Circuit and Electromagnetic System Design Notes

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Impedance-Matched Magic Tee

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Abstract

The magic tee is an important microwave-circuit element. While symmetry is crucial for combining and separating signals, it is not the only factor. For certain applications it is also important that certain reflections at the junction be small to maximize power throughput. This implies that appropriate impedance matching be applied.

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1. Introduction

The subject of magic tees (a special kind of hybrid junction or network) has an extensive literature. Here we cite but a few [5-8, 10]. There are many variations on this theme, but they all have a common feature: symmetry. It is symmetry which divides the electromagnetic waves into two kinds with respect to a symmetry plane: symmetric and antisymmetric [9]. This allows signals to be fed into (or out of) two ports on the symmetry plane (Δ port and Σ port) into the remaining two ports, without the Δ and Σ ports coupling to each other.

One of the uses of a magic tee is as part of a microwave-pulse-compression scheme [4]. When switching out the power by breaking the symmetry (such as by switch firing), it is important to switch out the energy as fast as possible so as to have the highest possible power into the load. (e.g., antenna). For this purpose one needs to minimize reflections at the magic tee. This can be accomplished by appropriate impedance matching.

2. Coaxial Magic Tee

A coaxial resonator (cavity) is discussed in [4] for pulse compression. In [4 (Fig. 3.1)] there is part of what is a magic tee, including the Σ port and side arms. Various forms of a coaxial magic tee are given in [5]. Such coaxial devices are appropriate for lower frequencies (hundreds of MHz and below) for which rectangular waveguide dimensions are large and unwieldy. Of course, one could also use ridged waveguides and variations on this.

Figure 2.1 gives a schematic for one version of a coaxial magic tee. This is a broadband form which illustrates the concept. Suppose a signal is fed in on the Δ port with differential impedance Z_Δ. For no reflection (full transmission into the two (terminated) side arms) we require

$$Z_\Delta = 2 Z_c \tag{2.1}$$

with Z_c as the characteristic impedance of each side arm. Similarly if we wish to match a signal from the Σ port with impedance Z_Σ we require

$$Z_c = 2 Z_\Sigma \tag{2.2}$$

Note by reciprocity a side-arm differential signal will match without reflection into the Δ port, and a side-arm common-mode signal will match without reflection into the Σ port.

If we combine the above two constraints we have

$$Z_\Delta = 2 Z_c = 4 Z_\Sigma \tag{2.3}$$

This is a significant range of impedances required. Of course, one can provide transformers with the Δ and Σ ports to make these ports matched to other more desirable impedances.

If we go to more-narrow-band schemes we can connect the Σ port directly to the two side arms (without the choke) and inductively couple the Δ port in a differential manner [5]. This might be more appropriate for the high-power schemes in [4]. In this case the impedance matching of the Σ port is the most important so as to dump the energy out the Σ port in approximately one round trip of a side arm (both dumping simultaneously).

