xi_i r(x_\tau) - \theta_i x_i & otherwise \end{cases} \tag{6}$$

Subsequently, by adjusting (6) to the PT function in (1) and by normalizing the rate of return function so that $\theta_i = 1$, without loss of generality for $x_i > 0$ we obtain $U_i(x_i) = [x_i(\xi_i r(x_\tau) - 1)]^{a_i}$. For notational purposes we denote:

$$\overline{r_i}(x_\tau) = (\xi_i r(x_\tau) - 1)^{a_i} \tag{7}$$

If the CPR collapses, only users who invested into the safe resource will transmit, which prior to normalization refers to $\theta_i(1 - x_i) < \theta_i$. Thus, the utility obtained in case the CPR survives, is $U_i(x_i) = {x_i}^{a_i} \overline{r_i}(x_\tau)$. For any $r(x_\tau)$ increasing, concave, twice continuously differentiable, and greater than 1, $\forall x_\tau \in [0, 1]$, then for any $a_i \in (0, 1]$, $\overline{r_i}(x_\tau)$ is also increasing, concave, twice continuously differentiable, and positive [17].

At this point, we introduce the stochastic probabilistic framework under which the PT model has been designed. Failure probability reflects how close the CPR comes to failure and is increasing with the total investment, with $p(x_\tau = 1) = 1$ indicating a certain collapse of the CPR. Hence, the PT utility of the problem is formulated as:

$$U_i(x_i) = \begin{cases} {x_i}^{a_i} \overline{r_i}(x_\tau) & with\ prob.\ (1 - p(x_\tau)) \\ -k_i x_i^{a_i} & with\ prob.\ p(x_\tau) \end{cases} \tag{8}$$

where the first outcome of (8) denotes user gains from his investment when the CPR survives, and the second illustrates the negative return in case of CPR failure, due to the fact that total investment surpassed the size of the resource, leading to failure. Acknowledging the probabilities of failure for the CPR the expected PT utility can be written as follows:

$$\mathbf{E}(U_i) = {x_i}^{a_i} \overline{r_i}(x_\tau)(1 - p(x_\tau)) - k_i x_i^{a_i} p(x_\tau) \tag{9}$$

Consequently, users' objective is to maximize the expected utility in (9), which can be formally expressed as follows:

$$\begin{gathered} max \mathbf{E}(U_i) = max\{x_i^{a_i} f_i(x_\tau)\},\ \forall i \in \aleph \\ s.t.\ x_i \in [0, 1] \end{gathered} \tag{10}$$

where $f_i(x_\tau) = \overline{r_i}(x_\tau)(1 - p(x_\tau)) - k_i p(x_\tau)$ is defined as the effective rate of return. For convenience, the key parameters used throughout the paper are summarized in Table I.

IV. Common Pool Resource Game Approach

A. Formulation and Solution

Towards treating the above optimization problem and obtaining a stable solution, we model it as a Fragile CPR game [17], since as mentioned before it is assumed to be: 1) non excludable, meaning that all users may access the available resources, and 2) rivalrous, i.e., users compete over claiming those scarce resources, where greedy utilization may eventually drive the system to an outcome of collapse, popularized by [18] as "Tragedy of the Commons". Users have the dilemma of investing only to a commonly accessible resource (i.e., CPR) with high returns under however a high collapse risk, or split their operation including a safe resource, which despite its guaranteed operation status does not deliver gains of comparable magnitude. We consider a Fragile CPR game under the following properties:

TABLE I: List of Parameters

Symbol	Description
R	Network radius
$\aleph$	User set
x_i	Player specific investment to unlicensed band
N_l	Users transmitting via the licensed band
N	Total number of users
W_l	Licensed band spectrum
W_u	Unlicensed band spectrum
γ_i	Signal to Interference plus Noise Ratio
P_i'	Transmission power via the licensed band
P_i''	Transmission power via the unlicensed band
x_τ	Aggregate player investment to unlicensed band
G_i	Channel gain
$r(x_\tau)$	Rate of return function
$\overline{r_i}(x_\tau)$	PT related rate of return
$p(x_\tau)$	Unlicensed band failure probability
a_i, β_i	Sensitivity parameters
k_i	Risk aversion parameter
$f_i(x_\tau)$	Effective rate of return
$g(x_\tau)$	Player specific optimal nonzero investment
Q	Pure Nash Equilibrium Support
R_i	User data rate
$BR_i(x_{-\tau})$	Player best response
S_i	Player investment strategy set
S_i'	Modified player investment strategy set
$\overline{S}'_{-i}$	Space of total investment excluding player i

