Mathematics Notes

Note 32

PROLATE AND OBLATE SPHEROIDAL WAVE FUNCTIONS OF COMPLEX ARGUMENT

R. Jeffrey Lytle

Frederick J. Deadrick

Lawrence Livermore Laboratory
Livermore, California

October 24, 1973

Abstract

This report describes the spheroidal coordinate system and the solutions to the scalar Helmholtz wave equation in the spheroidal coordinate system. The methods of generating, and sample numerical results for, the eigenvalues, angle eigenfunctions, and radial eigenfunctions for complex expansion parameters are presented.

Contents

Introduction

The spheroidal coordinate system and the spheroidal wave functions are encountered in a wide variety of problems in engineering, physics, and mathematics. Some areas in which these functions have found application are image enhancement[1]; filter design[2]; optimization studies[3]; signal representation[4]; terminal velocities of drops or bubbles in a fluid[5]; buckling of shells under pressure[6], theory of weak inclusions in stress-concentration calculations[7]; attenuation of electromagnetic waves by rain[8]; electromagnetic radiation,[9] reception,[10] diffraction,[11] and scattering[12] by spheroids and circular apertures; heat conduction,[13] acoustic radiation,[14] diffraction,[15] and scattering[16] by spheroids and circular apertures; vibrations of celestial bodies[17]; effect of gravitational and magnetic torques on the two-body problem[18]; and use of the doubly orthogonal and doubly complete property of the spheroidal angle functions.[19] For a more detailed history of the applications of the spheroidal functions before 1957, the reader is referred to Carson Flammer's book, Spheroidal Wave Functions.[20]

Because of this wide range of applications of the spheroidal functions, there has been much interest in generating numerical results for the oblate and prolate spheroidal eigenvalues and eigenfunctions. Contributions since 1957 have included: a matrix method of determining the prolate eigenvalues,[21] a summary of the various approaches,[22] asymptotic expansions of the prolate eigenvalues and eigenfunctions,[23] tables of the prolate spheroidal eigenvalues and eigenfunctions,[24] tables of the prolate spheroidal radial eigenfunctions,[25] tables of oblate spheroidal functions,[26] tables of prolate and oblate spheroidal functions,[27,28] a computer program for spheroidal eigenvalues and eigenfunctions with complex expansion parameters,[29] eigenvalue results for complex expansion parameters,[30] a generalization of the matrix eigenvalue method to include oblate eigenvalues,[31] and a computer program to compute oblate spheroidal functions.[32]

This report describes the spheroidal coordinate system and the solutions to the scalar Helmholtz wave equation in the spheroidal coordinate system. The methods of generating, and sample numerical results for, the eigenvalues, angle eigenfunctions, and radial eigenfunctions for complex expansion parameters are presented. A computer program listing the procedures described herein is available upon request from the authors.

A detailed description of the spheroidal coordinate system and the spheroidal wave equations is not attempted here; rather, only an outline is presented. For more detailed expositions, the reader should consult one of the standard general works on the subject.[20,22,33] Specifically, it is assumed that the reader has available to him either Flammer's Spheroidal Wave Functions (Ref. 20) or the NBS Handbook of Mathematical Functions by Abramowitz and Stegun (Ref. 22). The notation and the normalization used here are those of Flammer, except that we denote the semifocal length as ℓ instead of using Flammer's d/2.

Spheroidal Coordinate Systems

Footballs and cigars are typical prolate spheroids; raindrops and doorknobs are typical oblate spheroids. The prolate extreme is an infinitesimally thick, finite-length needle, and the oblate extreme is an infinitesimally thick disk; for both shapes, the opposite of these extremes is the sphere.

The prolate and oblate spheroidal coordinate systems consist of the confocal hyperbolas and ellipses comprising the two-dimensional elliptic coordinate system rotated about one of the axes of the ellipse. Rotation about the major axis produces the prolate spheroidal system; rotation about the minor axis produces the oblate spheroidal system.

Figure 1 illustrates the pertinent variables in the prolate coordinate system. If the axis of revolution is designated as the z axis, the rectangular and the prolate spheroidal coordinate systems are related via

$$x = \ell \sqrt{(\xi^2 - 1)(1 - \eta^2)} \cos \phi, \tag{1}$$

$$y = \ell \sqrt{(\xi^2 - 1)(1 - \eta^2)} \sin \phi, \tag{2}$$

$$z = \ell \xi \eta, \tag{3}$$

where ℓ is the semifocal distance, ϕ is the angle common to both the cylindrical and the spheroidal systems, ξ describes the system of confocal ellipses ($\xi \geq 1$), and η describes the system of confocal hyperbolas ($|\eta| \leq 1$). The semimajor axis, b, is equal to $\ell\xi$, whereas the semiminor axis, a, is equal to $\ell \sqrt{\xi^2 - 1}$.

Figure 2 illustrates the pertinent variables in the oblate coordinate system. With the axis of revolution designated as the z axis, the rectangular and the oblate spheroidal coordinate systems are related via

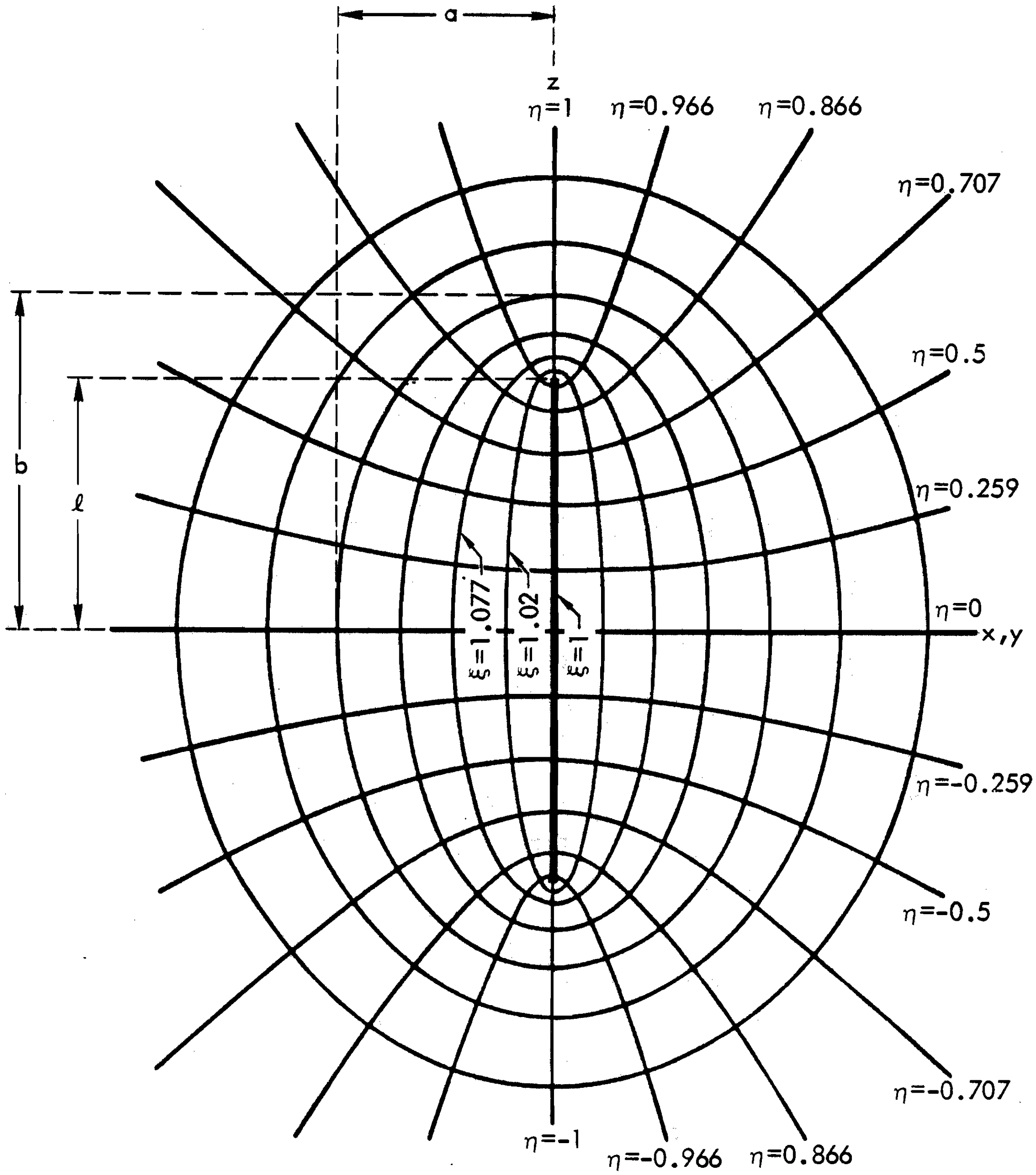

Fig. 1. The prolate spheroidal coordinate system. ℓ = semifocal length, a = semiminor axis, b = semimajor axis.