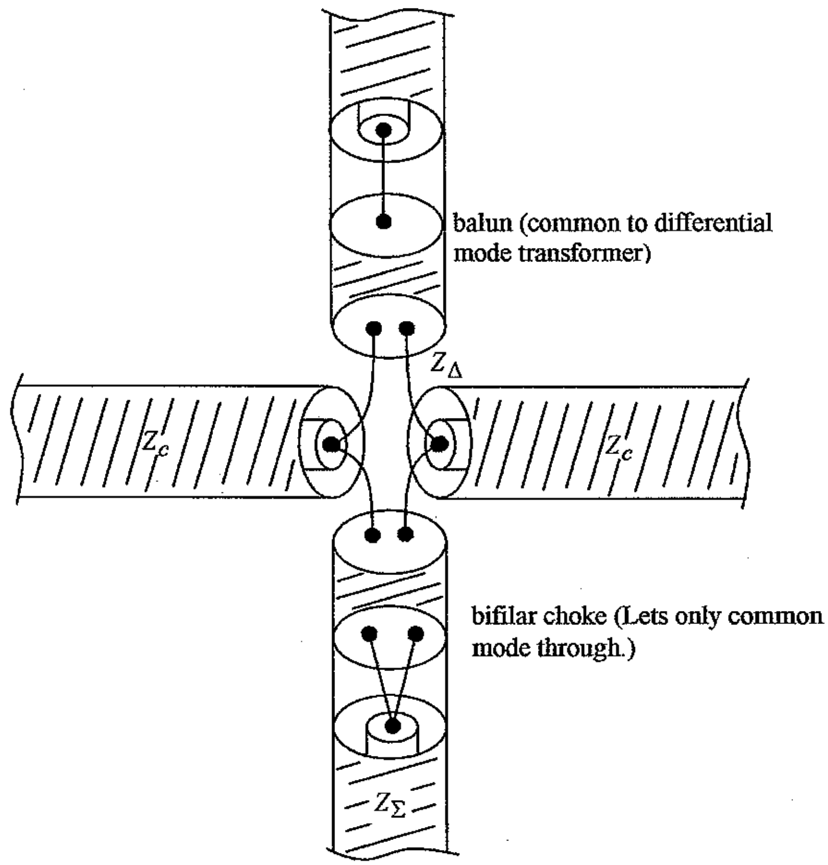
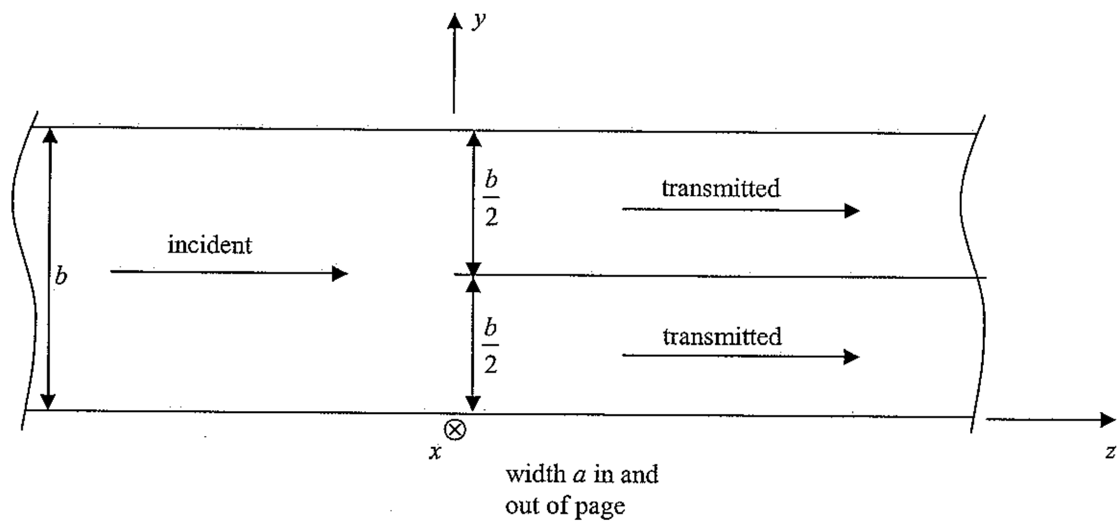


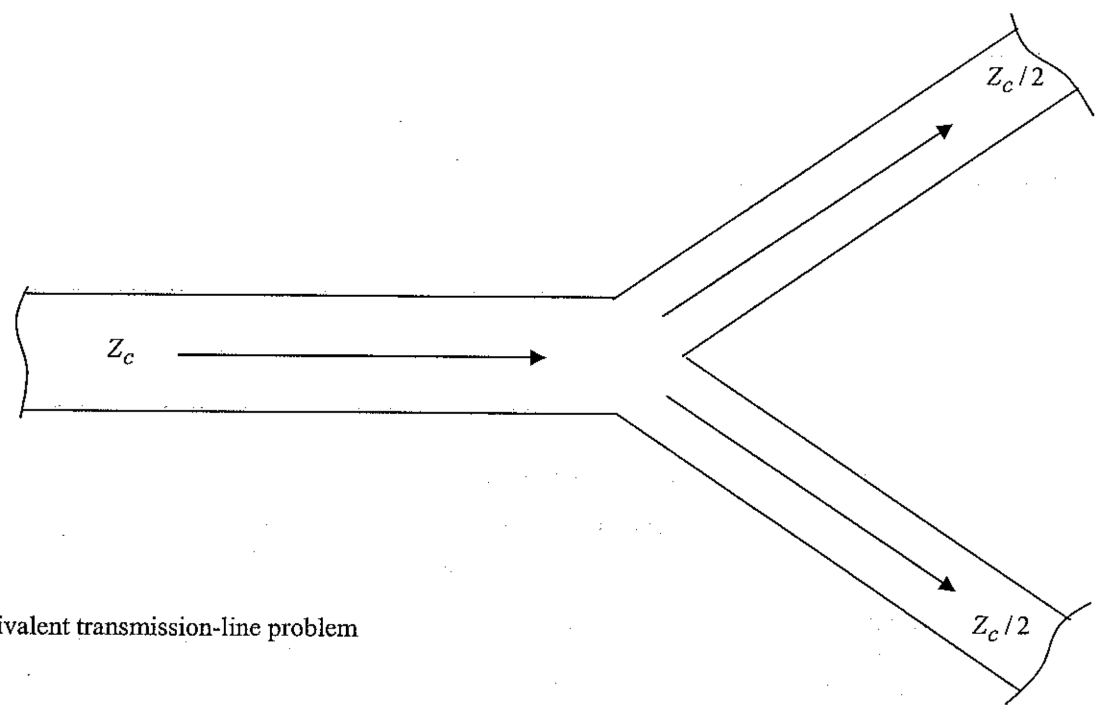
Figure 2.1 Impedance-Matched Coaxial Magic Tee