Assumption 1:

1) The failure probability $p(x_\tau)$ is a convex, strictly increasing, and twice differentiable function of the normalized total investment $x_\tau = [0, 1)$ and $p(1) = 1$.
2) The rate of return $\overline{r_i}(x_\tau)$ in (7) is monotonic (increasing), concave, twice continuously differentiable, and positive $\forall x_\tau \in [0, 1]$.
3) The strategy set of each player i is defined as: $S_i = [0, 1], \forall i \in \aleph$.

Without loss of generality we adopt failure probability from aggregate user investment equal to $p(x_\tau) = {x_\tau}^2$. Subsequently, the effective rate of return is defined accordingly as:

$$f_i(x_\tau) = \overline{r_i}(x_\tau)(1 - {x_\tau}^2) - k_i {x_\tau}^2 \tag{11}$$

Let $br_i(x_{-\tau})$ =argmax$\mathbf{E}(U_i(x_\tau, x_{-\tau}))$, $br_i : \overline{S}_{-i} \rightrightarrows S_i$ be the best response correspondence of player i, where $\overline{S}_{-i}$ denotes the space of total investment by the players, excluding player i. Please note that in the following we use the terms user and player interchangeably. Any best response $br_i(x_{-\tau}) \in br_i(x_{-\tau})$, $0 \leq br_i(x_{-\tau}) < 1$ implies that the user i may invest only in the safe resource (i.e., $br_i(x_{-\tau}) = 0$), or have a strictly positive return by investing in the CPR. Best response $br_i(x_{-\tau})$ cannot be equal to one, otherwise it will lead to a certain failure of the CPR.

Theorem 1 (Existence of PNE): For each player of the Fragile CPR game, $G = [\aleph, \{S_i\}_{i\in\aleph}, \{U_i\}_{i\in\aleph}]$ there exists a player specific value $z \in [0, 1]$ such that $br_i(x_{-\tau}) = 0, \forall x_\tau \geq z$ and an interval $\ell \subset [0, z]$ such that $br_i(x_{-\tau}) > 0, \forall x_\tau < z$ with each best response $br_i(x_{-\tau}) \in br_i(x_{-\tau})$ satisfying $br_i(x_{-\tau}) + x_{-\tau} \in \ell$ [17].

Proof of Theorem 1: For $br_i(x_{-\tau}) > 0$ the best response investment according to the first order condition of (9) is:

$$\frac{\partial \mathbf{E}(U_i)}{\partial x_i} = {x_i}^{a_i - 1} \phi(x_\tau) = 0 \tag{12}$$

where $\phi(x_\tau) = [x_i f_i'(x_\tau) + a_i f_i(x_\tau)]$. Towards proving that (12) equals zero, we examine $\phi(x_\tau)$. By applying Bolzano Theorem for $f_i'(x_\tau)$, we observe that $f_i'(0) > 0$ and $f_i'(1) < 0$. Since $f_i(x_\tau)$ in (10) is concave in x_τ, $f_i'(x_\tau)$ is monotone and decreasing, thus there exists a unique global maximum $y \in (0, 1)$ such that $f_i'(x_\tau) < 0, \forall x_\tau \in [y, 1]$. Similarly, we apply again Bolzano Theorem for $f_i(x_\tau)$, where $f_i(y) > 0$ (since $f_i(0) > 0$) and $f_i(1) < 0$. Hence, there exists a $z \in (y, 1)$ such that $f_i(x_\tau) > 0, \forall x_\tau \in [y, z]$. Subsequently, there exists the player specific interval $[y, z]$ where $\phi(x_\tau) = 0$ (due to the opposite signs of $f_i(x_\tau)$ and $f_i'(x_\tau)$), with subsequently all the best responses of the players being positive. We denote the new modified space $[y, z]$ as $S_i' \subset \ell \subset S_i$.

Lemma 1: The best response correspondence $BR_i(x_{-\tau})$ is single valued $\forall x_\tau \in S_i'$.

Proof of Lemma 1: $\mathbf{E}(U_i)$ has a critical point in the modified space S_i', guaranteeing the existence of nonzero best responses. Given that $\overline{r_i}(x_\tau)$ is increasing, $\mathbf{E}(U_i)$ is concave, thus the critical point in S_i' is a unique maximum of $\mathbf{E}(U_i)$.

