Circuit and Electromagnetic
System Design Notes
Note 2

12 Sep 1966

2.5 Megagauss From a Capacitor Discharge

by
D. W. Forster
J. C. Martin
Atomic Weapons Research Establishment

ABSTRACT

A short programme of work has been conducted with two capacitor banks capable of delivering megamp short circuit currents. The work was completed in April 1965. The aim of the programme was to demonstrate the feasibility of generating MGs magnetic flux densities in small volumes for times of the order of one microsecond, and hence to achieve energy densities comparable with those obtained by explosives but with a non-destructive system of reasonable repetition rate. With an 11 kJ bank the maximum field obtained in a volume of about 0.1 cc was 1.5 MGs and with a 60 kJ bank, the maximum field in a volume of 0.3 cc was 2.5 MGs. The design of the larger bank is described in detail and certain considerations relating to the effects of high fields on the generating coils are discussed.

1. 60 kJ Capacitor Bank

1.1. General.

The well known requirements for a capacitor bank to be used for the type of investigation reported herein are demonstrated by considering the energy stored in a field generating coil fed from such a bank.

The peak current in such a system is given approximately by:

$$I_m \sim \frac{V_o}{R + \left(\frac{L_e + L_c}{C}\right)^{1/2}} \tag{1}$$

where V_o is initial bank voltage,
R is total circuit resistance,
C is total capacitance,
L_e is external circuit inductance,
L_c is coil inductance.

The flux density B is related to current by the energy relation:

$$\int_v \frac{B^2}{2\mu_o} \cdot dv = \frac{L_c I_m^2}{2} \tag{2}$$

or, to a first approximation:

$$L_c I_m^2 \sim \frac{B_m^2}{\mu_o} \cdot v \qquad (3)$$

where v is total coil volume.

Hence,

$$B_m^2 v \propto \frac{L_c V_o^2}{\left[R + \left(\frac{L_e + L_c}{C}\right)^{1/2}\right]^2} \qquad (4)$$

To generate the maximum field in a given volume hence requires a resistance low compared with the dynamic impedance of the system, an external circuit inductance which is low compared with that of the coil and a high initially stored energy. These criteria have been built into the system to be described.

1.2. General Description.

The design of the capacitor bank is sketched in Figures 1 and 2. Care has been taken in making connection to each capacitor to utilise the very low inductance of these units to the full. The resulting design of the strip lines had led to an extremely low inductance bank, which in turn enables experiments of the type discussed later to be made with a relatively small system.

1.2.1. A total of 28 capacitors is used, with individual characteristics of

$C = 20\ \mu F$ a $15\ kV$
$L \sim 2.5\ nH$
$R \sim 5\ m\Omega$

The bank is constructed in two blocks of 14 units each, the final characteristics into a wide short circuit being

$C = 560\ \mu F$ a $15\ kV$
$L \sim 0.9\ nH$
$R \sim 0.5\ m\Omega$
Short circuit current ~ 9.0 MA

1.2.2. Current monitoring is achieved by use of a low resistance section in the lower line of each of the two bank halves. Each 'shunt' is constructed from 6 brass strips, .003" thick, 6" wide and 6" long, giving a total measured resistance of 0.126 mΩ. The strips are folded (see Figure 3) to minimise inductive components, and the monitoring leads are brought out

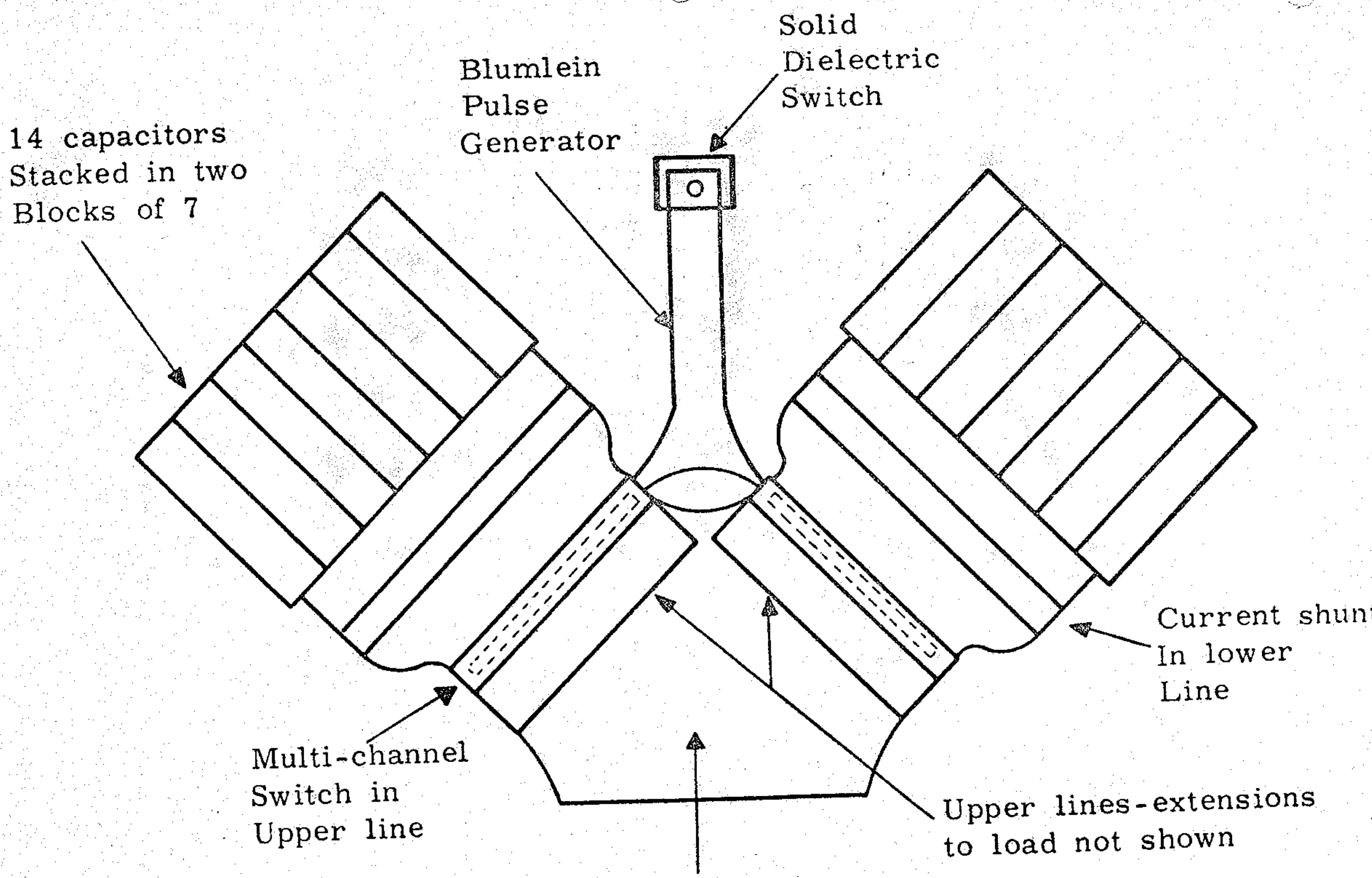
Solid
Dielectric
Switch
Blumlein
Pulse
Generator
14 capacitors
Stacked in two
Blocks of 7
Current shunt
In lower
Line
Multi-channel
Switch in
Upper line
Upper lines-extensions
to load not shown
Lower line

