Introduction to Independent Component Analysis

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Overview

- (Brief) ICA Intro
- ICA of fMRI
- Sorting/Calibration
- Validation
- Demo single-subject ICA

Overview

\[ X = A \times S \]

Modeling the Brain?

From "Science with a Smile" by Subramanian Raman

- "All models are wrong, but some are useful!"
- "I believe in ignorance-based methods because humans have a lot of ignorance and we should play to our strong suit."
  - Eric Lander, Whitehead Institute, M.I.T.

Independent Component Analysis

- Goal: Separate sources from a linear mixture
- Model:
  \[ X = AS \]
  - X: Mixture
  - A: Mixing coefficients
  - S: Sources
- \[ S = WX \]
- Assumptions
  - Linear mixing
  - Independence of sources
  - Non-gaussian sources

ICA vs PCA

Uncorrelated: \[ E\{y_1y_2\} = E\{x_1\}E\{y_2\} \]
Independent: \[ p(y_1, y_2) = p(y_1)p(y_2) \]
\[ \Rightarrow E\{h(x_1)h(y_2)\} = E\{h(x_1)\}E\{h(y_2)\} \]

PCA finds directions of maximal variance (using second order statistics)
ICA finds directions which maximize independence (using higher order statistics)
ICA vs. PCA

PCA finds directions of maximal variance (using second order statistics)

ICA finds directions which maximize independence (using higher order statistics)

ICA Example

- Mixing simple signals: sinus + chainsaw.

ICA Maximizes Nongaussianity

- Many real-world data sets have supergaussian distributions.
  - The random variables take relatively more often values that are very close to zero or are large.

FastICA demo (mixtures)
ICA of FMRI

General Linear Model

1. Model
(1 or more Regressors)

2. Data

3. Fitting the Model to the Data at each voxel

\[ y(j) = \beta_x + \sum_{i} \beta_i x_i(j) + e(j) \]

Regression Results

A little more detail

ICA Example

ICA Halloween (Un)Mixer!

\[ X = A \times S \]

- background
- Time
- candle 1
- candle 2
- candle 3
- Candle out
ICA of fMRI

The ICA model assumes the fMRI data, \( x \), is a linear mixture of statistically independent sources, \( s \):

\[
x = As
\]

\( p(x|x) = p(x)p(s) \)

The goal of ICA is to separate the sources given the mixed data and thus determine the \( s \) and \( A \) matrices.

ICA of fMRI Data

ICA of fMRI Data

Signal Types

Task related
Cardiac
Motion
Vasomotor oscillation/High order visual

Motion Artifact

Motion-related signal due to mouth movement from inferior temporal and orbitofrontal regions

Artifact Detection and Reduction

Note: PREPROCESSING MAY DIFFER FOR Art. Hunting Approach

Eye movements
N/2 Nyquist Ghost

Source: Christian Beckmann’s “Little Shop of MRI Horrors”: http://www.fmrib.ox.ac.uk/~beckmann/homepage/academis/littleshop/
Spatial versus Temporal ICA

• Does it matter?
• Why is spatial ICA more common?
• Some examples:

Temporally and Spatially low-correlated Components

<table>
<thead>
<tr>
<th>SICA</th>
<th>SPM</th>
<th>TICA</th>
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Spatially Dependent Components

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A few points

• ICA is not model-free! (but does make no assumptions about shape of time course)
• Fishing vs. Hypothesizing
  • ICA is a data-driven approach, but that does not mean you should automatically associate it with fishing (flexible analysis vs. hypothesis-based study)
  • The key here is what are the questions being asked, and what is the approach that will be used to answer these questions
• There are tools available for asking focused questions about an ICA of fMRI analysis

Uses of ICA

• Improving fit to task-related components
• Find areas of ‘activation’ which respond in a more complex way to an external stimulus
• Artifact Reduction/Filtering
• Examination of temporally coherent, but not necessarily task-related components
• Data exploration of unpredicted structure

Sorting
Ambiguities of ICA: Sorting/Scaling

• ICA is modeling the data as a linear combination of images and time courses
• Why is sorting necessary?
  • Permutation ambiguity: \( X = AS = (AP^{-1})(PS) \)
• Why is scaling/calibration necessary?

\[
data = tc_1 \ast im_1 + tc_2 \ast im_2 + E
\]
\[
data = (a \ast tc_1) \ast \left( \frac{1}{a} \ast im_1 \right) + (b \ast tc_2) \ast \left( \frac{1}{b} \ast im_2 \right) + E
\]

Types of Sorting

• Temporal Sorting
• Correlation
• Multiple regression
• Others? (skew, kurtosis, power spectra, etc.)
• Spatial Sorting
• Correlation (w/ mask or SPM)
• Maximum value (w/i mask)
• Multiple regression
• Multi-variate sorting
  • SVM Approaches (Formisano)

Example

Number of Components

• Too many -> over-splitting of the components
• Too few -> over-clumping of the components
• How to choose?
  • Between 20 and 40 appears to be a reasonable choice for typical fMRI experiment
  • Tools for estimating this number are available in GIFT and other ICA software programs (AIC/MDL/BIC)
  • Post-ICA clustering is also used to address this issue

Number of Components (Order Selection)

\[
AIC(N) = -2M \left( X - \hat{X} \right) \mathbb{E} \left[ \hat{\theta} \right] + \frac{1}{2} \left( 1 + XK + \frac{1}{2} (N-1) \right)
\]
\[
MDL(N) = -M \left( K - \hat{N} \right) \mathbb{E} \left[ \hat{\theta} \right] + \frac{1}{2} \left( 1 + XK + \frac{1}{2} (N-1) \right) \ln M
\]

\[
AIC = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\hat{\lambda}_i - \lambda_i}{\hat{\lambda}_i} \right)
\]


Correction for correlated samples \([V. D. Calhoun, T. Adali, and V. D. Calhoun, "Sample Dependence Corrected for Order Selection in fMRI Analysis," in Proc. AIBI, Washington, DC, 2005.]\)
Validation

- **Algorithm Differences**

- **Algorithm & Preprocessing Differences**

- **Cluster Validation**

- **Test/retest Performance**
Clustering of five algorithms using ICASSO

Infomax, FICA1, FICA2, FICA3, JADE

Three Review Articles

Three Review Articles

A Few Software Packages

- The ICA-DTU toolbox
  - FastICA
  - Non-negative ICA algorithms
  - MatLab specific with demo data
- FMRIB Software Library, which includes the ICA tool MELODIC
  - FastICA
  - Complete Package
- AnalyzeFMRI
  - FastICA
- BrainVoyager
  - FastICA
  - Complete Package

Group ICA of fMRI Toolbox (GIFT)

- FMRILAB
  - Multiple ICA algorithms
  - Not fMRI specific although one fMRI example included
- ICA
  - Component Estimation
  - Detrend level
  - Scaling
  - Sorting
  - Component Explorer
  - Orthogonal Viewer
  - Composite Viewer
  - Examine Regression Parameters
  - Artifact Removal Tool

Demo

- Single-subject ICA
  - Component Estimation
  - Defaults file
    - Detrend level
    - Scaling
    - Sorting
  - Component Explorer
  - Orthogonal Viewer
  - Composite Viewer
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