Lemma 2: The best response correspondence $BR_i(x_{-\tau})$ is continuous $\forall x_\tau \in S_i'$.

Proof of Lemma 2: Based on Berge's Maximum Theorem $BR_i(x_{-\tau})$ is upper hemicontinuous and proven to be single valued from Lemma 1. This applies to each player $i \in \aleph$ by symmetry.

Hence, for each player $i \in \aleph$ there exists a PNE for the Fragile CPR game G in the modified strategy space $S_i' \subset \ell, \forall x_\tau \in [y, z] \subset [0, 1]$.

Theorem 2 (Uniqueness of PNE): The Fragile CPR game G admits a unique PNE, $x^* = \{x_i^*\}_{i\in\aleph}$ for the total investment $x_\tau^* = \sum_{I=1}^{N} x_i^*$.

Proof of Theorem 2: Towards proving the uniqueness of game's PNE, we utilize the following two propositions:

Proposition 1: For simplicity of the proof, we define:

$$g(x_\tau) = -\frac{a_i f(x_\tau)}{f'(x_\tau)} \tag{13}$$

where $g(x_\tau)$ represents the optimal nonzero investment of a player and satisfies:

$$g(BR(x_{-\tau}) + x_{-\tau}) = BR(x_{-\tau}),\ when\ BR(x_{-\tau}) > 0 \tag{14}$$

It is easily proven that $\frac{\partial g(x_\tau)}{\partial x_\tau} = -a_i + a_i \frac{f(x_\tau)}{f'(x_\tau)^2} f''(x_\tau) < 0, \forall x_\tau \in S_i'$

Proposition 2: We determine the set of players with nonzero investment as the support of a PNE [17], as follows:

$$Q \triangleq \{i \in \aleph | x_\tau^* < z\} \tag{15}$$

We initially assume that the game admits two PNEs with $x_{\tau 1}^*$ and $x_{\tau 2}^*$ with supports Q_1, Q_2 for each PNE, respectively. Without loss of generality, we consider $x_{\tau 2}^* > x_{\tau 1}^*$, with supports stemming from (15) to be $Q_2 \subseteq Q_1$, respectively. The optimal nonzero investments of all players for the above PNEs are shown below:

$$\sum_{i \in Q_1} g_i(x^*_{\tau 1}) = x^*_{\tau 1},\ and \sum_{i \in Q_2} g_i(x^*_{\tau 2}) = x^*_{\tau 2} \qquad (16)$$

For the optimal nonzero investments we have $\sum_{i \in Q_2} g_i(x^*_{\tau 1}) \leq x^*_{\tau 1} < x^*_{\tau 2} = \sum_{i \in Q_2} g_i(x^*_{\tau 2})$. However from Proposition 1, $g_i(x_\tau)$ is a strictly decreasing function, and given that $x^*_{\tau 1} < x^*_{\tau 2}$, then $g_i(x^*_{\tau 1}) > g_i(x^*_{\tau 2})$ which contradicts the previous conclusion. Hence, we conclude that the PNE of the game is unique, and $x^*_{\tau 1} = x^*_{\tau 2}$.

Definition 1: The vector $\mathbf{x}^* = \{x^*_i\}_{i \in \aleph}$ in the modified strategy set S'_i is a Pure Nash Equilibrium of the Fragile CPR game G, if for each user i the following holds:

$$U_i(\mathbf{x}^*) \geq U_i(x_i), \forall x_i \in S'_i \qquad (17)$$

B. Convergence

In this part, we discuss game's G convergence to its unique PNE $\mathbf{x}^* = \{x^*_i\}_{i \in \aleph}$. Fragile CPR games are categorized as Weak Strategic Substitutes games with aggregation [19], modeled as below:

Definition 2: A game $\Gamma = [\aleph, \{S'_i\}_{i \in \aleph}, \{U_i\}_{i \in \aleph}]$ is a Weak Strategic Substitutes game with aggregation, if the corresponding player utilities are defined as $u_i : S'_i \times \overline{S'}_{-i} \rightarrow \mathbb{R}$ and the the best response correspondence satisfies the following:

1) $BR_i(x_{-\tau})$ =argmax$\mathbf{E}(U_i(x_\tau, x_{-\tau}))$, $\forall x_-\tau \in \overline{S'}_{-i}$
2) $BR_i(x_{-\tau})$ is continuous on $\overline{S'}_{-i}$
3) $BR_i(x_{-\tau})$ is decreasing in $\overline{S'}_{-i}$

Properties 1 and 2 already hold as proven in Lemmas 1 and 2 respectively, therefore, in the following we focus only on the 3rd property.