FIG 1

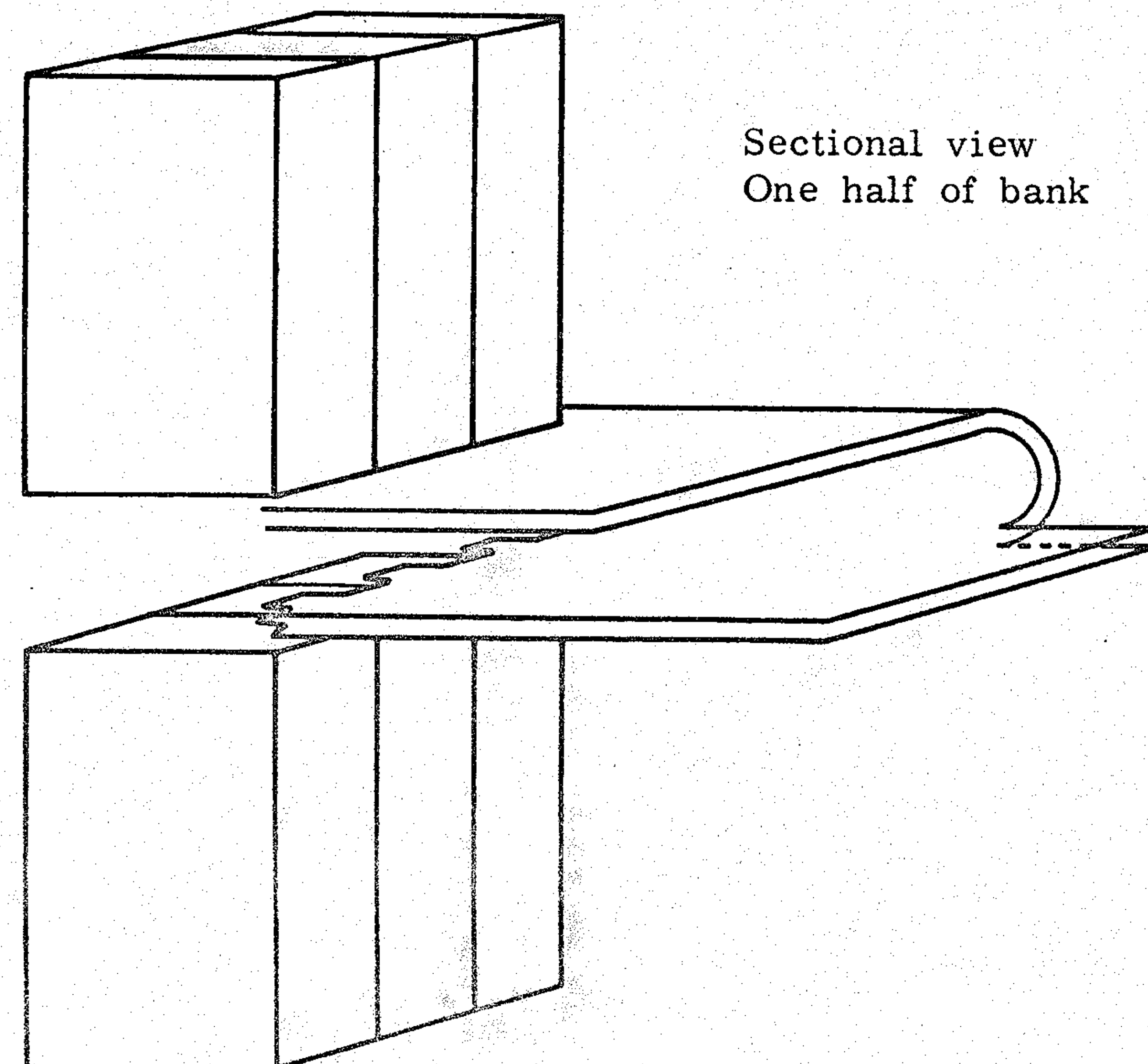

FIG. 2

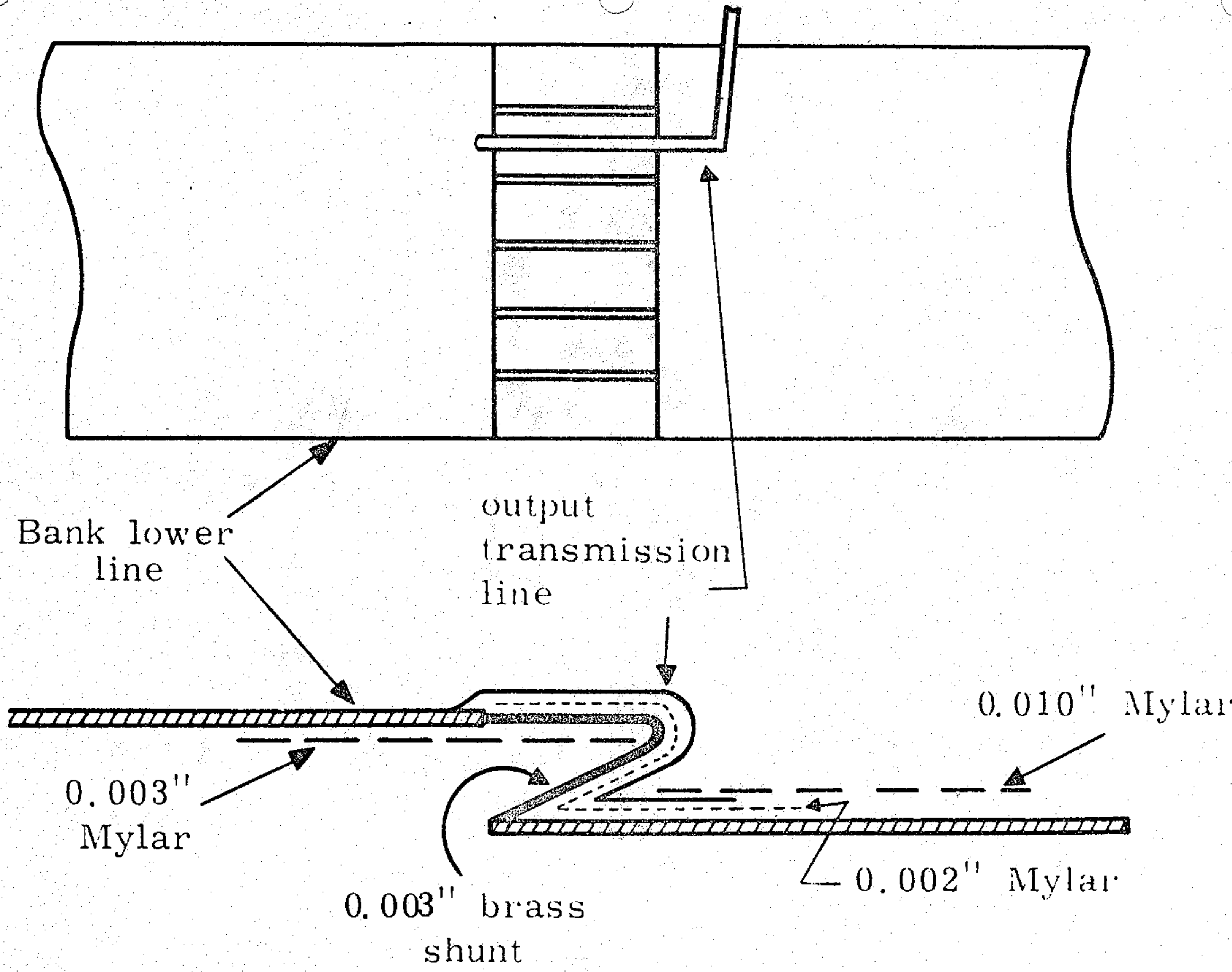
Bank lower
line
output
transmission
line
0.003"
Mylar
0.003" brass
shunt
0.010" Mylar
0.002" Mylar

