Review of Last Lecture

- Basic MOS Transistor
- MOSFET Operations
- Cutoff, Linear, and Saturation Regions in MOSFET
- NMOS and PMOS Structures
Today's Lecture

- Device Model for Linear Region
- Device Model for Saturation Region
- Channel Length Modulation

I-V Characteristic of MOSFET

![I-V Characteristic Diagram]

- V_{gs}
- V_{ds}
- I_{ds}
- V_{gs} = 0 V
- V_{gs} = 1.0 V
- V_{gs} = 1.5 V
- V_{gs} = 2.0 V
- V_{gs} = 2.5 V

![Graph with I-V Characteristics]
Device Operation: Linear (Ohmic) Region

Question: What is the MOS current equation in linear (or ohmic) region?

\[ I_D = \mu_C W \left( V_{GS} - V_T \right) \frac{V_{DS} - V_G}{2} \]

When: \( V_{DS} < V_{GS} - V_T \)

Channel Charge Density Calculation

What is the drain current in this configuration?

\[ Q = C_{ox} \Delta V = C_{ox} \left( V_{GS} - V_T \right) \]

Gate oxide capacitance per unit area

\[ C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} \]

Remember: \( V_T \) is the amount of gate voltage that you need to apply to “create” the channel.
Device Operation: Linear Region

\[ Q(y) = C_{ox} \Delta V(y) = C_{ox} (V_{GS} - V_T - V(y)) \]

\[ 0 < y < L \]

\[ I_D = \mu_n Q(y) E(y) W = \mu_n C_{ox} W (V_{GS} - V_T - V(y)) E(y) = \mu_n C_{ox} W (V_{GS} - V_T - V(y)) \frac{dV}{dy} \]

\[ I_D dy = \mu_n C_{ox} W (V_{GS} - V_T - V(y)) dV \Rightarrow \int_0^L I_D dy = \int_0^{V_{GS}} \mu_n C_{ox} W (V_{GS} - V_T - V(y)) dV \]

\[ I_D = \mu_n C_{ox} \frac{W}{L} \left( (V_{GS} - V_T) V_{GS} - \frac{V_{DS}^2}{2} \right) \]
**Question:** What is the MOS current equation in saturation region?

\[ I_{DS} = \frac{\mu C_{ox} W}{2} \left(V_{GS} - V_T\right)^2 \]

\[ I'_{DS} = I_{DS} \left(t + \lambda V_{DS}\right) \]

- \( V_{GD} - V_{GS} - V_{DS} \); so as \( V_{DS} \) increases \( V_{GD} \) will no longer exceed \( V_T \), thus the charge density in the channel near the drain will decrease.
- If \( V_{DS} = V_{GS} - V_T \) then \( V_{GD} = V_T \). At this operating point the charge density in the channel would diminish to zero right at the drain.
- When \( V_{DS} = V_{GS} - V_T \) the device is transitioning to saturation mode.

**Device Operation: Saturation**
Device Operation: Saturation

- As $V_{DS}$ increases beyond $V_{GS} - V_T$, the charge density in the channel reaches zero prior to reaching the drain. At this point mobile charges are injected into the depletion region and swept to the drain.

- The early termination of the channel is termed “pinch off”.

- $I_{DS}$ stops increasing with $V_{DS}$, and the device is said to be “saturated”.

Saturation Region Analogy

[Diagram showing a water analogy for the saturation region of a transistor, with water in, channel, and water out, and a separate diagram for saturated flow.]
**Device Current in Saturation Region**

- Since $I_{DS}$ does not increase with increasing $V_{DS}$ beyond $V_{DS} = V_{GS} - V_T$, one can find the equation for $I_{DS}$ in saturation by substituting $V_{DS} = V_{GS} - V_T$ into the $I_{DS}$ equation for linear mode:

  \[ I_{DS} = \mu_p C_m \frac{W}{L} \left[ (V_{GS} - V_T)\left(V_{GS} - V_T\right) - \frac{(V_{GS} - V_T)^2}{2} \right] \]

  \[ I_{DS} = \mu_p C_m \frac{W}{2L} (V_{GS} - V_T)^2 \]

- Our previous view of saturation is too simple. $I_{DS}$ will still have some $V_{DS}$ dependence for $V_{DS}$ values greater than $V_{GS} - V_T$.

- As $V_{DS}$ increases beyond $V_{GS} - V_T$ more and more of the channel becomes “punched off”. Thus the effective channel length ($L'$) is reduced by $\Delta L$.

- This $\Delta L$ is proportional to $\sqrt{V_{DS} - V_{DSAT}}$; however one will discover that $\frac{1}{L - k\sqrt{V_{DS} - V_{DSAT}}}$ is a fairly linear function. Therefore …

**Channel Length Modulation**

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Channel Length Modulation

- The effect of channel length modulation is typically modeled with an empirical linear factor $\lambda$.

- Thus the equation for $I_{DS}$ in saturation becomes:

$$I_{DS} = \mu_n C_{ox} \frac{W}{2L} (V_{GS} - V_T) \sqrt{1 + \lambda V_{DS}}$$

where $\lambda$ = “channel length modulation factor”

Device Operation: I-V curves

$$I_{DS} = K'_n \frac{W}{L} \left[ (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right] (1 + \lambda V_{DS})$$

$$V_{DS} < V_{GS} - V_T$$

$$V_{DS} > V_{GS} - V_T$$

$$K'_n = \mu_n C_{ox}$$

Linear region

Saturation region

$V_{GS}$

$I_{DS}$

$V_{DS}$

$I/\lambda$ 0