Lemma 3: The best response correspondence $BR_i(x_{-\tau})$ is decreasing $\forall x_\tau \in \overline{S'}_{-i}$.

Proof of Lemma 3: Let $x_{\tau 1} = BR_i(x_{-\tau 1})$ and $x_{\tau 2} = BR_i(x_{-\tau 2})$ with $x_{-\tau 2} > x_{-\tau 1} \in \overline{S'}_{-i}$. As introduced in Proposition 1, $g(x_\tau)$ is strictly decreasing, thus, according to (14), $x_{\tau 1} = g(x_{\tau_1} + x_{-\tau_1})$ and $x_{\tau 2} = g(x_{\tau_2} + x_{-\tau_2})$. From Theorem 1, it is stated that $BR_i(x_{-\tau}) + x_{-\tau} \in \ell$. Hence, if $x_{\tau 2} > x_{\tau 1}$ then $x_{\tau 2} = g(x_{\tau 2} + x_{-\tau 2}) < g(x_{\tau 1} + x_{-\tau 1}) = x_{\tau 1}$ which is contradicting.

Subsequently, the game's best response dynamics decrease in the aggregate strategies of the users, guaranteeing the convergence to the examined PNE.

V. Algorithm

In this section we present an iterative and low complexity algorithm reaching the game's unique PNE point, starting from any initial feasible set of investment values. The algorithm operates under a distributed manner, with each user to declare his behavioral specific parameters and selections for the upcoming transmission. The *"ALLURE-U"* algorithm, i.e., **Al**location of **L**icensed and **U**nlicensed **Re**sources under probabilistic **U**ncertainty, acts as the common interface between users and the ISP dynamically managing spectrum among the licensed and unlicensed bands, and integrates users' decision part with the resource allocation process.

Initially users configure their topological and behavioral parameters in terms of exhibiting a risk seeking or averse behavior towards CPR failure. According to NOMA standards the ISP applies the SIC technique and observes users' transmission based on their choice to invest in each band. The ISP collects all the information from the power allocation of each user to the unlicensed band, and broadcasts (alongside the interference) the failure probability. Users depending on their unique preferences (determined by the parameters a_i and k_i) reconsider their decisions under the rising probability of collapse of the CPR, and may accept to transmit also with the licensed band which however provides lower bandwidth, or decide to keep transmitting only via the unlicensed band.

Eventually the algorithm functions under two potential outcomes. The first being the convergence of the system to the PNE point, where all users manage to optimize their transmission given the strategies of the rest of the players, while on the other side the algorithm may terminate its operation due to CPR collapse from over-exploitation. In the latter case, only the users who partially invested to the licensed band manage to transmit, whilst the rest of the users reach a state of non operation, despite devoting all of their transmission power to the unlicensed band. Due to the non-centralized nature of the algorithm and the parallel execution of the decisions, as well as the low data exchange requirements, the algorithm converges very fast to the PNE point or identifies the collapse of the resource in very few steps, i.e. in practice in less than 10 iterations. The simplified arithmetic calculations performed, establish the convergence of the ALLURE-U framework with minimal demand for computational complexity. The algorithm was tested in an Intel(R) Core(TM) i7-7500U CPU @ 2.70 GHz 2.90 GHz laptop with 8.00 GB of available RAM and its average run time per user was less than $0.3msec$. The above result ensures the adaptability and applicability of the proposed scheme to realistic implementations, where low duration timeslots (i.e $0.5msec$) are regarded.