FIG. 3

non-inductively in the form of a strip transmission line. The shunts are calibrated by firing the bank into a wide short circuit and using the resulting waveform to obtain current, inductance and resistance. The current deduced in this way is within 5% of that obtained from the voltage drop across the measured shunt resistance.

1.3. Switching.

To fulfil the requirements of a low inductance switch which is easily replaceable, a system has been developed which utilises the multiple breakdowns of a solid dielectric in a strip line configuration, each half of the bank being switched separately and simultaneously. Details of the switch are shown in Figure 4.

It consists of a .00025" thick aluminium strip which acts as a trigger electrode, sandwiched between two mylar strips .005" and .002" thickness, the whole being assembled with a controlled silicone grease layer, to exclude as much air as possible, and inserted between heavily backed main line extensions. The trigger strip is held at a voltage determined by the mylar thickness ratio such that each dielectric sheet is equally stressed in the static condition.

The trigger pulse is generated by a low impedance (0.4 Ω), 24" long strip line in a Blumlein configuration, giving a pulse approximately 6 nsec long and 30 kV or so in magnitude into each switch line impedance of about 2 Ω. The direction of the trigger pulse is such that the potential of the trigger strip is taken away from that of the electrode backing the thicker (.005") mylar. As the trigger pulse travels along the switch, field enhancement along the edge of the strip causes breakdown in the .005" mylar, switching time being of the order of 1 ns. Because of this finite resistive phase of the switch channel, the pulse continues to travel along the switch, its trailing edge being eroded with each breakdown. At each point, as the trigger electrode is connected to the main electrode of the .005" mylar, overvolting of the thinner sheet occurs with subsequent rupture. The energy carried by these ruptures is sufficient to initiate intense shock waves which, in turn, enhance the original breakdown channel in the .005" mylar. The result is a fast multi-channel switch which may have as many as 50 separate switch channels.

2. Field Generating Coils and Field Probes

2.1. Field Generating Coils.

2.1.1. Coils have been constructed from various thicknesses of copper sheet (see Figure 5), each coil and its tapered lead-in conductors being formed from one piece of material, i.e. without joins of any form. A single sheet of .005" mylar, made

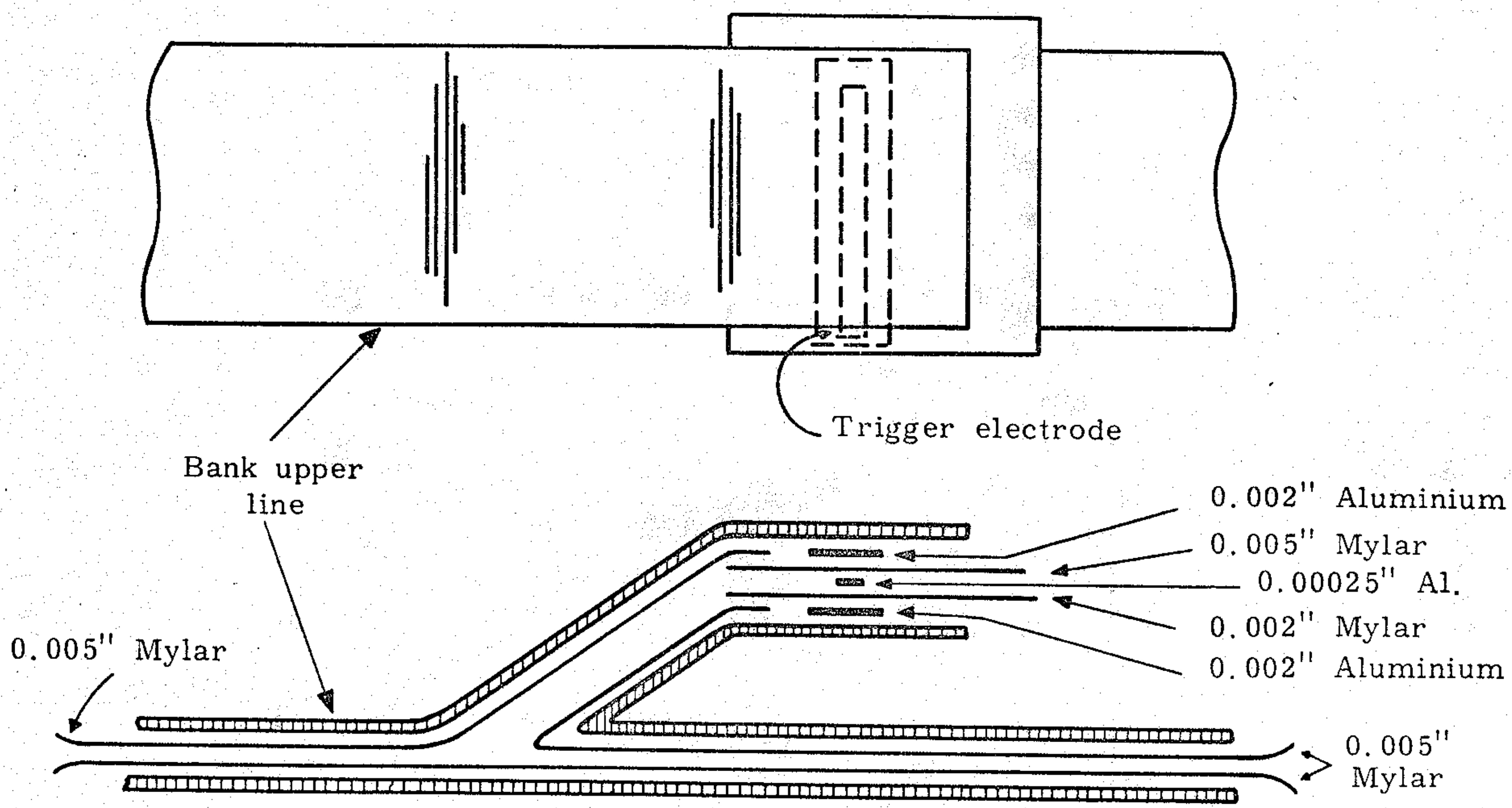
Trigger electrode
Bank upper line
0.002" Aluminium
0.005" Mylar
0.00025" Al.
0.002" Mylar
0.002" Aluminium
0.005" Mylar
0.005" Mylar