3. Rectangular-Waveguide Magic Tee

To begin our discussion, consider the problem illustrated in Fig. 3.1A. Similar to the discussion in [1, 2] we can insert perfectly conducting sheets (septa) perpendicular to the electric field (and parallel to the broad walls) of the $H_{1,0}$ mode of a rectangular waveguide without disturbing the fields. A wave incident from the left will propagate to the right without reflection.



A. Waveguide bifurcation



B. Equivalent transmission-line problem

Fig. 3.1 Waveguide Bifurcation

Considering two equal guides each of height $b/2$ and width a, we will have equal powers flowing down each guide. This property is analogous to the transmission-line problem in Fig. 3.1B. In this case, a transmission line of characteristic impedance Z_c is matched without reflection into the series combination of two transmission lines, each of characteristic impedance $Z_c/2$. This suggests that we can ascribe impedances to the waveguides in Fig. 3.1A with the same series combination property.

A commonly used parameter of a waveguide mode is the modal impedance, which for the $H_{1,0}$ mode is [8, 10]

$$
\begin{aligned}
Z_w &= Y_w^{-1} = \frac{k_0}{k} Z_0 && \text{(modal impedance)} \\
k &= \left[k_0^2 - \left[\frac{\pi}{a} \right]^2 \right] && \text{(propagation constant)} \\
k_0 &= \frac{\omega}{c} = \frac{2\pi f}{c} && \qquad (3.1) \\
Z_0 &= \left[\frac{\mu}{\varepsilon} \right]^{1/2} \equiv \text{wave impedance of plane wave in uniform isotropic medium} \\
c &= \left[\mu\varepsilon \right]^{-1/2} \equiv \text{propagation speed in infinite uniform medium}
\end{aligned}
$$

This gives the ratio of the electric field (transverse) to the transverse part of the magnetic field. Above cutoff it is real valued.

What we need is what we may call a guide impedance which brings in the guide dimensions, a and b. Thinking of the ratio of a voltage to a current we might construct something like

$$
\begin{aligned}
V &= -Eb \\
I &= -Ha && \qquad (3.2) \\
Z_g &= \frac{V}{I} = \frac{E}{H}\frac{b}{a} = Z_w \frac{b}{a} && \text{(guide impedance)}
\end{aligned}
$$

Of course, E and H vary across the guide cross section proportional to $\sin(\pi\, x/a)$. So one might want to choose some average of E and H on appropriate integration paths. One may then choose some positive constant times the above formula. As long as one is consistent this will not modify the basic results. For the present let us take this constant as unity. See also the discussion in [3 (Section IV)].

Now take the configuration in Fig. 3.1A. Bend the two half-height right-side guides away from each other in two 90° E-plane bends. This deforms into the Δ port and two side ports of the magic tee in Fig. 3.2. With the constraint