VI. Performance Evaluation

A. Evaluation Setting and Parameters

In this section we provide a series of numerical results evaluating the performance and the inherent attributes of the proposed prospect theoretic framework for a dual band wireless network operating under NOMA. Initially, we focus on a basic scenario where users exhibit common risk averse behavior (i.e. homogeneous population) regarding their preferences for the unlicensed band over the licensed one, in order to gain some insight about the process of optimal user power investment in each band, as well as the corresponding utility obtained. Subsequently, the effect of user heterogeneity is investigated in terms of both loss aversion and sensitivity parameters, by assuming that some users may modify their behavior towards a more risk seeking attitude, therefore impacting their competitor user decisions, and the overall system operation as a whole. Furthermore, the case of aggressive

Algorithm 1 ALLURE-U: Allocation of Licensed and Unlicensed Resources under probabilistic Uncertainty

Require:
User number N, specific constants k_i, a_i and topological coordinates

$ite \leftarrow 1$
$collapse^{(ite)} \leftarrow 0$
$convergence^{(ite)} \leftarrow 0$
$CPR\ Size \leftarrow unlicensed\ bandwidth$
Calculate and sort channel gains G_i according to SIC
Assign random value to initial user investment $x_i^{(ite)}$
while $convergence^{(ite)}$=0 **and** $collapse^{(ite)}$=0 **do**
  Split transmission power $P_i^{\prime(ite)}, P_i^{\prime\prime(ite)}$ per band according to $x_i^{(ite)}$ and calculate intracell interference;
  Calculate utility $U_i^{(ite)}$ according to (9)
  for all $x_i \in [0,1]$ **do**
    x_i^{*}=$argmax_{x_i} U_i$
    if $U_i > U_i^{(ite)}$ **then**
      $x_i^{(ite+1)} \leftarrow x_i^{*}$ **and** $U_i^{(ite+1)} \leftarrow U_i$
    end if
  end for
  Calculate $normalized\ x_\tau = \dfrac{\sum_1^N x_i^{(ite+1)}}{N}$
  if $\sum R_i^{unlicensed} > CPR\ Size$ **then**
    $collapse^{(ite+1)} \leftarrow 1$
  end if
  if $x_i^{(ite+1)} - x_i^{(ite)} < \epsilon$ **then**
    $convergence^{(ite+1)} \leftarrow 1$
  end if
  $ite \leftarrow ite + 1$
end while
return
User investment to unlicensed resource x_i and $collapse$ to confirm if the band still operates

user investment leading to failure of the CPR due to the potential over-exploitation of the unlicensed band by the users, is also examined and evaluated. Finally, a direct comparison of our proposed prospect theoretic framework (i.e. ALLURE-U Algorithm) against a conventional EUT based framework is performed, clearly demonstrating its superiority, and stressing the fact that the realistic modeling empowered by PT, properly reflects user behavioral preferences against risk, indicating the deviations that the EUT based approaches fail to capture.

The uplink of a NOMA wireless network of radius $R = 2.5km$ is considered, with 20 users being accommodated within its range. System's total bandwidth is set at $W = 4MHz$, where for demonstration purposes 90% is allocated to the unlicensed band and the remaining 10% to the licensed band [20]. The user transmission power is set to $P_{max} = 0.2Watts$, with all users splitting it between the two bands.

B. Common User Behavior - Homogeneous Population

Originally, we examine users' investment considering common risk aversion and sensitivity parameters (i.e. $k_i = k = 20$

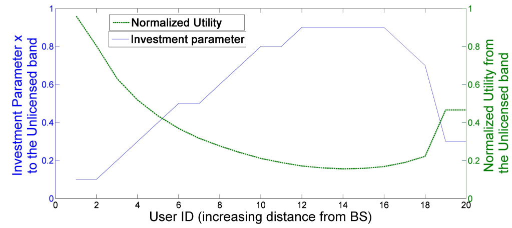

Fig. 2: Investment parameter x_i and normalized utility per user

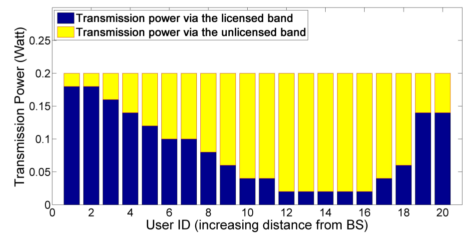

Fig. 3: Power allocation among bands per user

and $a_i = a = 0.1$, $\forall i \in \aleph$). This suggests a homogeneous perception of users against resource failure, hence the main factors affecting users' transmission being their channel conditions and sensed interference. Fig. 2 illustrates each user i investment x_i, as well as his normalized utility obtained from his transmission to the unlicensed band. We observe that users with intermediate distance from the BS almost fully transmit under the unlicensed band, whereas users closer to or further away from the BS choose to split their transmission power to both bands. The above stems from the application of the SIC technique in NOMA, where users with medium distance from the BS sense significant interference while they are also affected by their own channel conditions, which impacts their corresponding utility. This drove their decision towards transmitting almost fully via the unlicensed band given its higher bandwidth capacity and potential. On the contrary, the users closer to the BS due to their superior channel conditions, and the most distant ones who sense lower interference due to SIC, share the advantage of a less strenuous transmission extracting broader utility surplus. Hence, they invest less in the unlicensed band and transmit via the licensed band as well, ensuring their operation even in case of CPR failure. Accordingly, Fig. 3 depicts the users' transmission power allocation between unlicensed and licensed bands.