FIG. 4

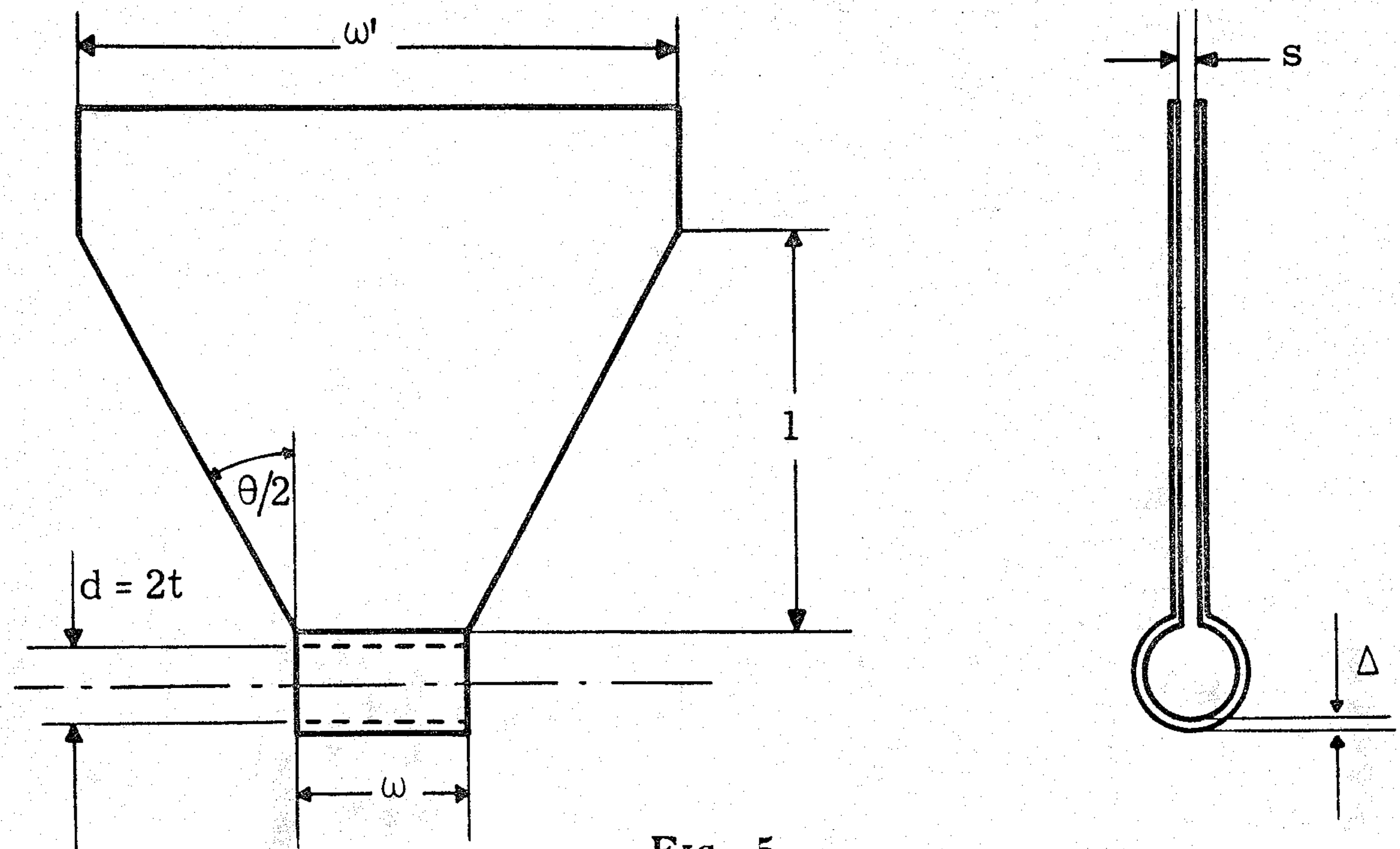

FIG. 5

to conform to the inner surfaces of the coil and lead ins, is interleaved with the main bank insulation (Figure 6). With those coils of higher inductance, almost the full bank voltage of 15 kV appears initially across the coil, and field enhancement at the corners formed at the junction of coil and feeds can result in breakdown. The coil assemblies are thus immersed in transformer oil contained in a polythene bag surrounding both coil and field probe.

2.1.2. A lower limit to the thickness of the coil material is set by joule heating. The so-called 'blow up criterion' relates the Action integral $\int_o^t I^2 \, dt$ and sample area of cross section A to quantities which are solely a function of the material through which the current I is flowing. For copper, approximately

$$\int_o^t \frac{I^2 \, dt}{A^2} < 1.6 \times 10^9 \quad (1) \qquad (5)$$

for the material to remain coherent up to time t for I in ampere, t in second and A in square centimeter.

We are interested in a value for A which will maintain the material in a coherent form for times greater than that taken by the current to reach its maximum value I_m. With the 60 kJ bank and coils of the type used, this leads to

$$\left.\begin{array}{l} \int_o^{t_m} I^2 \, dt \sim 10^7 \\ \text{and hence} \\ A > 8 \times 10^{-2} \ \text{cm}^2 \end{array}\right\} \qquad (6)$$

For example, a coil of 1.5 cm width must have a thickness greater than ~.021" to remain coherent up to time t_m. In all work with the 60 kJ bank, a thickness of .049" was used.

This simple approach is adequate for gaining a rough idea of the lower limit to coil thickness and is valid for flux densities below about 2 MGs. However, at about this level

(1) This assumes that the energy coefficient of copper is independent of temperature up to its boiling point.