$$x = \ell\sqrt{(\xi^2 + 1)(1 - \eta^2)}\cos\phi, \tag{4}$$

$$y = \ell\sqrt{(\xi^2 + 1)(1 - \eta^2)}\sin\phi, \tag{5}$$

$$z = \ell\xi\eta, \tag{6}$$

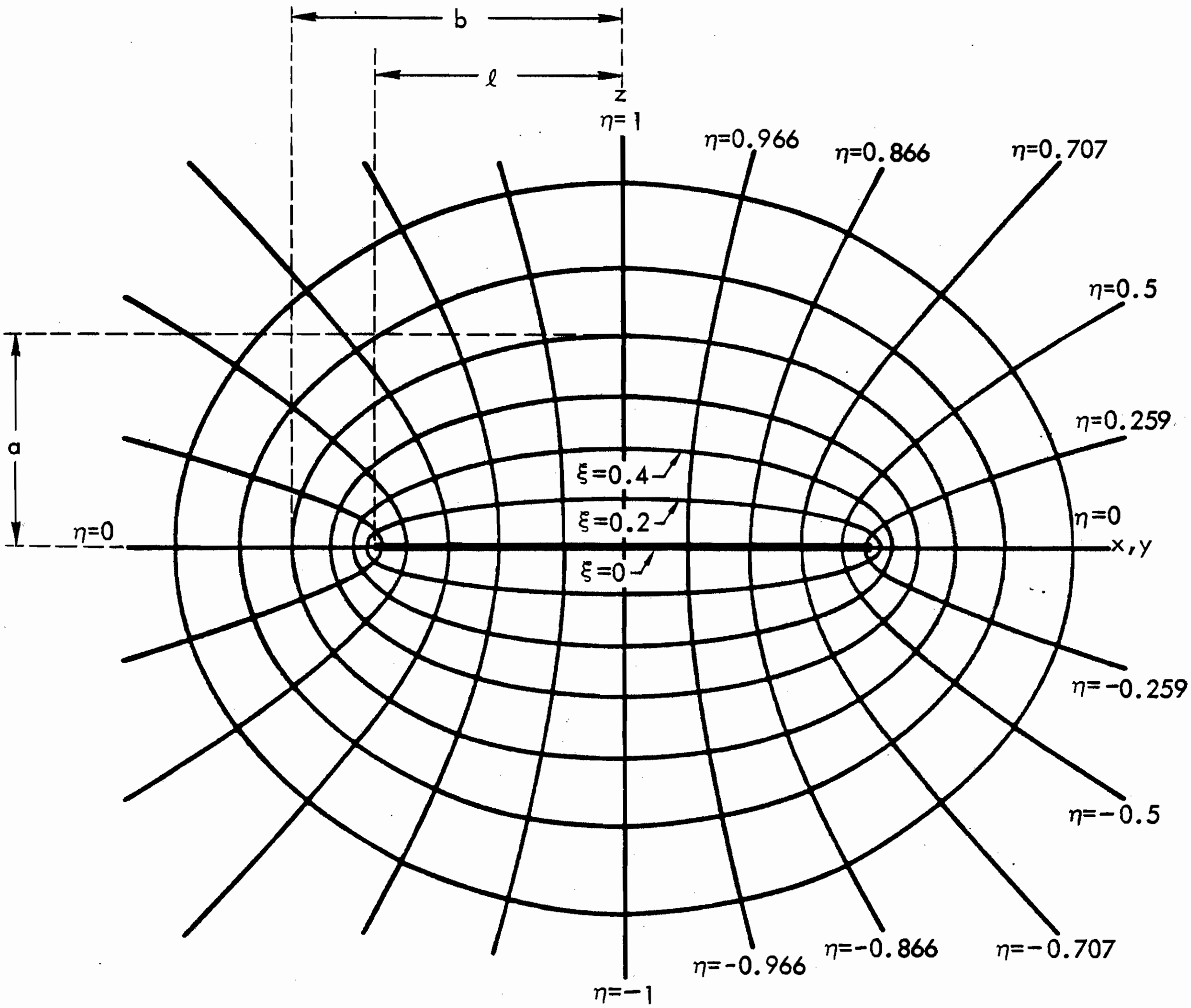

Fig. 2. The oblate spheroidal coordinate system. ℓ = semifocal length, a = semiminor axis, b = semimajor axis.

where ℓ is the semifocal axis, ϕ is the angle common to both the cylindrical and the spheroidal systems, ξ describes the system of confocal ellipses ($0 \leq \xi < \infty$), and η describes the system of confocal hyperbolas ($|\eta| \leq 1$). The semiminor axis, a, is equal to $\ell\xi$, whereas the semimajor axis, b, is equal to $\ell\sqrt{\xi^2 + 1}$.

Scalar Wave Equation in Spheroidal Coordinates

The scalar wave equation is

$$(\nabla^2 + k^2)\psi = 0, \tag{7}$$

where k^2 is in general complex (to include energy dissipation as well as propagation). In spheroidal coordinates, this equation is

$$\frac{\partial}{\partial \xi}(\xi^2 \pm 1)\frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta}(1 - \eta^2)\frac{\partial}{\partial \eta} + \frac{\xi^2 \pm \eta^2}{(\xi^2 \pm 1)(1 - \eta^2)}\frac{\partial^2}{\partial \phi^2} + c^2(\xi^2 \pm \eta^2)\psi = 0, \tag{8}$$

where the upper sign corresponds to the oblate and the lower to the prolate spheroidal system and

$$c^2 = k^2 \ell^2. \tag{9}$$

Note that the oblate equation can be obtained from the prolate equation by the transformations $\xi \rightarrow \pm i\xi$, $c \rightarrow \mp ic$. Also note that $\psi(c) = \psi(-c)$.

Solutions of Eq. (8) are obtainable using separation of variables, that is, assuming

$$\psi = S_{mn}(c,\eta)\, R_{mn}(c,\xi) \begin{matrix}\cos\\ \sin\end{matrix}\, m\phi \tag{10}$$

for the prolate system, and

$$\psi = S_{mn}(-ic,\eta)\, R_{mn}(-ic,i\xi) \begin{matrix}\cos\\ \sin\end{matrix}\, m\phi \tag{11}$$

for the oblate system. The angle functions thus satisfy

$$\frac{d}{d\eta}\left[(1 - \eta^2)\frac{dS_{mn}(c,\eta)}{d\eta}\right] + \left(\lambda_{mn} - c^2\eta^2 - \frac{m^2}{1 - \eta^2}\right) S_{mn}(c,\eta) = 0, \tag{12}$$

$$\frac{d}{d\eta}\left[(1 - \eta^2)\frac{dS_{mn}(-ic,\eta)}{d\eta}\right] + \left(\lambda_{mn} + c^2\eta^2 - \frac{m^2}{1 - \eta^2}\right) S_{mn}(-ic,\eta) = 0, \tag{13}$$

and the radial functions satisfy

$$\frac{d}{d\xi}\left[(\xi^2 - 1)\frac{dR_{mn}(c,\xi)}{d\xi}\right] - \left(\lambda_{mn} - c^2\xi^2 + \frac{m^2}{\xi^2 - 1}\right) R_{mn}(c,\xi) = 0, \tag{14}$$

$$\frac{d}{d\xi}\left[(\xi^2 + 1)\frac{dR_{mn}(-ic, i\xi)}{d\xi}\right] - \left(\lambda_{mn} - c^2\xi^2 - \frac{m^2}{\xi^2 + 1}\right) R_{mn}(ic,i\xi) = 0. \tag{15}$$

The separation constants λ_{mn} and m are the same in the prolate equation pair, Eqs. (12) and (14), and in the oblate equation pair, Eqs. (13) and (15). The prolate and oblate separation constants λ_{mn} and m are not necessarily the same, however.

Eigenvalues

The eigenvalues m are the set of integers m = 0, 1, 2, ... for most problems of interest. It is assumed throughout this report that interest is restricted to those situations requiring m to be an integer. The eigenvalues λ_{mn} are different for the prolate and oblate systems. Although there are a variety of methods for determining λ_{mn}, this report will be concerned with only three methods: a power-series expansion, an asymptotic expansion, and a matrix method.