C. Differentiating User Behavior - Heterogeneous Population

In this subsection we investigate a scenario under which some of the worse performing users become less risk averse and decide to excessively invest in the CPR. For demonstration purposes we consider that users 11-13 increase their sensitivity parameter a_i from 0.1 to 0.8, while the rest of the users retain their original risk perceptions. Specifically, in Fig. 4 we present achieved users' data rates in a comparative manner, under the heterogeneous scenario considered here (right graph) and the homogeneous one considered in previous subsection (left graph), aiming in particular at studying the impact of the

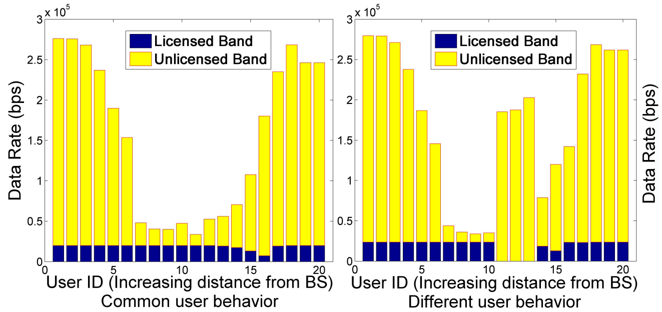

Fig. 4: Data rates with (a) common user behavior and (b) diverging 11-13 user behavior

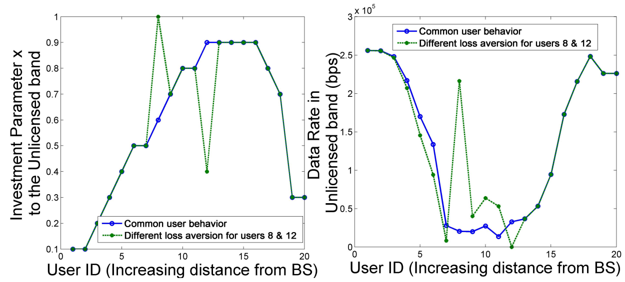

Fig. 5: (a) Investment parameter x_i and (b) data rate per user under changing k_i for users 8 and 12

increase of the sensitivity parameter a_i. All other things being equal, users 11-13 significantly improve their data rates due to fully investing their power to the higher capacity unlicensed band (no transmission with the licensed band). On the other hand, the rest of the users of the network are also impacted by this change of behavior of users 11-13, with data rates for some of them deteriorating (e.g., users 9 and 10) in the unlicensed band, while on the contrary a positive effect is observed through the rise in data rate per user in the licensed band. Subsequently, we adjust the loss aversion parameter k_i selectively for users 8 and 12, such as to eliminate the risk aversion for user 8, while increases it tenfold for user 12. As depicted in Fig. 5, user's 8 investment to the CPR reaches 100%, while user 12 becomes more conservative compared to the original scenario with his investment to the CPR dropping from approximately 90% to 40%.

D. Collapse of the Unlicensed band

Please note that in the previous subsections, all experiments were conducted under conditions which did not endanger the operation of the unlicensed band cumulatively. In the following, we address the case where aggregate investment possibly exceeds spectrum's capacity to address user demand, and therefore CPR may fail. Accordingly, we investigate the network's performance as users' average sensitivity gradually increases. Based on the results presented in Table II, an increase in the average user sensitivity (i.e., parameter a_i) results in higher investment x_i by all users to the CPR. Users become more risk seeking, with the number of users of the licensed band dropping from twenty to just four, at the moment that the unlicensed band collapses. The unlicensed bandwidth utilization initially rises with higher investment as expected, however it eventually fails when demand surpasses the available spectrum. Therefore, only the four users who kept transmitting in the licensed band will still operate, whereas the remaining 80% of users investing to the unlicensed band eventually fail to transmit.