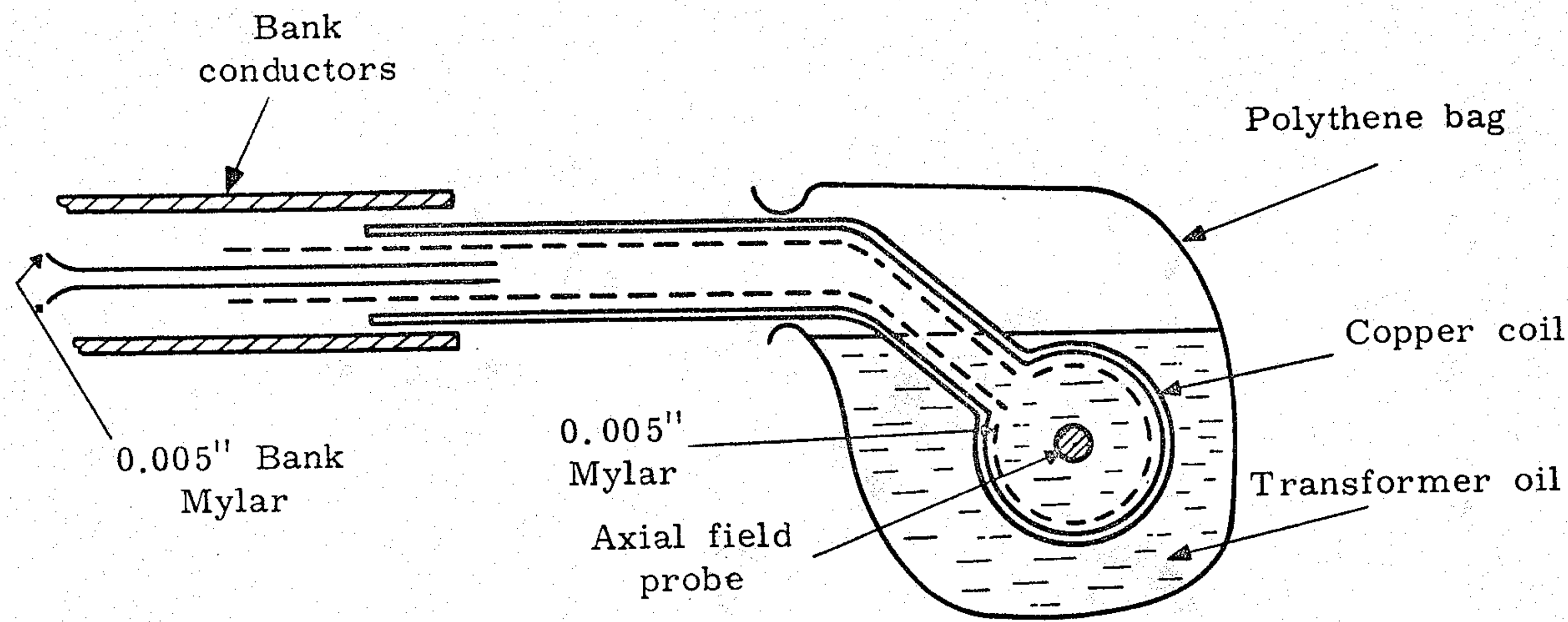
Bank
conductors
Polythene bag
Copper coil
0.005'' Bank
Mylar
0.005''
Mylar
Axial field
probe
Transformer oil

FIG. 6

approximate calculations show that the presence of rapidly moving vaporisation fronts begin to affect the situation and at flux densities greater than 3 MGs the application of such joule heating calculations to the metal as a whole will not be valid.

2.1.3. The effective load produced by the coil assembly.

As shown in Figure 5, the lead-in conductors are tapered from a wide bank connection to the coil. An analysis of this configuration shows that the inductance and resistance of the tapered section may be approximated by:

$$L_{taper} \sim \frac{4\pi s}{\theta} \cdot \ln\left(\frac{w'}{w}\right) \tag{7}$$

$$R_{taper} \sim \frac{2\rho}{\theta \cdot \delta} \cdot \ln\left(\frac{w'}{w}\right) \tag{8}$$

where dimensions are in cm (See Figure 5).

ρ is resistivity (Ω-cm)

δ is skin depth (effective).

The inductance of the coil itself, assuming a uniform current distribution on the inside surface of the coil, is given approximately by:

$$L_{coil} \sim \frac{40r^2}{w + r} \text{ nH} . \tag{9}$$

To give some idea of the magnitudes involved compared with those pertaining to the bank under short circuit conditions, Table I gives these quantities as a function of coil width for:

$w' = 30$ cm

$r = 0.25$ cm

$\delta = 1.8 \times 10^{-2}$ cm

$l = 15$ cm

$s = 3.8 \times 10^{-2}$ cm.

TABLE I

w cm	L_{taper} nH	L_{coil} nH	R_{total} mΩ
0.5	1.0	3.3	0.73
1.0	0.8	2.0	0.50
1.5	0.75	1.4	0.42
2.0	0.7	1.1	0.37

2.1.4. Finally, mechanical motion of the coil during the current pulse resulting from J × B forces produces an effective resistance (dL/dt) which is a function of time but has a maximum in the interval 0 - t_m at t_m of:

$$\left.\frac{dL}{dt}\right|_{t_m} \sim \frac{0.8\ B_m^2 t_m}{\sigma\Delta} \cdot f\left(\frac{w}{r}\right) m\Omega \tag{10}$$

where t_m is time to peak (μsec)

B_m is peak flux density (MGs)

σ is metal density (g/cc)

and f(w/r) is a function of the coil aspect ratio:

$$f\left(\frac{w}{r}\right) = \frac{1}{1 + \frac{w}{r}}\left[2 - \frac{1}{1 + \frac{w}{r}}\right] \tag{11}$$

For example, for an unbacked copper coil in which:

$$\left.\begin{aligned} w &= 1.5 \\ r &= 0.25 \\ \Delta &= 0.15 \\ B_m &= 2.5 \\ t_m &= 1.8 \end{aligned}\right\}\ dL/dt \text{ at } t_m \sim 1.8\ m\Omega$$

This represents the maximum dL/dt attained during the first quarter cycle of current and hence, the mean effective value will be somewhat lower.

At first sight, this approach immediately suggests that dL/dt values can be reduced to insignificant levels by using a suitably massive coil assembly. However, for flux densities in excess of 2 MGs or so, shock hydrodynamic effects become important and the local particle velocity sets a lower limit to the expansion velocity and hence dL/dt. At lower flux densities and thus pressures this limit is well below that which makes dL/dt terms significant and here backing coils with high density materials has obvious advantages, but at 5 MGs, for example, the particle velocity is around 1.6×10^5 cm/s and the resulting dL/dt for a coil with an aspect ratio of 2 will have a lower limit of about 3.5 mΩ.