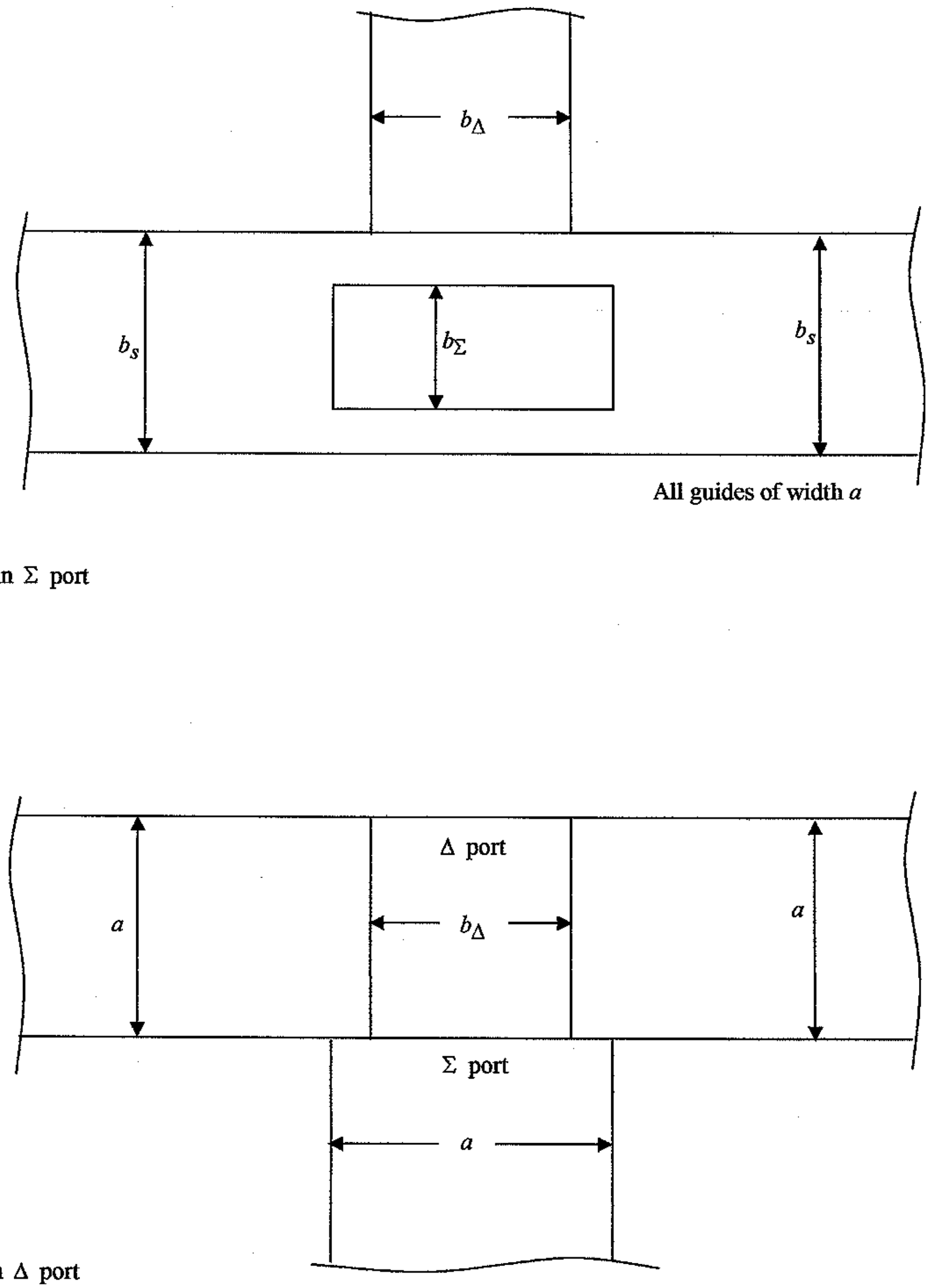
$$b_{\Delta} = 2\, b_s \tag{3.3}$$

the Δ port is matched into the two side ports. Of course, there are various parasitic elements associated with the junction now, being not as "clean" as in Fig. 3.1A. So some tuning elements may help [6].

Now consider the parallel combination of two transmission lines each of characteristic impedance $2\, Z_c$. This matches to a wave propagating on a transmission line of characteristic impedance Z_c. Applying this to the Σ port we can constrain

$$b_s = 2\, b_{\Sigma} \tag{3.4}$$

to achieve a match. Again, parasitics at the junction may be significant. Furthermore, considering the match to the Σ port as two H-plane bends in the side guides, the mode matching is not so "neat" as in Fig. 3.1A.



A. View in Σ port

B. View in Δ port

Fig. 3.2 Rectangular-Waveguide Magic Tee

4. Transforming to Standard Waveguide dimensions

The constraints on waveguide heights discussed in the previous section lead to nonstandard heights. Particularly if both constraints are applied, we have

$$b_\Delta = 2\,b_s = 4\,b_\Sigma \tag{4.1}$$

implying a factor of four variation in heights. The largest of these, b_Δ, is limited to not much more than $a/2$ (depending on frequency) so that this waveguide is not overmoded.

The nonstandard heights can be transitioned to standard height ($a/2$) by means of transformers [8]. One can smoothly vary the height over several wavelengths to achieve a broadband match (low reflection). One can accomplish this by a quarterwave transformer in a much shorter, but more narrow-band, device. In this case [3 (Section IV)] a waveguide of length $\lambda/4$ with height as the geometric mean of the two heights (to be matched) is used. In the present case this gives a factor of $\sqrt{2}$ times the smaller height for the transformer height.

Taking one end of the quarter-wave transformer as right at the magic-tee junction, one can perhaps avoid some extreme cases of guide height by increasing or decreasing the extreme values by a factor of $\sqrt{2}$. This still leaves the question of design details in the junction itself, still maintaining the symmetry plane.

170 (10)

5. Concluding Remarks

This paper has presented some design considerations for improvements in magic tees, at least for some applications. In particular the intent is to reduce certain reflections at the tee junction. These basic considerations lead to further questions concerning details of the magic-tee design.

References

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