E. Comparison with EUT

It is noted that one of the key differentiating factors of our work and PT in general, is that it considers the user behavioral deviations from the traditional EUT, where the latter regards users as neutral agents who aim at selfishly maximizing their utility reflecting their degree of satisfaction from the QoS they receive. In this subsection, we present a comparative case between our proposed prospect theoretic framework and an EUT based approach, with the latter characterized by the absence of user risk perceptions. From Table III, it is clearly shown that EUT instructs users to invest in the CPR due to the higher available spectrum. Therefore, users' aggregate investment exceeds the threshold that the band can tolerate and since the utilization demand surpasses 100%, it collapses. On the contrary, according to our framework users follow a risk averse attitude and adopt a more conservative position considering the fragility of the CPR, and therefore decreasing the potential of failure.

In Fig. 6 we compare users' transmission power among our proposed prospect theoretic approach (i.e. ALLURE-U Algorithm) and two different cases of EUT realization, where in the first one the band collapses (fails) while in the second one though it remains active it still operates with almost full utilization. ALLURE-U algorithm delivers lower power levels compared to both EUT cases, since users do not exhaust their full power transmission to the unlicensed band, thus protecting it from potential over-exploitation. On the other hand, under EUT realizations, users transmit at higher power in order to extract additional bandwidth and return from the available spectrum. This reverses (cancels) the positive impact of SIC for users far from the BS, while simultaneously increases the probability of CPR failure since users lack the sophistication of considering the potential risks in their final decisions.

TABLE II: Unlicensed Band Collapse

Mean a_i	**Mean** x_i	**Licensed Users**	**Unlicensed Spectrum Utilization**	**Unlicensed Band Status**
0.10	0.58	20	74.86%	*active*
0.20	0.61	17	86.25%	*active*
0.24	0.74	9	97.02%	*active*
0.39	0.91	4	108.99%	*fails*

TABLE III: Unlicensed Band Transmission: PT vs EUT

System Model	**Mean Power (Watt)**	**Unlicensed Spectrum Utilization**	**Unlicensed Band Status**
EUT	0.1467	101.65%	*fails*
PT	0.1160	74.86%	*active*

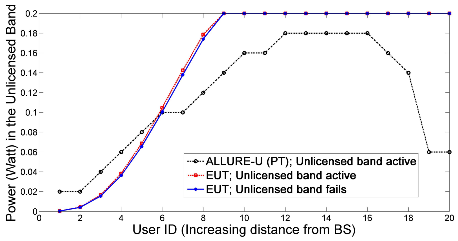

Fig. 6: Transmission power per user ID under PT and EUT

VII. Concluding Remarks

In this paper the subject of spectrum management in a wireless network, supporting both licensed and unlicensed bands and operating under NOMA access technology, is investigated. Aiming at a pragmatic approach towards resource management and decision making within this setting, the unlicensed band is considered as a common pool of resource, subject to the risk of collapse from potential over-exploitation, with the failure probability rising with user aggregate investment. In order to more accurately reflect user behavior under risk, the above problem was formulated under a prospect theoretic perspective, which in contrast to traditional models adopting EUT, inherently considers and values users' risk aversion and probability weighting between gains and losses.

Taking into account the potential interdependence of users' decisions with respect to their transmissions, the problem was treated as a non-cooperative CPR game among the users, while its convergence to a unique Pure Nash Equilibrium has been proven, and an algorithm (i.e. ALLURE-U) that determines the optimal power investment of each user to the corresponding bands in a distributed manner, was designed. Detailed numerical results were presented highlighting the operation and superiority of the proposed framework, while providing useful insights about user decisions under realistic conditions and behaviors, and demonstrating the benefits stemming from the proper shared optimization of both bands.

Within the emerging user-centric 5G era, the paradigm shift towards decentralized and ad-hoc network designs, where users interdependent behaviors and decisions play a key role in system's self optimization, highlights the dynamics and potential of prospect theoretic approaches in several aspects of communication systems and related decision making processes. Telecommunication markets are transforming rapidly with the rise of Internet of Things, incorporating hybrid access schemes such as Wi-Fi and small cell or femto cell integration, device-to-device (D2D) and machine-to-machine (M2M) communications, therefore the design of agile techniques for efficient spectrum sharing and support of multi-tier architectures offering service diversification and user heterogeneity, become of high practical and research importance.

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