Two of the coils which gave flux densities in excess of 2.5 MGs were backed with lead, the results showing no significant increase in peak field. This is consistent with a rough calculation on the above lines which predicts a decrease in dL/dt of about 30% at most and a consequent increase in peak current of the order of 7%.

2.2. Magnetic Field Probes.

2.2.1. The design of the field probe used in these experiments is sketched in Fig. 7. Capacitive coupling to the probe is kept to a minimum and the leads taken out from the probe in a manner which reduces the flux coupling to them to a minimum.

We are interested in measuring rates of change of flux density which can be as high as 2×10^{12} gauss/s. Initially, probe coils were made with effective diameters which gave voltages of a few hundred volts with these values of dB/dt. The small physical size of these coils, which made construction tedious, and the problem of pick-up with such relatively low signal levels, led to the design of Figure 7. Despite the high voltages developed (~5 kV), insulation was perfectly satisfactory.

A typical coil had an effective area of about 0.10 cm^2 and 3 turns providing an output voltage of 3×10^{-9} volt per gauss/s.

2.2.2. Calibration.

Accurate measurement of the probe dimensions becomes unnecessary if a known dB/dt is available for calibration. For this purpose, a subsidiary calibration test set up was used: this, in fact, was a miniature version of the main experiment. A 1 μF, 10 kV low inductance capacitor was switched via a current shunt and solid dielectric switch into a standard and accurately measured field generating loop of about 0.5 cm internal diameter and 3.5 cm width. The shunt was calibrated by measurement on the short circuit waveform of the test set up and from the known current the axial magnetic field was obtained using equation (12) given below.

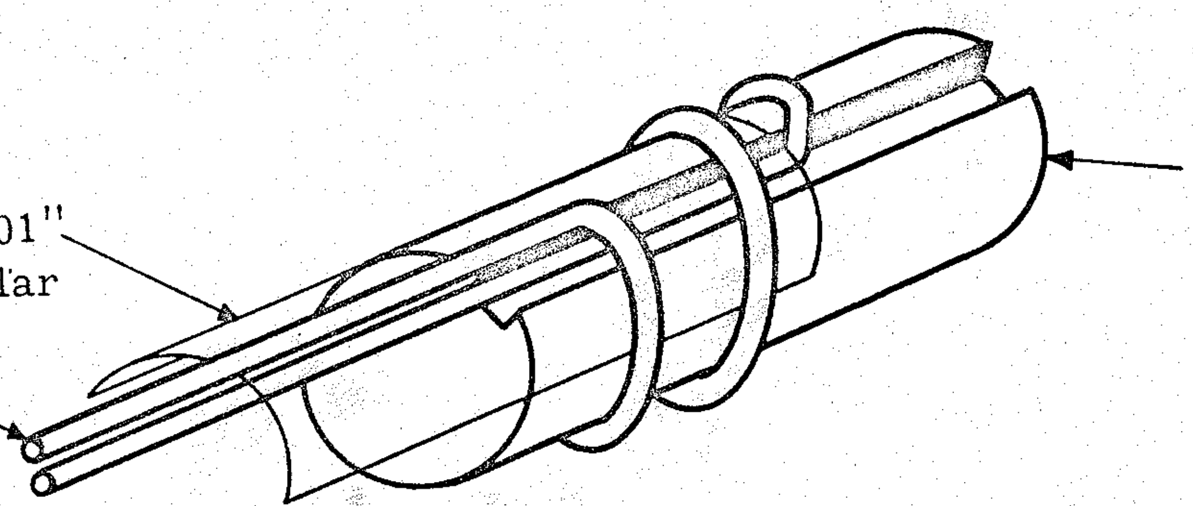
0.125'' Diameter
Slotted perspex
Former
0.001''
Mylar
0.006''
Copper
wire

FIG. 7

2.2.3. Response Time.

The response time of the field probes is governed by the ratio of their inductance to the impedance of the cable which they feed. For a three turn coil on a 1/8" diameter former, the inductance is less than 50 nH and hence the rise time is less than 1 ns into a 50 Ω cable. This is obviously negligible compared with a quarter period of the main bank current waveform.

3. Field Measurements

3.1 With a knowledge of the field probe response, graphical integration of the probe voltage yields the axial magnetic field generated within the field coil. A typical record, enlarged from the original Polaroid print, is shown in Figure 8.

The field measured in this way may be compared with that calculated for a uniform current density on the inner coil surface, assuming no extraneous effects. Obviously, several factors may enter here: non-uniform current distribution axially, current diffusion in the metal and the possible presence of magnetic shocks, coil expansion due to J × B forces, and the generation of current carrying plasma in the subsequent void (which may to some extent offset the reduction in axial field produced by diffusion). No attempt has been made to investigate these effects, apart from non-uniform current distribution and motion of the metal as a whole. The results show no large departures from the predicted values and we may draw the conclusion that, to a first approximation, up to about 3 MGs in the geometries under consideration, such effects make no significant contribution to axial field.

3.2 Measurements with both 11 kJ and 60 kJ banks have been combined in Figure 9. For a coil of width w and inner diameter d, the axial flux density at the centre of the coil is given by:

$$B_{zo} = \frac{1.26\ I}{(w^2 + d^2)^{1/2}} \times \alpha\left(\frac{w}{d}\right) \text{gauss} \qquad (12)$$

where $\alpha(w/d)$ is a correction factor for axial non-uniformity of current and is shown in Table II. This factor was determined experimentally and it was found at the same time that the inductance given by equation (9) was at most 10% in error.

In Figure 9, the results have been normalised by plotting B_{zo} against

$$I_m\left[1 + \left(\frac{w}{d}\right)^2\right]^{-1/2} \times \alpha\left(\frac{w}{d}\right)$$

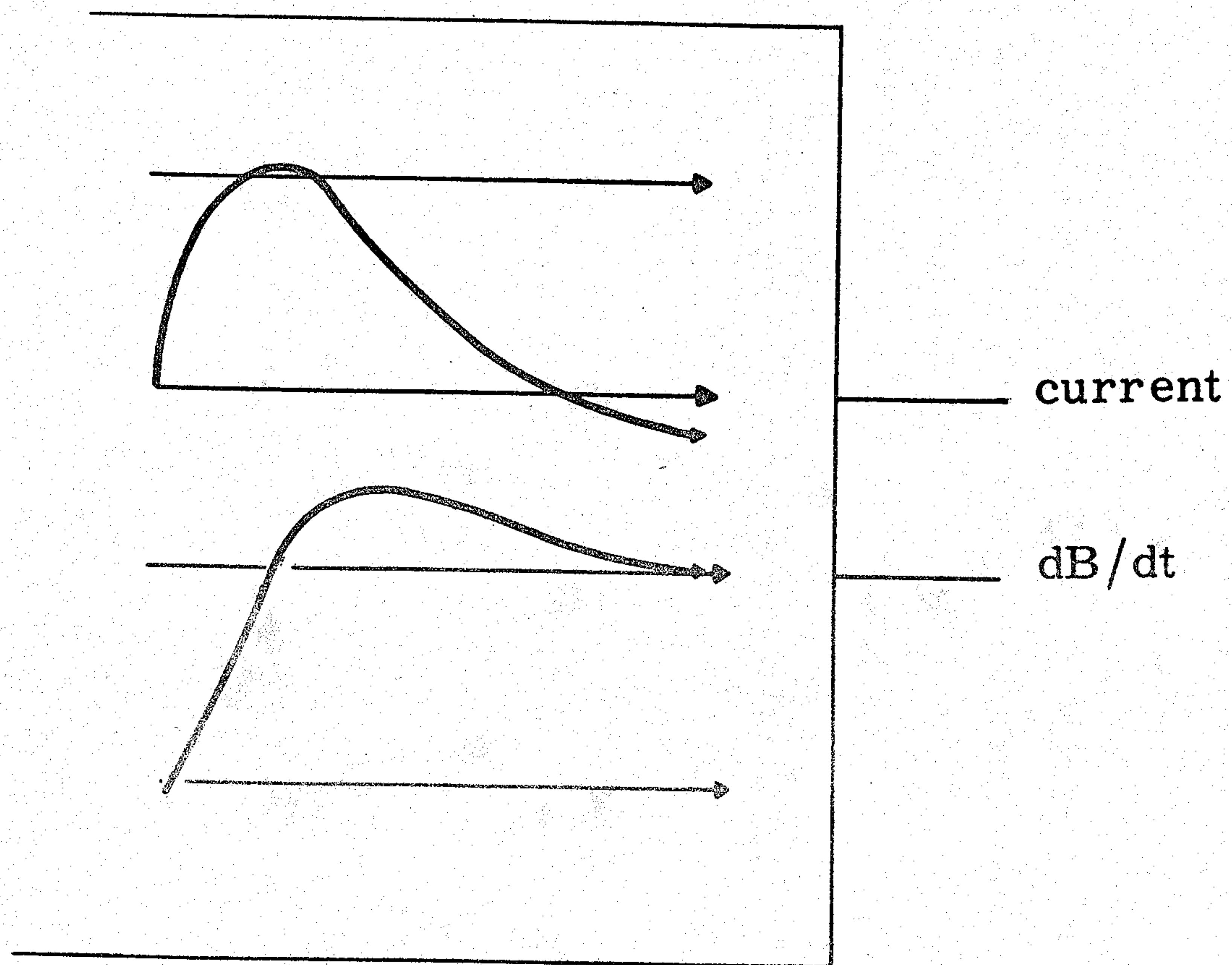

Fig. 8

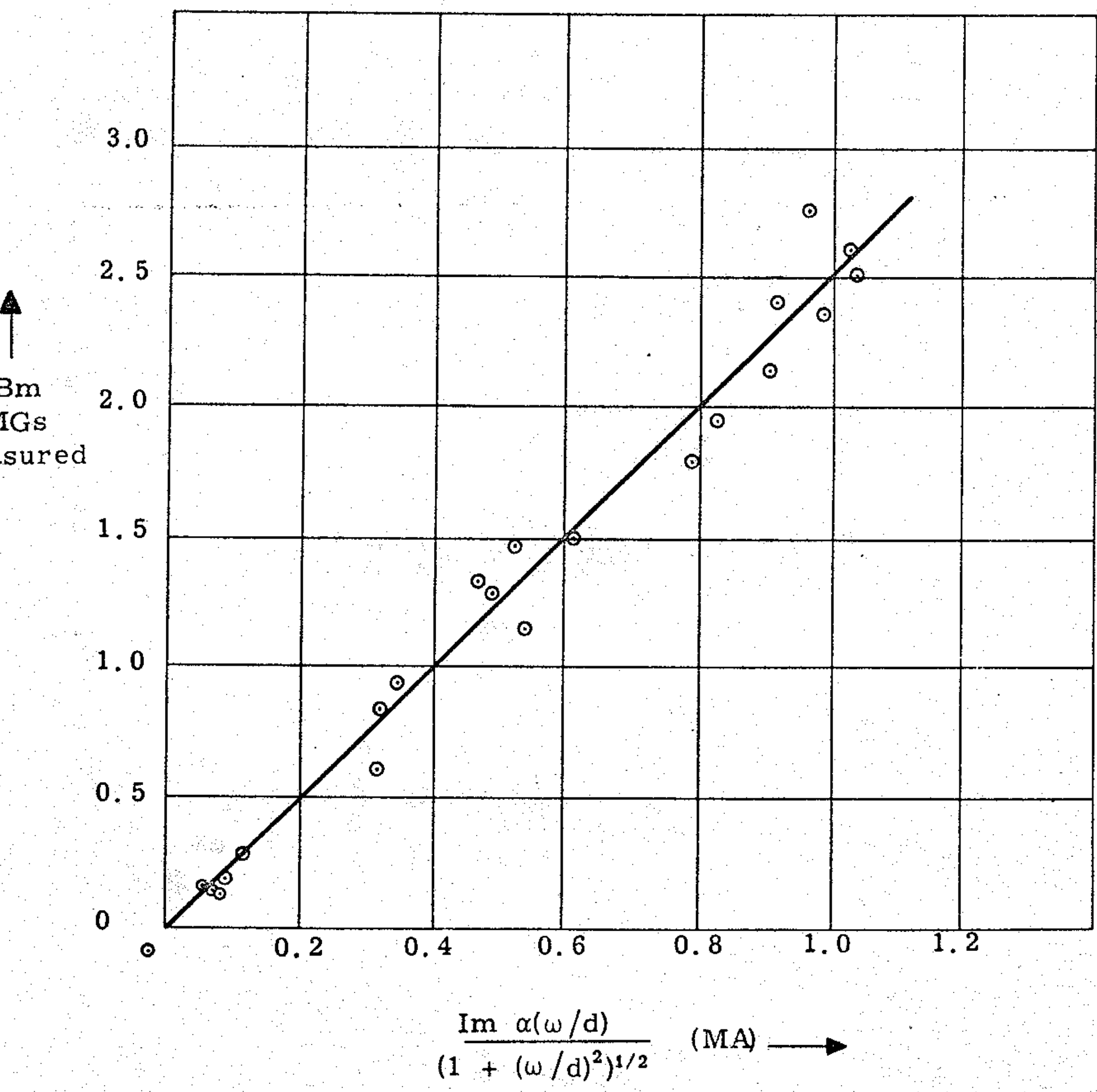
3.0
2.5
2.0
1.5
1.0
0.5
0
Bm
MGs
Measured
0.2
0.4
0.6
0.8
1.0
1.2
$\frac{\text{Im } \alpha(\omega/d)}{(1 + (\omega/d)^2)^{1/2}}$ (MA)

FIG. 9

all coils having a diameter of 0.5 cm.

TABLE II

w/d	1.0	3.0	10
$\alpha(w/d)$	0.80	0.84	0.94

Account may be taken of the effect of coil expansion up to time t_m. To a first order, if we assume that the flux density is distributed uniformly throughout the coil volume, then if k is the ratio of the flux density to current, the increase in d at time t_m when the current is a maximum is given by:

$$\delta d \sim \frac{10^{-2}k^2}{4\pi\sigma\Delta} \int_o^{t_m} \left[\int_o^t I^2 (t)\ dt \right] dt \tag{13}$$

where k is in Gs/A.

σ in g/cc.

Δ in cm.

A first approximation to the integral may be obtained by assuming a sinusoidal current increase. In this case (13) becomes:

$$\delta d \sim \frac{1.2 \times 10^{-2}k^2 I_m^2 t_m^2}{\sigma\Delta} \tag{14}$$

where I_m is the peak current in amps and t_m in seconds. The resulting correction to be applied to (12) in the case of real coils varies in the cases considered from less than 1% to 6%. A second effect of the motion is to make the current distribution more uniform than in the above case and hence the corrections in Table II should probably be reduced slightly.

3.3. Energy Transfer.

For flux densities around 2.5 MGs, the energy transferred to the generating coil was typically about 15% of that initially stored in the bank.

For a simple R - L - C circuit, this energy transfer ratio is solely a function of β, where $\beta = 2Z/R$ and $Z = \sqrt{L/C}$. This enables an approximate estimate to be made of the effective value of dL/dt by deducing β and hence R from the energy stored in the coil inductance at measured current maximum. The values of dL/dt obtained in this way may be compared with those calculated from equation (10). For example, in a typical case, direct calculation from equation (10) gave a maximum dL/dt of 1.6 mΩ, at current peak, whereas estimation from the energy transfer ratio gave about 1.3 mΩ, the latter representing a mean effective value over the first quarter cycle of current. This reasonable agreement between the two methods, even though they are not completely independent, helps to increase confidence in the approach taken to examine the behaviour of coils in the low megagauss field region.

3.4. Summary.

Particulars of those experiments which gave the highest flux densities have been averaged for three different coils and the results summarised in Table III. Column 4 gives the flux densities measured by axial probe and column 5 gives the values calculated from equation (12) and the measured maximum current.

TABLE III

Coil Width (cm)	Coil Inside Diameter (cm)	Total Max. Current (MA)	Measured Max. Flux Density (MGs)	Max. Flux Density calculated from Current (MGs)
0.8	0.5	2.4	2.51	2.60
1.0	0.5	2.75	2.51	2.45
1.5	0.5	3.5	2.60	2.35

3.5. Possible Future Extensions.

There are two obvious ways in which this work could be extended:

(a) towards higher fields, using a replaceable coil of very simple construction which is destroyed every time;

(b) to accept lower fields but to produce them in systems where the coil survives for a number of shots.

Considering the possibility of producing increased fields by means of condenser banks, if the effective volume of the magnetic field is 1 cc, then the energy in the coil is 200 kJ if the field were 7 MGs. In view of the fact that the energy must be delivered before the coil has time to fly apart, it is unreasonable to expect that more than 10% to 15% energy transfer could be achieved. A fairly detailed analysis of the problem, including the hydrodynamics of the coil, indicates that with a 1 megajoule bank of advanced design a field of 7 MGs is achievable in a volume of the order of 1cc. The bank has to have a very low inductance and be capable of delivering energy at the rate of about 0.5 MMW into a matched load in order to build the field up before the coil expands too much. This approach will of necessity be a single shot one, but by prefabricating the very simple coil and its feed transmission line, several shots a day should be possible.

In addition to a reasonable rate of experimental shots compared with explosive generators, the condenser approach has the advantage that the mass of the exploding coil is small and its energy easily dissipated. In the experiment quoted above it has been found possible to have experimental equipment within a few inches of the coil and have it remain intact; something that is impossible with an explosive approach involving as it does large masses of high explosive.

The cost of such a bank will be less than £30,000 and while this is a large sum, the system can be used to produce several thousand high field pulses as well as doing other interesting experiments.

With regard to the possibility of producing a multishot field coil working in the 1 to 2 MGs range, this becomes feasible in principle if the duration of the magnetic field pulse is reduced by a couple of orders of magnitude. Up to fields of about 1 MGs the copper on the inside of the coil does not vaporise and while it will melt, as the layer is only of the order of 2×10^{-3} cm. thick, it will cool by thermal conduction in a time of a few microseconds and the surface should survive. The field at which melting sets in is independent of the duration of the pulse, but the depth of the layer melted is pulse length dependent. The main phenomenon destroying the coil is the mechanical impulse given to it and this is dependent on pulse length. For a single pulse of duration 50 ns, the energy given to a coil 1/2 cm in diameter, 1 1/2 cm long, is only about 2 joule and providing the copper of the coil is backed by a strong material, this energy can be easily contained without exceeding the elastic limit of the materials. It is not necessary to have a truly uni-directional current pulse; since the energy deposited goes approximately as the current squared, it is enough that the second current pulse after the first should be 60% or less in amplitude. The difficulty of this approach is that the energy must be delivered to the coil in the form of a high voltage pulse. The inductance of the coil with leads is at least 3 nH and for the peak current of 1.3 MA to be established in a sufficiently fast time a di/dt of 5×10^{13} A/s. is required and this means the open circuit voltage of the generator must be at least 150 kV. Indeed, to allow for various losses, the pulse generator should provide an open circuit voltage of about 1/4 MV with an internal impedance of 0.2 Ω, for a pulse length of the order of 50 ns. With modern techniques such a modulator is not difficult to build and therefore the construction of a multishot coil working at the level of 1 MGs would appear to be feasible.

Summarising, it appears possible to obtain in volumes of the order of 1 cc:

(a) pulse fields of 1 MGs, as in a multishot bench top system, storing about 1 kJ;

(b) fields of 7 MGs, with a system using a condenser bank of 1 MJ, capable of being fired a few times a day;

and these are to be compared with:

(c) explosive systems which should be expected to give up to 50 MGs, with a low rate of firing and using hundreds of pounds of H.E.

While it should be stressed that the estimates of cost are very rough indeed, it is considered that the cost per shot of the above systems are, very approximately:

(a) £1/2 per shot;

(b) £20 per shot;

(c) £1,000 per shot, or more,

when averaged over a period of about a year's use.

Acknowledgments

The authors wish to express their thanks to the Director, Atomic Weapons Research Establishment, Aldermaston, for permission to publish this work; to Mr. K. J. Shaw, who built the capacitor bank and performed many of the experiments; and to Mr. P. G. Carpenter for his experimental assistance throughout.