POWER SERIES

For the prolate system, it is well known that a power-series representation for $\lambda_{mn}(c)$ is

$$\lambda_{mn} = \sum_{k=0}^{\infty} \ell_{2k} c^{2k};$$

$$\ell_0 = n(n + 1);$$

$$\ell_2 = \frac{1}{2}\left[1 - \frac{(2m - 1)(2m + 1)}{(2n - 1)(2n + 3)}\right];$$

$$\ell_4 = \frac{-(n - m + 1)(n - m + 2)(n + m + 1)(n + m + 2)}{2(2n + 1)(2n + 3)^3(2n + 5)} + \frac{(n - m - 1)(n - m)(n + m - 1)(n + m)}{2(2n - 3)(2n - 1)^3(2n + 1)};$$

$$\ell_6 = (4m^2 - 1)\left[\frac{(n - m + 1)(n - m + 2)(n + m + 1)(n + m + 2)}{(2n - 1)(2n + 1)(2n + 3)^5(2n + 5)(2n + 7)} - \frac{(n - m - 1)(n - m)(n + m - 1)(n + m)}{(2n - 5)(2n - 3)(2n - 1)^5(2n + 1)(2n + 3)}\right];$$

$$\ell_8 = 2(4m^2 - 1)^2 A + \frac{1}{16} B + \frac{1}{8} C + \frac{1}{2} D;$$

$$A = \frac{(n - m - 1)(n - m)(n + m - 1)(n + m)}{(2n - 5)^2(2n - 3)(2n - 1)^7(2n + 1)(2n + 3)^2} - \frac{(n - m + 1)(n - m + 2)(n + m + 1)(n + m + 2)}{(2n - 1)^2(2n + 1)(2n + 3)^7(2n + 5)(2n + 7)^2};$$

$$B = \frac{(n-m-3)(n-m-2)(n-m-1)(n-m)(n+m-3)(n+m-2)(n+m-1)(n+m)}{(2n-7)(2n-5)^2(2n-3)^3(2n-1)^4(2n+1)}$$

$$- \frac{(n-m+1)(n-m+2)(n-m+3)(n-m+4)(n+m+1)(n+m+2)(n+m+3)(n+m+4)}{(2n+1)(2n+3)^4(2n+5)^3(2n+7)^2(2n+9)};$$

$$C = \frac{(n-m+1)^2(n-m+2)^2(n+m+1)^2(n+m+2)^2}{(2n+1)^2(2n+3)^7(2n+5)^2}$$

$$- \frac{(n-m-1)^2(n-m)^2(n+m-1)^2(n+m)^2}{(2n-3)^2(2n-1)^7(2n+1)^2};$$

$$D = \frac{(n-m-1)(n-m)(n-m+1)(n-m+2)(n+m-1)(n+m)(n+m+1)(n+m+2)}{(2n-3)(2n-1)^4(2n+1)^2(2n+3)^4(2n+5)}. \tag{16}$$

For the oblate system, the power-series representation for $\lambda_{mn}(-ic)$ is

$$\lambda_{mn} = \sum_{k=0}^{\infty} (-1)^k \ell_{2k} c^{2k}, \tag{17}$$

where the ℓ_k's are the same as in Eq. (16). Note that Eq. (17) is obtained from Eq. (16) by the transformation $c \rightarrow -ic$.

ASYMPTOTIC EXPANSION

An asymptotic expansion for the prolate $\lambda_{mn}(c)$ is

$$\lambda_{mn}(c) = cq + m^2 \frac{1}{8}(q^2+5) - \frac{q}{64c}(q^2+11-32m^2) - \frac{1}{1024c^2}\Big[5(q^4+26q^2+21)$$

$$- 384m^2(q^2+1)\Big] - \frac{1}{c^3}\left[\frac{1}{128^2}(33q^5+1594q^3+5621q) - \frac{m^2}{128}(37q^3+167q) + \frac{m^4}{8}q\right]$$

$$- \frac{1}{c^4}\Big[\frac{1}{256^2}(63q^6+4940q^4+43327q^2+22470) - \frac{m^2}{512}(115q^4+1310q^2$$

$$+ 735) + \frac{3m^4}{8}(q^2 + 1)\Big] - \frac{1}{c^5}\Big[\frac{1}{1024^2}(527q^7 + 61529q^5 + 1043961q^3 + 2241599q)$$

$$- \frac{m^2}{32 \cdot 1024}(5739q^5 + 127550q^3 + 298951q) + \frac{m^4}{512}(355q^2 + 1505q) - \frac{m^6 q}{16}\Big] + O(c^{-6}) ;$$

$$q = 2(n - m) + 1. \tag{18}$$

The asymptotic expansion for the oblate $\lambda_{mn}(-ic)$ is

$$\lambda_{mn} = -c^2 + 2c(2\nu + m + 1) - 2\nu(\nu + m + 1) - (m + 1) + \Lambda_{mn}, \tag{19}$$

$$\Lambda_{mn} = \frac{\beta_1^{mn}}{c} + \frac{\beta_2^{mn}}{c^2} + \frac{\beta_3^{mn}}{c^3} + \frac{\beta_4^{mn}}{c^4} + \ldots ;$$

$$\beta_1^{mn} = -2^{-3}q(q^2 + 1 - m^2);$$

$$\beta_2^{mn} = -2^{-6}\left[5q^4 + 10q^2 + 1 - 2m^2(3q^2 + 1) + m^4\right];$$

$$\beta_3^{mn} = -2^{-9}q\left[33q^4 + 114q^2 + 37 - 2m^2(23q^2 + 25) + 13m^4\right];$$

$$\beta_4^{mn} = -2^{-10}\left[63q^6 + 340q^4 + 239q^2 + 14 - 10m^2(10q^4 + 23q^2 + 3) + m^4(39q^2 - 18) - 2m^6\right].$$

$\nu = \frac{1}{2}(n - m)$ and $q = n + 1$ for $(n - m)$ even,

$\nu = \frac{1}{2}(n - m - 1)$ and $q = n$ for $(n - m)$ odd;

MATRIX METHOD

The matrix method of Weeks[21] requires determining the eigenvalues of an infinite tridiagonal matrix. By truncating the matrix to a $N \times N$ matrix, one can determine approximate results for the λ_{mn}. The matrix equations to be solved for the prolate system are given below (for the oblate matrix equations, let $c \rightarrow -ic$ in Eqs. (20)-(27)):

For the λ_{0n} with n even,

$$\begin{bmatrix} D_0-\lambda & E_0 & 0 & 0 & \cdots \\ E_0 & D_2-\lambda & E_2 & 0 & \\ 0 & E_2 & D_4-\lambda & E_4 & \\ \cdot & 0 & E_4 & D_6-\lambda & \end{bmatrix} \begin{bmatrix} a_0 \\ a_2 \\ a_4 \\ a_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} . \qquad (20)$$

For the λ_{0n} with n odd,

$$\begin{bmatrix} D_1-\lambda & E_1 & 0 & 0 & \cdots \\ E_1 & D_3-\lambda & E_3 & 0 & \\ 0 & E_3 & D_5-\lambda & E_5 & \\ \cdot & \cdot & E_5 & D_7-\lambda & \end{bmatrix} \begin{bmatrix} a_1 \\ a_3 \\ a_5 \\ a_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} . \qquad (21)$$

For the λ_{1n} with n even,

$$\begin{bmatrix} F_2-\lambda & G_2 & 0 & \cdots & \\ G_2 & F_4-\lambda & G_4 & 0 & \cdots \\ 0 & G_4 & F_6-\lambda & G_6 & \\ \cdot & 0 & G_6 & F_8-\lambda & \end{bmatrix} \begin{bmatrix} b_2 \\ b_4 \\ b_6 \\ b_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} . \qquad (22)$$

For the λ_{1n} with n odd,

$$\begin{bmatrix} F_1-\lambda & G_1 & 0 & \cdots \\ G_1 & F_3-\lambda & G_3 & 0 & \cdots \\ 0 & G_3 & F_5-\lambda & G_5 & \cdot \\ \cdot & 0 & G_5 & F_7-\lambda \end{bmatrix} \begin{bmatrix} b_1 \\ b_3 \\ b_5 \\ b_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} . \tag{23}$$

In the above matrix equations,

$$D_s = s(s+1) + c^2 \frac{2s^2(2s+3) - 1}{(2s-1)(2s+1)(2s+3)}, \tag{24}$$

$$E_s = c^2 \frac{(s+1)(s+2)}{2s+3} \sqrt{\frac{1}{(2s+1)(2s+5)}}, \tag{25}$$

$$F_s = s(s+1) + c^2 \frac{(2s^2 + 2s - 3)}{(2s-1)(2s+3)}, \tag{26}$$

$$G_s = \frac{c^2}{2s+3} \sqrt{\frac{s(s+1)(s+2)(s+3)}{(2s+1)(2s+5)}} . \tag{27}$$

Standard matrix analysis procedures can be used to determine the eigenvalues of the above matrices. It is noted that as s becomes large, the matrix equation becomes strongly diagonal, i.e.,

$$D_s \to s(s+1), \; E_s \to O(c^2), \tag{28}$$

$$F_s \to s(s+1), \; G_s \to O(c^2). \tag{29}$$

To insure numerical accuracy, one should truncate the matrix at $s = N^2 \gg |c^2|$.

All the preceding procedures yield approximate results for λ_{mn}. It is sometimes desirable to obtain more accurate eigenvalues. This is possible by using Bouwkamp's iteration method. For the details of this method, one is referred to Flammer's text.[20]

For informational purposes, it is noted that $\lambda_{mn}(c) = \lambda_{mn}(-c)$ and $\lambda_{mn}(\bar{c}) = \overline{\lambda_{mn}(c)}$, where the bar denotes complex conjugate.

Angle Eigenfunctions

Because of the similarity of Eq. (12), for small c, to the differential equation for associated Legendre functions, the prolate angle functions $S_{mn}(c,\eta)$ can easily be determined in terms of expansions of Legendre functions via

$$S_{mn}(c,\eta) = \sum_{r=0}^{\infty} d_{2r}^{mn}(c)\; P_{m+2r}^{m}(\eta) \text{ for } n - m \text{ even,} \tag{30}$$

$$S_{mn}(c,\eta) = \sum_{r=0}^{\infty} d_{2r+1}^{mn}(c)\; P_{m+2r+1}^{m}(\eta) \text{ for } n - m \text{ odd.} \tag{31}$$

Similar remarks hold for the oblate angle functions $S_{mn}(-ic,\eta)$. The discussion and formulae presented below are given in terms of the prolate functions; to make the same results valid for the oblate functions, replace c by -ic.

The prolate expansion coefficients $d_r^{mn}(c)$ satisfy the recursion relation

$$\frac{(2m+r+2)(2m+r+1)c^2}{(2m+2r+3)(2m+2r+5)}\, d_{r+2}^{mn}(c) + \Big[(m+r)(m+r+1) - \lambda_{mn}(c)$$

$$+ \frac{2(m+r)(m+r+1) - 2m^2 - 1}{(2m+2r-1)(2m+2r+3)}\, c^2\Big]\, d_r^{mn}(c)$$

$$+ \frac{r(r-1)c^2}{(2m+2r-3)(2m+2r-1)}\, d_{r-2}^{mn}(c) = 0 \quad (r \geq 0). \tag{32}$$

An alternate way of writing Eq. (32) is via a continued fraction expansion involving N_r^m, where

$$N_r^m = -\frac{(2m+r)(2m+r-1)c^2}{(2m+2r-1)(2m+2r+1)}\, \frac{d_r^{mn}}{d_{r-2}^{mn}} \quad (r \geq 2). \tag{33}$$

(Note: this equation is in error by a minus sign in both Flammer[20] and Abramowitz and Stegun.[22])

The continued fraction expansion equivalent to Eq. (32) is

$$N_{r+2}^m = \gamma_r^m - \lambda_{mn} - \frac{\beta_r^m}{N_r^m} \quad (r \geq 2), \tag{34}$$

$$N_2^m = \gamma_0^m - \lambda_{mn},$$ (35)

$$N_3^m = \gamma_1^m - \lambda_{mn},$$

where

$$\gamma_r^m = (m + r)(m + r + 1) + \frac{1}{2} c^2 \left[1 - \frac{4m^2 - 1}{(2m + 2r - 1)(2m + 2r + 3)}\right] \quad (r \geq 0), \tag{36}$$

$$\beta_r^m = \frac{r(r - 1)(2m + r)(2m + r - 1)c^4}{(2m + 2r - 1)^2(2m + 2r - 3)(2m + 2r + 1)} \quad (r \geq 2). \tag{37}$$

As $r \to \infty$, $N_r^m/N_{r-2}^m \to 0$, and thus a backward recursion generation with $N_L^m = 0$, where $L \gg n$, yields accurate results for the lower order N_r^m.

Knowing the values of N_r^m, one knows the ratio of d_r^{mn}/d_{r-2}^{mn} via Eq. (33), and thus the expansion coefficients d_r^{mn} are known except for a normalization constant. The normalization used is that of Flammer, with the result that for $n - m$ even,

$$d_0^{mn}(c) = \frac{\dfrac{(-1)^{\frac{n-m}{2}} (n + m)!}{2^{n-m}\left(\frac{n - m}{2}\right)!\left(\frac{n + m}{2}\right)!}}{\alpha_{0m} + \alpha_{2m}\dfrac{d_2^{mn}(c)}{d_0^{mn}(c)} + \alpha_{4m}\dfrac{d_4^{mn}(c)}{d_2^{mn}(c)}\dfrac{d_2^{mn}(c)}{d_0^{mn}(c)} + \ldots}, \tag{38}$$

and for $n - m$ odd

$$d_1^{mn}(c) = \frac{\dfrac{(-1)^{\frac{n-m-1}{2}} (n + m + 1)!}{2^{n-m}\left(\frac{n - m - 1}{2}\right)!\left(\frac{n + m + 1}{2}\right)!}}{\alpha_{1m} + \alpha_{3m}\dfrac{d_3^{mn}(c)}{d_1^{mn}(c)} + \alpha_{5m}\dfrac{d_5^{mn}(c)}{d_3^{mn}(c)}\dfrac{d_3^{mn}(c)}{d_1^{mn}(c)} + \ldots}, \tag{39}$$

where for s even,

$$\alpha_{sm} = \frac{(-1)^{s/2} (s + 2m)!}{2^s \left(\frac{s}{2}\right)!\left(\frac{s + 2m}{2}\right)!}, \tag{40}$$

and for s odd,

$$\alpha_{sm} = \frac{(-1)^{\frac{s-1}{2}} (s + 2m + 1)!}{2^s \left(\frac{s - 1}{2}\right)! \left(\frac{s + 2m + 1}{2}\right)!} . \tag{41}$$

The quantity $d_r^{mn}(c)/d_{r-2}^{mn}(c)$ is known via Eq. (33) once the N_r^m are evaluated. The expansion coefficients $d_r^{mn}(c)$ are evaluated in the above manner. Use of the expansion coefficients $d_r^{mn}(c)$ in Eq. (30) enables one to evaluate the prolate angle functions $S_{mn}(c,\eta)$. The oblate angle functions $S_{mn}(-ic,\eta)$ are evaluated using the above expressions and replacing c by -ic.

A constant that frequently occurs in theoretical developments is the quantity N_{mn}. It occurs in the orthogonality relation between the angle functions, that is,

$$\int_{-1}^{+1} S_{mn}(c,\eta)\, S_{mn'}(c,\eta)\, d\eta = N_{mn}\, \delta_{nn'} . \tag{42}$$

Thus, by definition,

$$N_{mn} = 2 \sum_{r=0}^{\infty} \frac{(2r + 2m)! \left[d_{2r}^{mn}(c)\right]^2}{(4r + 2m + 1)(2r)!} \quad \text{for } n - m \text{ even,} \tag{43}$$

$$N_{mn} = 2 \sum_{r=0}^{\infty} \frac{(2r + 1 + 2m)! \left[d_{2r+1}^{mn}(c)\right]^2}{(4r + 2m + 3)(2r + 1)!} \quad \text{for } n - m \text{ odd.} \tag{44}$$

Radial Eigenfunctions

The radial prolate spheroidal wave functions $R_{mn}(c,\xi)$ satisfy Eq. (14). There are two linearly independent solutions of this second-order linear differential equation. A common representation of functions is in terms of the radial functions of the first kind, $R_{mn}^{(1)}(c,\xi)$, and radial functions of the second kind, $R_{mn}^{(2)}(c,\xi)$. The prolate radial function of the first kind is represented by

$$R_{mn}^{(1)}(c,\xi) = \frac{2^{n+m} d_0^{mn}(c)\, c^m m! \left(\frac{n - m}{2}\right)! \left(\frac{n + m}{2}\right)!}{(2m + 1)(n + m)! \sum_{r=0}^{\infty} d_{2r}^{mn}(c) \frac{(2m + 2r)!}{2r}} \sum_{r=0}^{\infty} d_{2r}^{mn}(c)\, P_{m+2r}^{m}(\xi) \tag{45}$$

for n - m even, and

$$R_{mn}^{(1)}(c,\xi) = \frac{2^{n+m} d_1^{mn}(c)\, c^{m+1} m! \left(\frac{n-m-1}{2}\right)! \left(\frac{n+m+1}{2}\right)!}{(2m+3)(n+m+1)! \sum_{r=0}^{\infty} d_{2r+1}^{mn}(c) \frac{(2m+2r+1)!}{(2r+1)!}} \sum_{r=0}^{\infty} d_{2r+1}^{mn}(c) P_{m+2r+1}^{m}(\xi) \tag{46}$$

for n - m odd. It is noted that by definition, since $\xi \geq 1$, the Legendre function $P_n^m(\xi)$ is

$$P_n^m(\xi) = (\xi^2 - 1)^{m/2} \frac{d^m P_n(\xi)}{d\xi^m} . \tag{47}$$

The prolate radial function of the second kind is represented by

$$R_{mn}^{(2)}(c,\xi) = \frac{(2m-1)(m)!\,(n+m)!\, c^{m-1}}{2^{n-m}(2m)! \left(\frac{n-m}{2}\right)! \left(\frac{n+m}{2}\right)! \, d_{-2m}^{mn}(c) \sum_{r=0}^{\infty} d_{2r}^{mn}(c) \frac{(2m+2r)!}{(2r)!}}$$

$$\left[\sum_{\substack{r=-2m,\\ r \text{ even}}}^{\infty} d_r^{mn}(c) Q_{m+r}^m(\xi) + \sum_{\substack{r=2m+2,\\ r \text{ even}}}^{\infty} d_{\rho/r}^{mn}(c) P_{r-m-1}^m(\xi) \right] \tag{48}$$

for n - m even, and by

$$R_{mn}^{(2)}(c,\xi) = \frac{(2m-3)(2m-1)(m)!\,(n+m+1)!\, c^{m-2}}{2^{n-m}(2m)! \left(\frac{n-m-1}{2}\right)! \left(\frac{n+m+1}{2}\right)! \, d_{-2m+1}^{mn}(c) \sum_{r=0}^{\infty} d_{2r+1}^{mn}(c) \frac{(2m+2r+1)!}{(2r+1)!}}$$

$$\left[\sum_{\substack{r=-2m+1,\\ r \text{ odd}}}^{\infty} d_r^{mn}(c) Q_{m+r}^m(\xi) + \sum_{\substack{r=2m+1,\\ r \text{ odd}}}^{\infty} d_{\rho/r}^{mn}(c) P_{r-m-1}^m(\xi) \right] \tag{49}$$

for n - m odd. By definition, the Legendre function $Q_n^m(\xi)$ is

$$Q_n^m(\xi) = (\xi^2 - 1)^{m/2} \frac{d^m Q_n(\xi)}{d\xi^m} . \tag{50}$$

The quantities $d_r^{mn}(c)$ for $r < 0$ and $d_{\rho/r}^{mn}(c)$ are defined below.

The quantities $d_r^{mn}(c)$ for $r \leq 0$ are defined by the recursion relation

$$C_{r-2}^{m} d_{r-2}^{mn} = -A_{r+2}^{m} d_{r+2}^{mn} - B_r^{mn} d_r^{mn} , \tag{51}$$

with

$$A_r^m(c) = \frac{(2m + r)(2m + r - 1)}{(2m + 2r - 1)(2m + 2r + 1)} c^2 , \tag{52}$$

$$B_r^{mn}(c) = \left[(m + r)(m + r + 1) - \lambda_{mn}(c) + \frac{2(m + r)(m + r + 1) - 2m^2 - 1}{(2m + 2r - 1)(2m + 2r + 3)} c^2 \right] , \tag{53}$$

$$C_r^m(c) = \frac{(r + 1)(r \; 2)}{(2m + 2r + 1)(2m + 2r + 3)} c^2 . \tag{54}$$

Equation (51) should be used to determine the $d_r^{mn}(c)$ for $r \geq -2m$, as these are needed in Eqs. (48) and (49). Backward recursion is to be used. The values $d_{\rho/-2m-1}^{mn}$ and $d_{\rho/-2m-2}^{mn}$ are evaluated using the continued fraction expansions

$$d_{\rho/-2m-1}^{mn} = - d_{-2m+1}^{mn} \frac{c^2}{(-2m + 1)(-2m + 3)} \cfrac{1}{B_{-2m-1}^{mn} - \cfrac{C_{-2m-3}^{m} A_{-2m-1}^{m}}{B_{-2m-3}^{mn} - \cdots}} \tag{55}$$

for $n - m$ odd, and

$$d_{\rho/-2m-2}^{mn} = -d_{-2m}^{mn} \frac{c^2}{(2m + 1)(-2m + 1)} \cfrac{1}{B_{-2m}^{mn} - \cfrac{C_{-2m-2}^{m} A_{-2m}^{m}}{B_{-2m-2}^{mn} - \cdots}} \tag{56}$$

for $n - m$ even.

The values of $d_{\rho/r}^{mn}$ for $r < -2m - 2$ are evaluated using the continued fraction relation

$$d_{\rho/r}^{mn} = - \cfrac{d_{\rho/r+2}^{mn} A_{r+2}^{m}}{B_r^{mn} - \cfrac{C_{r-2}^{m} A_r^{m}}{B_{r-2}^{mn} - \cdots}} . \tag{57}$$

With the use of the above defining formulae, one can calculate the prolate spheroidal functions of the first kind, $R_{mn}^{(1)}(c,\xi)$, and the second kind, $R_{mn}^{(2)}(c,\xi)$. It is helpful to present the asymptotic behavior of these functions for large $c\xi$. By definition,

$$R_{mn}^{(1)}(c,\xi) \xrightarrow[c\xi\to\infty]{} \frac{1}{c\xi} \cos\left[c\xi - \frac{(n+1)\pi}{2}\right], \tag{58}$$

$$R_{mn}^{(2)}(c,\xi) \xrightarrow[c\xi\to\infty]{} \frac{1}{c\xi} \sin\left[c\xi - \frac{(n+1)\pi}{2}\right]. \tag{59}$$

From the above asymptotic behavior, it is seen that radial functions of the third and fourth kind,

$$R_{mn}^{(3)}(c,\xi) = R_{mn}^{(1)}(c,\xi) + iR_{mn}^{(2)}(c,\xi) \tag{60}$$

and

$$R_{mn}^{(4)}(c,\xi) = R_{mn}^{(1)}(c,\xi) - iR_{mn}^{(2)}(c,\xi), \tag{61}$$

are useful in that their asymptotic behavior is that of inward- and outward-traveling waves (for an assumed time variation of exp $(+i\omega t)$).

The Wronskian relating $R_{mn}^{(1)}$ and $R_{mn}^{(2)}$ is useful for checking the relative numerical accuracy of these functions. The Wronskian relation is

$$R_{mn}^{(1)} \frac{dR_{mn}^{(2)}}{d\xi} - R_{mn}^{(2)} \frac{dR_{mn}^{(1)}}{d\xi} = \frac{1}{c(\xi^2 - 1)}. \tag{62}$$

Another useful check on the accuracy of the prolate spheroidal functions is the fact that for $m = 1$ and $c = \frac{p\pi}{2}$,

$$R_{1n}^{(1)}\left(\frac{p\pi}{2}, \xi\right) = \frac{\cos\left\{\frac{\pi}{2}\left[p\xi - (n+1)\right]\right\}}{\frac{p\pi}{2}\sqrt{\xi^2 - 1}}, \tag{63}$$

$$R_{1n}^{(2)}\left(\frac{p\pi}{2}, \xi\right) = \frac{\sin\left\{\frac{\pi}{2}\left[p\xi - (n+1)\right]\right\}}{\frac{p\pi}{2}\sqrt{\xi^2 - 1}}, \tag{64}$$

if n is even when p is even, and n is odd when p is odd. Equations (63) and (64) are valid only for prolate functions, not for oblate functions.

The oblate radial functions are obtained from the above prolate radial function equations, Eqs. (45), (46), (48), and (49), by letting $c \to -ic$ and $\xi \to i\xi$.

There are various other expansions of spheroidal radial functions, e.g., expansion in terms of power series of ξ, $\sqrt{\xi^2 - 1}$, spherical Bessel functions, and cylindrical Bessel functions. These alternate expansions are not discussed in this report. The interested reader is referred to the text by Flammer.[20]

Sample Computational Results

The computational algorithm for the eigenvalues and eigenfunctions is given in the preceding part of the report. The computer program has been written so that c can be complex (enabling one to consider energy dissipation problems). Both prolate and oblate spheroidal function results have been generated. The results for positive real c have been checked against available tables of prolate spheroidal eigenvalues, expansion coefficients, and wave functions, and an excellent comparison has been obtained in all cases (at least six-place accuracy). The results for negative imaginary c have been checked against available tables of oblate spheroidal eigenvalues, expansion coefficients, and wave functions. Again, at least a six-place comparison has been obtained.

No simple check was available for the general complex values of c results other than the Wronskian relationship between the radial functions. Results obtained by using this relationship indicated at least six-place accuracy in the radial functions for the ranges considered.

No such check is available for the angle function of the first kind; however, because of the assumed accuracy in the method of computation of the Legendre functions, and an assumed six-place accuracy in the expansion coefficients, a five place accuracy is estimated for the angle functions.

The program as written can be used to calculate angle functions of the first kind and radial functions of the first and second kinds, for $|c| \lesssim 15$, for $m = 0$ or $m = 1$, and $n \lesssim 29$. Lower-order function values can be obtained for larger values of c.

Computational results for $\lambda_{0n}(c)$ and $\lambda_{1n}(c)$ are given in the Appendix (Table A1) for $c = 1 + i1$, $10 + i10$, $5 + i0$, and $0 + i5$. It is noted that $\lambda_{mn} \to n(n + 1)$ for large n. The results shown were calculated using the matrix method of Weeks,[21] Eqs. (20)-(27). There is no difficulty in using this method for complex values of c.

The expansion coefficients $d_r^{mn}(c)$ and $d_{\rho/n}^{mn}(c)$ can be calculated using Eqs. (33)-(41) and (51)-(57). Sample results are given in the Appendix (Table A2) for $c = 1 - i1$, $5 + i0$, and $0 + i5$.

By using Eqs. (30) and (31), numerical results for the angle eigenfunctions can be computed. The behavior of the angle eigenfunction dependence upon c and η is illustrated in Fig. 3. Note that the results in Fig. 3 for $0 \le c^2 \le 50$ are indicative of prolate angle function dependence upon η in a lossless medium (Im c = 0). The results in Fig. 3 for $-50 \le c^2 \le 0$ are indicative of the oblate angle function dependence upon η in a lossless medium (Im c = 0).

Figure 4 depicts the oblate ($-50 \le c^2 \le 0$) and prolate ($0 \le c^2 \le 50$) radial function dependence upon ξ for a lossless medium (Im c = 0). Note that the ξ scale is different for the oblate ($0.1 \le \xi \le 1.0$) and prolate ($1.01 \le \xi \le 1.1$) functions shown in Fig. 4. This is as required (see Figs. 1 and 2), since $\xi \ge 0$ for the oblate system and $\xi \ge 1$ for the prolate system. A lower prolate bound of $\xi = 1.01$ (rather than $\xi = 1.0$) and a lower oblate bound of $\xi = 0.1$ (rather than $\xi = 0.0$) have been used, since the radial function of the second kind, $R_{mn}^{(2)}(c,\xi)$, becomes large as $\xi \to 1$ (for prolate) and $\xi \to 0$ (for oblate).

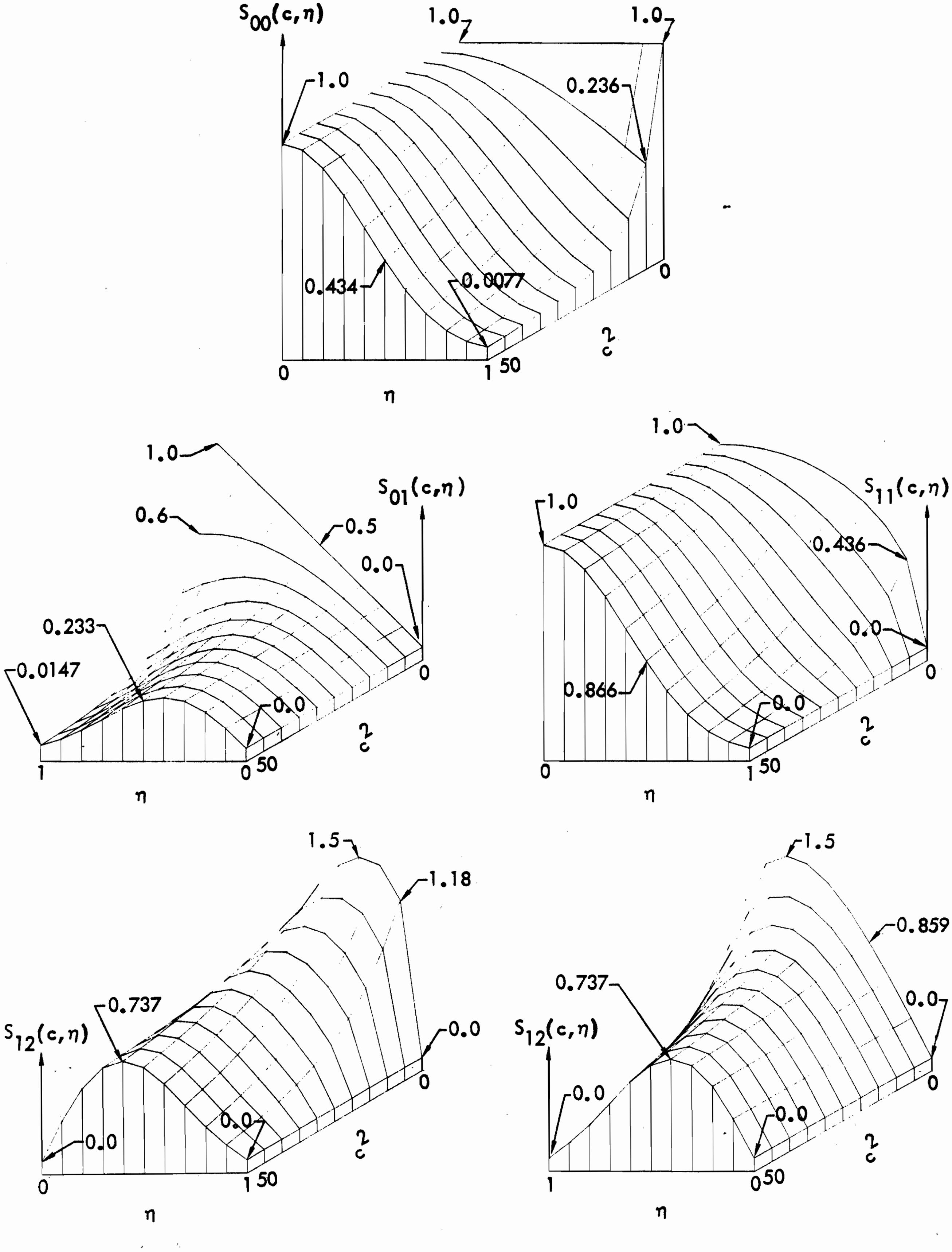

Fig. 3. Angle eigenfunction dependence upon c and η.

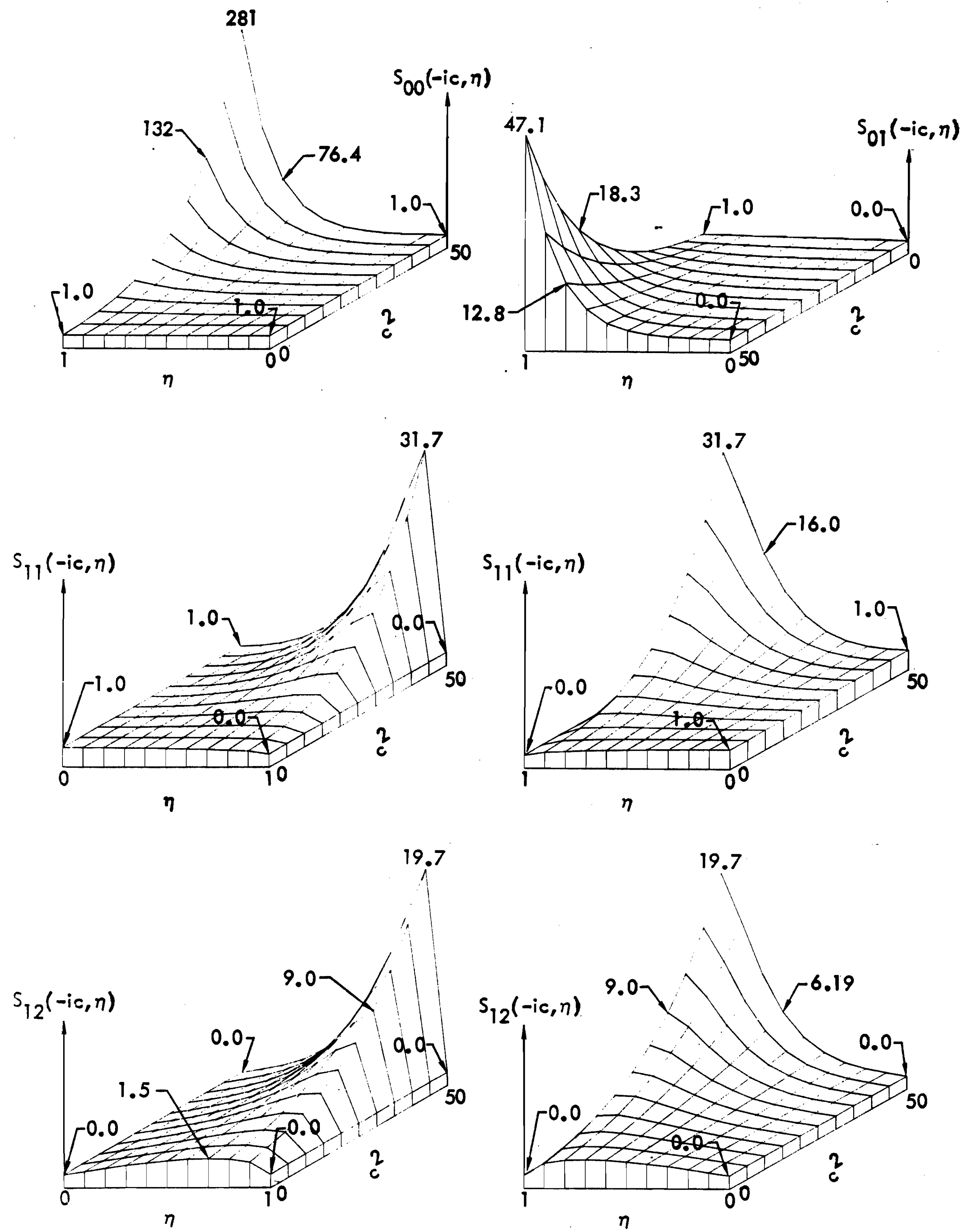

Fig. 3. (continued)

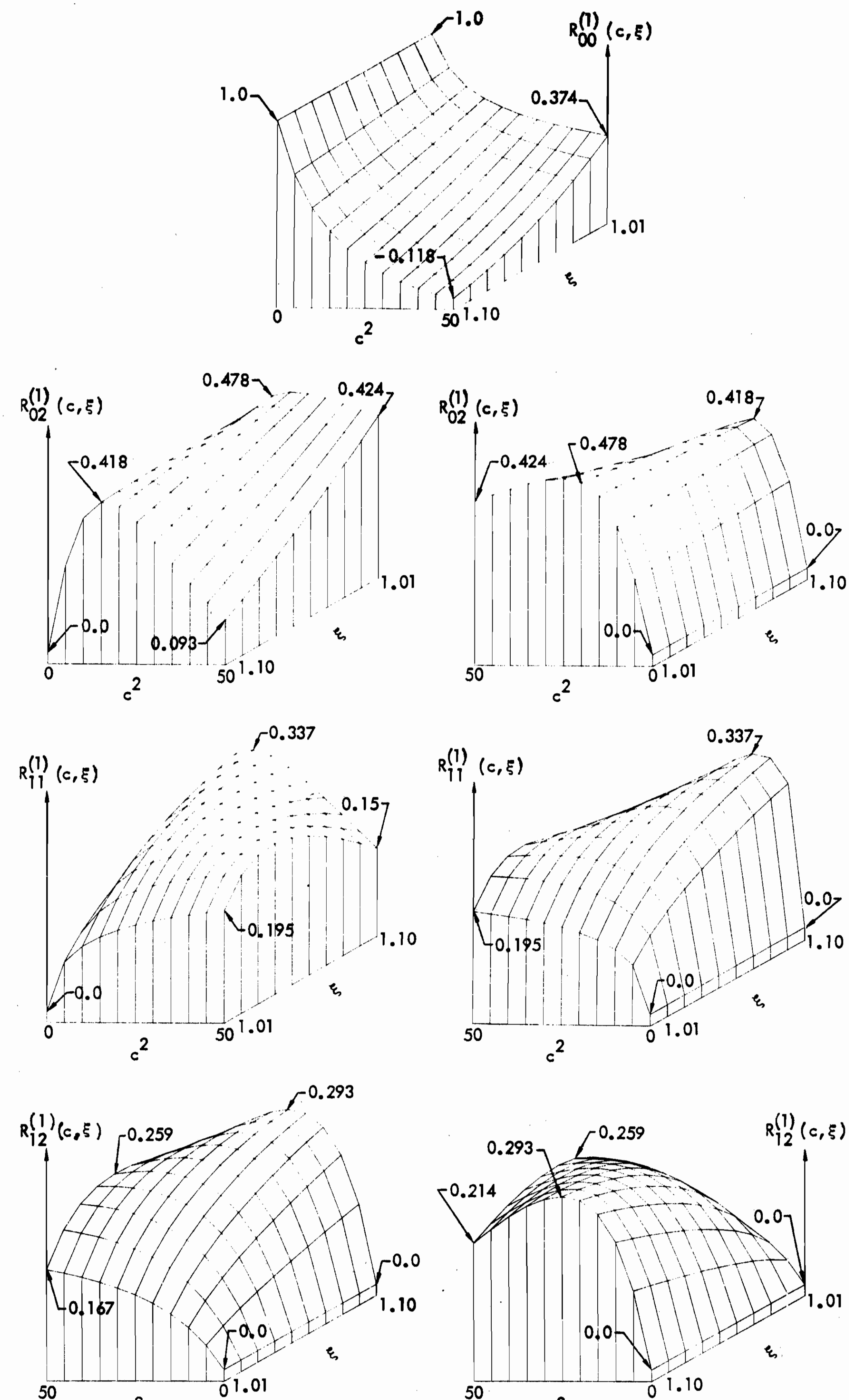

Fig. 4. Radial function dependence upon ξ.

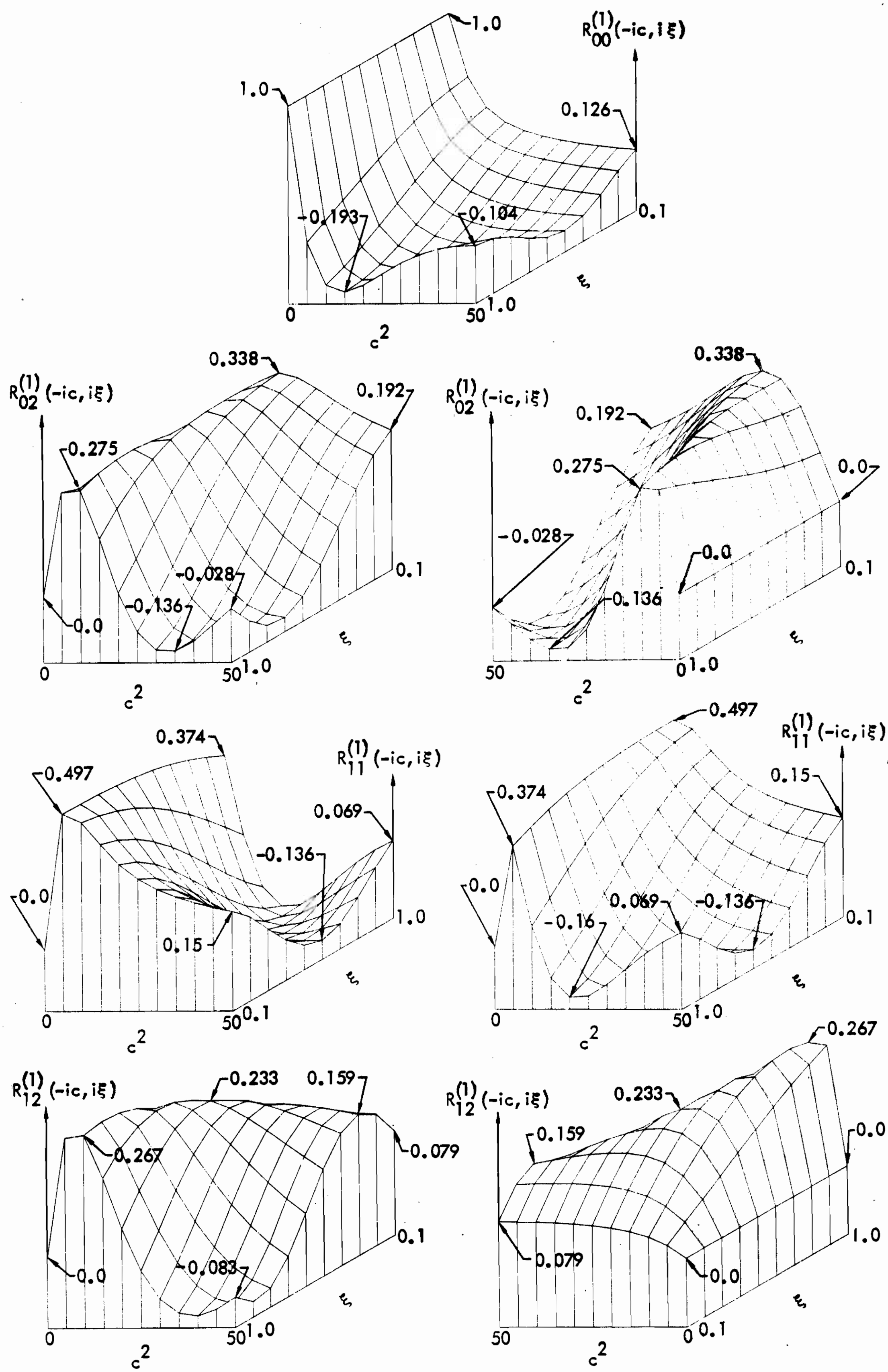

Fig. 4. (continued)

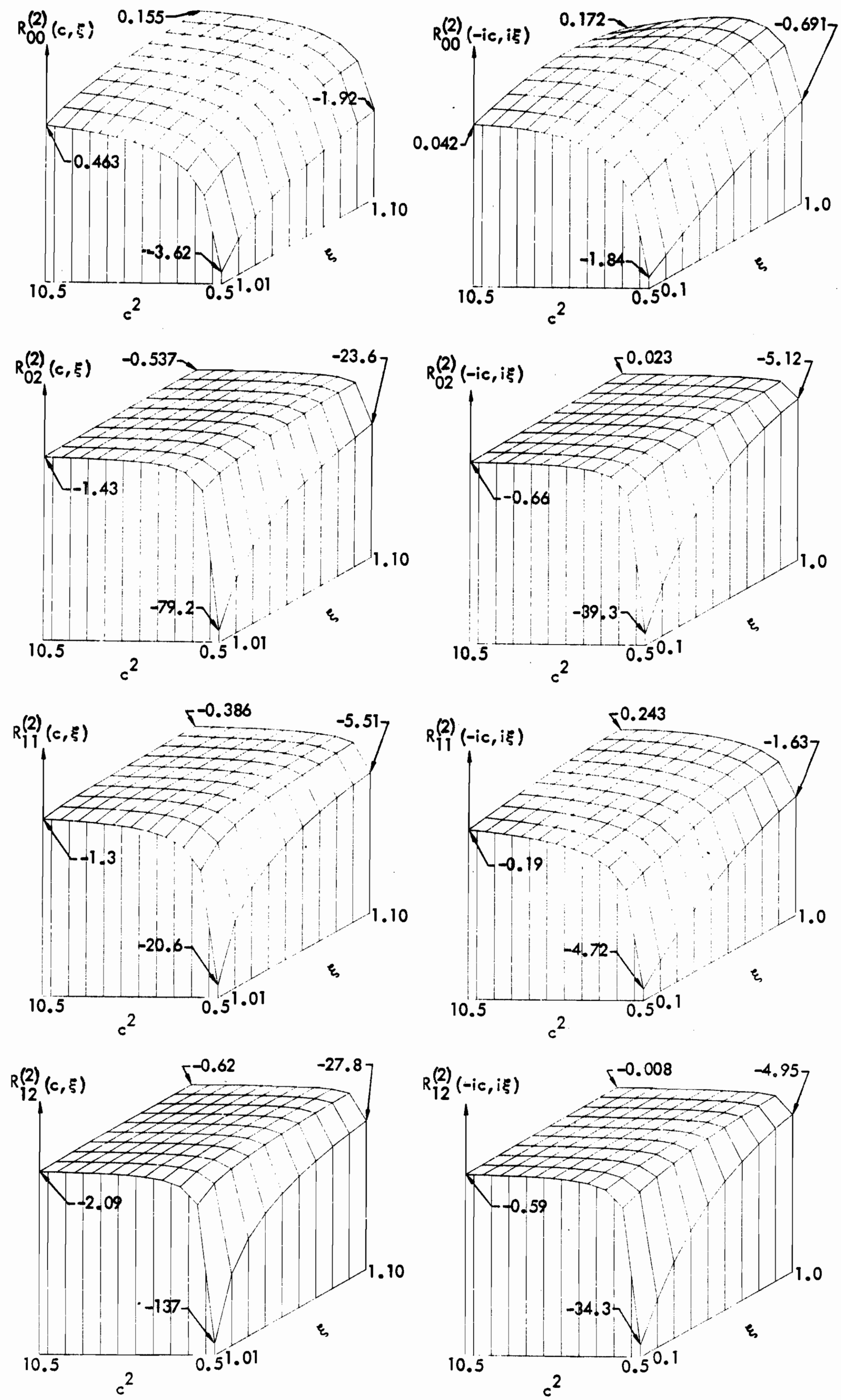

Fig. 4. (continued)

References

1. Henry A. Brown, "Effect of Truncation on Image Enhancement by Prolate Spheroidal Functions," J. Opt. Soc. Am. 59 (February 1969).
2. Gabor C. Temes, "The Prolate Filter: An Ideal Lowpass Filter with Optimum Step-Response," J. Franklin Inst. 293, 77 (1972).
3. D. R. Rhodes, "On an Optimum Line Source for Maximum Directivity," IEEE Trans. Antennas Propagat. AP-19, 485 (1971).
4. Ray H. Pettit, "Signal Representation by Prolate Spheroidal Wave Functions," IEEE Trans. Aerosp. Electron. Syst. AES-1, 39 (1965).
5. M. Hnatow, R. Pfeffer, and J. Happel, "Sedimentation of Two Prolate Spheroids in Close Proximity," Can. J. Chem. Eng. 47 (February 1969).
6. D. A. Danielson, "Buckling and Initial Postbuckling Behavior of Spheroidal Shells under Pressure," AIAA J. 7 (5) (May 1969).
7. W. T. Chen, "Axisymmetric Stress Field Around Spheroidal Inclusions and Cavities in a Transversely Isotropic Material," Trans. ASME J. Appl. Mech 35E, 770 (1968).
8. Tomohiro Oguchi, "Attenuation of Electromagnetic Wave Due to Rain with Distorted Raindrops. II," Tokyo, Japan, Radio Research Laboratories Journal 11, 19 (1964).
9. James R. Wait, "Theories of Prolate Spheroidal Antennas," Radio Sci. 1, 475 (1966).
10. Clayborne D. Taylor, "On the Exact Theory of a Prolate Spheroidal Receiving and Scattering Antenna," Radio Sci. 2, 351 (1967).
11. Carson Flammer, "The Vector Wave Function Solution of the Diffraction of Electromagnetic Waves by Circular Disks and Apertures. I. Oblate Spheroidal Vector Wave Functions. II. The Diffraction Problems," J. Appl. Phys. 24, 1218 (1953).
12. K. M. Siegel, F. V. Schultz, B. H. Gere, and F. B. Sleator, "The Theoretical and Numerical Determination of the Radar Cross Section of a Prolate Spheroid," IEEE Trans. Antennas Propagat. AP-4, 266 (1956).
13. A Haji-Sheikh and E. M. Sparrow, "Transient Heat Conduction in a Prolate Spheroidal Solid," Trans. ASME J. Heat Transfer 88C, 331 (1966).
14. Philip M. Morse and Herman Feshbach, Methods of Theoretical Physics, Part II (McGraw-Hill, New York, 1953).
15. R. D. Spence, "The Diffraction of Sound by Circular Disks and Apertures," J. Acoust. Soc. Amer. 20, 380 (1948).
16. R. D. Spence and Sara Granger, "The Scattering of Sound from a Prolate Spheroid," J. Acoust. Soc. Amer. 23, 701 (1951).
17. Sarva Jit Singh and Ari Ben-Menahem, "Eigenvibrations of the Earth Excited by Finite Dislocations. II. Spheroidal Oscillation," Geophys. J. Roy. Astron. Soc. 17, 333 (1969).

18. Robert L. Holland and Hans J. Sperling, "A First-Order Theory for the Rotational Motion of a Triaxial Rigid Body Orbiting an Oblate Primary," Astron. J. 74 (3) (April 1969).

19. D. Slepian and H. Pollak, "Prolate Spheroidal Wave Functions, Fourier Analysis and Uncertainty, II," Bell Syst. Tech. J. 40, 43 (1961).

20. Carson Flammer, Spheroidal Wave Functions (Stanford University Press, 1957).

21. Walter L. Weeks, Prolate Spheroidal Wave Functions for Electromagnetic Theory, University of Illinois, Antenna Laboratory, Tech. Rept. 38 (1959).

22. M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, National Bureau of Standards Applied Mathematics Series 55 (1965).

23. D. Slepian, "Some Asymptotic Expansions for Prolate Spheroidal Wave Functions," J. Math. Phys. 44, 99 (1965).

24. H. E. Hunter, D. B. Kirk, T. B. A. Senior, and E. R. Wittenberg, Tables of Prolate Spheroidal Functions for m = 0, Vols. I and II, University of Michigan Radiation Laboratory, Rept. 07133-1-T (1965).

25. C. Chang and C. Yeh, The Radial Prolate Spheroidal Functions, University of Southern California, School of Engineering, USCEE Rept. 166 (1966).

26. S. Hanish, C. Shely, and R. Baier, Tables of Spheroidal Wave Functions, