

Distributed Estimation in a Power Constrained Sensor Network

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Abstract— This paper derives the optimum non-uniform quantization scheme for a distributed estimation problem based on noisy observations in a wireless sensor network. The optimal quantization scheme minimizes the mean squared estimation error subjected simultaneously to both a total system power as well as individual node power constraints. According to the derived optimal scheme, sensors with poor signal-to-noise ratios (SNR) refrain from transmitting their observations. The rest of the sensors transmit their quantized observations. The number of quantized bits depends on the SNR and the maximum power constraint at each node. The numerical results show that compared to a uniform power allocation scheme, the proposed scheme achieves significantly smaller Mean Squared Error (MSE) for the same average power consumption.

I. INTRODUCTION

Wireless distributed sensor networks have high potential to be important in number of applications such as border surveillance, environmental monitoring and survivor detection, to name a few. Even though there are several challenges in implementation of these networks, energy constraints are considered to be one of the important hard limitations, as the improvements in battery technologies is known to be very slow [1], [2].

Most of the wireless sensor networks are battery operated and these batteries may not be rechargeable on-site. In most applications replacing these batteries often may not also be possible. On the other hand it is important to maintain a certain quality of performance in a given sensor system. In most applications decisions will be made off-site depending on the observations. Thus to make correct decisions it is the quality of the information received by the decision maker is of concern. To accomplish these tasks we need to generate quality decisions, subjected to the available energy resource. Thus energy management in these networks is critical. We are motivated from the work in [3], where authors minimize the maximum total power consumption while keeping the MSE below an acceptable threshold. According to this scheme, however, sensors with better

observations may be forced to spend higher percentage of the total available power, and as a result may die out quickly. This will degrade the performance of the system as a whole.

In this paper, we focus on optimal quantization of sensor observations to minimize the mean squared estimation error subjected to constraints on both the total system power as well as the individual node power. We are motivated to impose both power constraints on the system because if the system is subjected to only a maximum total power constraint, under non-uniform quantization, it is possible for few sensors to spend more power than the rest of the sensors and run out of power quickly. These sensors may be located at critical locations and have better observations. As a result the reliability of the system may be compromised. On the other hand, if the system is subjected to only the individual power constraints, all of the sensors will try to operate at the maximum available power. Nodes with poor observations will also spend more power, although without contributing much to the final estimation performance. To overcome these problems, in the proposed scheme only the nodes which have *better* SNR will send quantized observations to the fusion center. The number of quantization levels is limited by their individual power constraints.

The rest of the paper is organized as follows: Section II formulates the optimization problem. The design of optimal quantization scheme is described in Section III. The numerical analysis is presented in Section IV and finally concluding remarks are given in Section V.

II. PROBLEM FORMULATION

We consider a sensor network with n nodes. Each node has an observation y_k given by

$$y_k = \theta + v_k, \quad k = 1, \dots, n.$$

where θ is a deterministic parameter to be estimated and v_k is the observation noise. We assume that the observation noise is zero mean and uncorrelated with

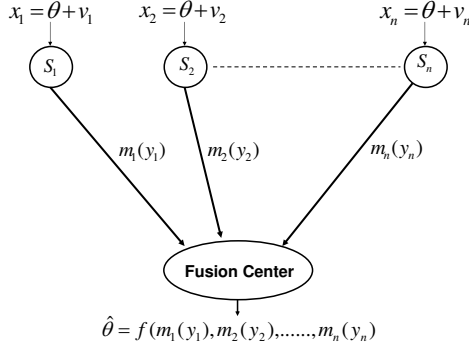


Fig. 1. Sensor Network Scheme

variance σ_k^2 but otherwise unknown. The observation range of the sensors is assumed to be $[-W, W]$. Each sensor locally quantizes the analog observation y_k into an unbiased discrete message $m_k(y_k)$ of length L_k bits as in [3]. These local estimates are transmitted to the fusion center over a wireless channel. The channel between the fusion center and each sensor is assumed to be corrupted by additive white Gaussian noise (AWGN). The channel is assumed to be lossy and the path loss is given by a_k (note that if the distance between the k^{th} sensor and the fusion center is d_k then $a_k \propto 1/d_k^r$, where r is the path loss exponent). At the fusion center final estimation is performed by combining these discrete messages. We assume that the fusion center has the knowledge of the observation noise variance σ_k^2 for all nodes and estimates θ by employing the quasi Best Linear Unbiased Estimator (BLUE) proposed in [3]. If the total channel distortion constant is p_0 , then an upper bound for the MSE can be given as [3],

$$D \leq (1 + p_0) \left(\sum_{k=1}^n \frac{1}{\sigma_k^2 + \delta_k^2} \right)^{-1},$$

where we have defined

$$\delta_k^2 = \frac{W^2}{(2^{L_k} - 1)^2}.$$

We further assume that M -ary QAM modulation is used with TDMA subjected to a bit error probability of p_b^k at sensor k . Thus, the average transmission power of sensor k can be given by the following expression [3], [4]:

$$P_k = c_k a_k B_s \ln \left(\frac{2}{p_b^k} \right) (2^{L_k} - 1),$$

where,

$$c_k = 2N_f N_0 G_0,$$

B_s is the sampling rate (assumed to be equal to the symbol transmission rate), N_f is the receiver noise figure, N_0 is the single sided power spectral density of the channel noise and G_0 is a system constant defined as in [5]. In this study we constrain the L^2 norm of both total and individual sensor powers P_T^2 and P_I^2 respectively, for computational reasons. Note that here we assume all nodes are subjected to the same individual maximum power constraint P_I^2 . Thus the design problem can be formulated as below:

$$\min (1 + p_0) \left(\sum_{k=1}^n \frac{1}{\sigma_k^2 + \delta_k^2} \right)^{-1}, \quad (1)$$

such that

$$\begin{aligned} \left(c_k a_k B_s \ln \left(\frac{2}{p_b^k} \right) (2^{L_k} - 1) \right)^2 &\leq P_I^2 \quad \text{for } k = 1, \dots, n, \\ \sum_{k=1}^n \left(c_k a_k B_s \ln \left(\frac{2}{p_b^k} \right) (2^{L_k} - 1) \right)^2 &\leq P_T^2, \\ L_k &\geq 0 \quad \text{for } k = 1, \dots, n. \end{aligned}$$

We assume that c_k 's, B_s 's, and p_b^k 's are the same for all the nodes. Further, if we define

$$\beta_k = (2^{L_k} - 1)^2,$$

and

$$a'_k = c_k B_s \ln \left(\frac{2}{p_b^k} \right) a_k,$$

the problem (1) could equivalently be written as the following:

$$\max \left(\sum_{k=1}^n \frac{\beta_k}{\sigma_k^2 \beta_k + W^2} \right), \quad (2)$$

such that

$$\begin{aligned} \sum_{k=1}^n a_k'^2 \beta_k &\leq P_T^2, \\ a_k'^2 \beta_k &\leq P_I^2 \quad \text{for } k = 1, \dots, n, \\ \beta_k &\geq 0 \quad \text{for } k = 1, \dots, n. \end{aligned}$$

III. OPTIMAL POWER ALLOCATION SOLUTION

If the square of the total system power is greater than the sum of the squares of the individual power constraints then even if all the sensors operate at their optimal power the square of the total power consumed by the system is equal to the sum of the squares of

the individual power constraints. Thus the effective total system power constraint can be given as,

$$\widetilde{P}_T^2 = \begin{cases} P_T^2 & \text{if } P_T^2 < \sum_{k=1}^n P_I^2 \\ nP_I^2 & \text{otherwise.} \end{cases}$$

The Lagrangian G corresponding to the problem (2) is:

$$G(L, \lambda_0, \mu_{1,k}, \mu_{2,k}) = \sum_{k=1}^n \frac{\beta_k}{\sigma_k^2 \beta_k + W^2} + \lambda_0 (\widetilde{P}_T^2 - \sum_{k=1}^n a_k'^2 \beta_k) + \sum_{k=1}^n \mu_{1,k} (P_I^2 - a_k'^2 \beta_k) + \sum_{k=1}^n \mu_{2,k} \beta_k.$$

where μ_M is as defined in (3) above and we have defined

$$g(k) = -\frac{W^2 \bar{a}_k'}{\sigma_k^2} a_k'^2 + \frac{\widetilde{P}_T^2 - P_I^2 + W^2 \bar{a}_k'^2}{\sigma_k^2} a_k' - P_I^2 \bar{a}_k',$$

where,

The KKT conditions for the above Lagrangian are given by

$$\frac{W^2}{(W^2 + \sigma_k^2 \beta_k)^2} - \lambda_0 a_k'^2 - \mu_{1,k} a_k'^2 + \mu_{2,k} = 0,$$

$$\lambda_0 (\widetilde{P}_T^2 - \sum_{k=1}^n a_k'^2 \beta_k) + \mu_{1,k} (P_I^2 - a_k'^2 \beta_k) + \mu_{2,k} \beta_k = 0.$$

Without losing any generality, suppose that the sensor that is furthest from the fusion center is labelled $k = 1$ and the one closest is labelled $k = n$, so that $a_1' > a_2' > \dots > a_n'$. Let us define

$$\mu_k = \frac{W \sum_{j=k}^n \frac{a_j'}{\sigma_j^2}}{\widetilde{P}_T^2 + W^2 \sum_{j=k}^n \frac{a_j'^2}{\sigma_j^2}} \quad \text{for } k = 1, \dots, n,$$

and the function $f(\cdot)$ as

$$f(a_k') = W a_k' \left(\frac{W \sum_{j=k}^n \frac{a_j'}{\sigma_j^2}}{\widetilde{P}_T^2 + W^2 \sum_{j=k}^n \frac{a_j'^2}{\sigma_j^2}} \right) = a_k' \mu_k. \quad (3)$$

Then there exist a unique M such that if $f(a_{M-1}') > 1$ and $f(a_M') < 1$ then $f(a_k') < 1$ for all k such that $M \leq k \leq n$: i.e. $M = \min \{k : f(a_k') < 1 \text{ and } k = 1, \dots, n\}$.

With these definitions, first let $\lambda_0 \neq 0$, $\mu_{1,k} = 0$, and $\mu_{2,k} = 0$. Then the conditions for a solution to the optimization problem can be written as,

$$\frac{W^2}{(W^2 + \sigma_k^2 \beta_k)^2} - \lambda_0 a_k'^2 = 0$$

$$\lambda_0 (\widetilde{P}_T^2 - \sum_{k=1}^n a_k'^2 \beta_k) = 0$$

$$(P_I^2 - a_k'^2 \beta_k) \geq 0.$$

$$\beta_k \geq 0$$

By using (3) the solution to the above conditions can then be written as

$$\beta_k = \begin{cases} 0 & \text{if } f(a_k') > 1 \\ \frac{\frac{W}{\sqrt{\mu_M a_k'}} - W^2}{\sigma_k^2} & \text{if } f(a_k') < 1 \text{ and } g(k) < 0 \end{cases}, \quad (4)$$

$$\bar{a}_k' = \sum_{\substack{j=M \\ j \neq k}}^n \frac{a_j'}{\sigma_j^2},$$

and

$$\bar{a}_k'^2 = \sum_{\substack{j=M \\ j \neq k}}^n \frac{a_j'^2}{\sigma_j^2}.$$

On the other hand, if we let $\lambda_0 = 0$, $\mu_{1,k} \neq 0$, and $\mu_{2,k} = 0$, then the conditions for a solution to the optimization problem become,

$$\frac{W^2}{(W^2 + \sigma_k^2 \beta_k)^2} - \mu_{1,k} a_k'^2 = 0$$

$$P_I^2 - a_k'^2 \beta_k = 0$$

$$\widetilde{P}_T^2 - \sum_{k=1}^n a_k'^2 \beta_k \geq 0$$

$$\beta_k \geq 0.$$

The solution to the above conditions is given by,

$$\beta_k = \left(\frac{P_I}{a_k'} \right)^2 \quad \text{for } k = 1, \dots, n. \quad (5)$$

Finally, by combining the solutions (4) and (5), the optimal number of quantization bits, L_k 's, can be given as

$$L_k = \begin{cases} 0 & \text{if } f(a_k') > 1 \\ \log_2 \left(\frac{1}{\sigma_k} \sqrt{\frac{W}{\sqrt{\mu_M a_k'}}} - W^2 + 1 \right) & \text{if } f(a_k') < 1 \\ & \text{and } g(k) < 0 \\ \log_2 \left(\frac{P_I}{a_k'} + 1 \right) & \text{if } f(a_k') < 1 \\ & \text{and } g(k) > 0. \end{cases}$$

IV. NUMERICAL ANALYSIS

In all numerical results we assume a sensor network with $n = 100$ and that the channel gain from each sensor to the fusion center is Rayleigh distributed. We also assume that the fusion center has the knowledge of sensor observation noise variances σ_k^2 for $k = 1, \dots, n$ which are taken as Chi-squared distributed with degree 1.

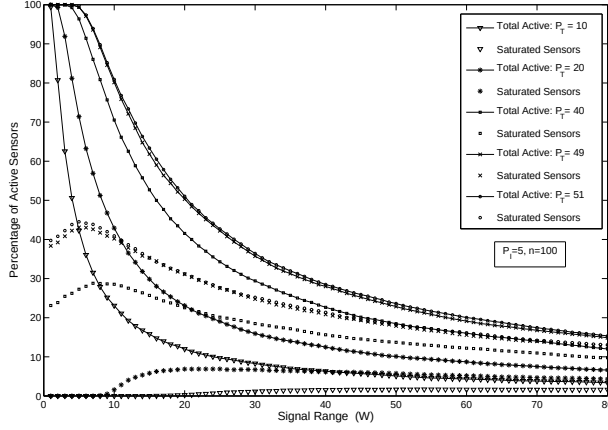


Fig. 2. Percentage of active sensors with different maximum total power constraints

In Fig. 2 the percentage of active sensors and the percentage of sensors operating at maximum power are shown for different signal ranges W and for different values of P_T while keeping $P_I = 5$. Figure 2 clearly shows that with the increase of total power more sensors become active. Further, the number of sensors operating at the maximum power level has a peak at relatively low values of W .

We next observe the MSE of the system. The results are given in Fig. 3. We can observe that the MSE performance improves both when P_T increases for a given P_I as well as when P_I increases for a given P_T . On the other hand, for a given P_I and P_T the performance degrades as W increases.

Finally, in order to assess the gain due to optimal bit allocation schemes, we can compare the proposed algorithm with simple uniform power allocation scheme. According to the uniform power allocation scheme power allocated to each node is given by

$$P_{uniform} = \begin{cases} P_I & \text{if } P_T^2 \geq \sum_{k=1}^n P_I^2 \\ \frac{P_T}{\sqrt{n}} & \text{otherwise} \end{cases}$$

Figure 4 plots the MSE performance for the optimal and uniform power allocation schemes. It is observed that the proposed optimal strategy outperforms the uniform

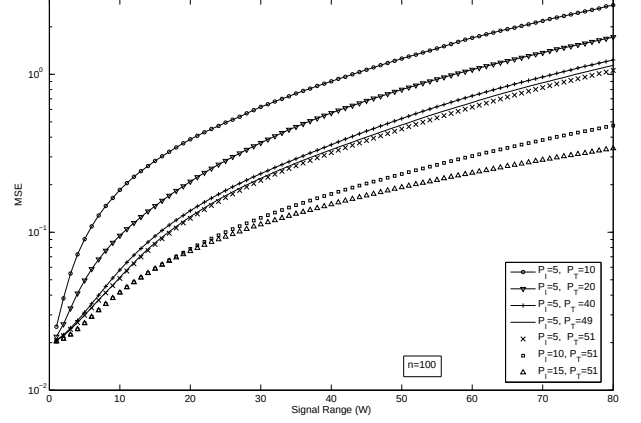


Fig. 3. Mean Squared Error

power allocation scheme for any given power constraint as expected. Especially, when the square of total system power constraint is larger than the sum of the squares of the individual power constraints, Even though the MSE performance does not improve significantly for large values of W , the power consumption has a great advantage when the optimal scheme is employed. This results in an increased life time for the system.

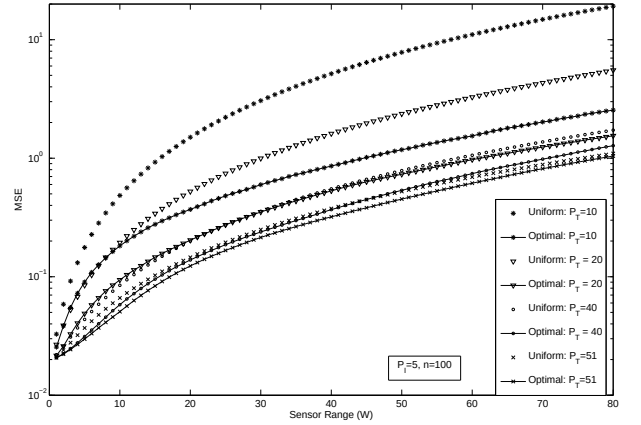


Fig. 4. Mean Squared Error Compared with Uniform Power Allocation

V. CONCLUSION

In this study we have derived the optimum quantization scheme that minimizes the MSE in a distributed estimation problem under strict power constraints. According to the optimal solution, if sensor observations are highly corrupted with noise and/or the channel from

nodes to the fusion center is poor, then the corresponding sensors do not transmit their observation to the fusion center thus saving energy. If the observations and channel are good, as assessed by a certain threshold, then sensors will transmit their quantized observation. The number of optimal quantization levels depends on the individual node power constraints as well as the global system power constraint. In particular, sensors with a large number of bits in optimal quantization limit their quantization to satisfy the maximum individual power constraint.

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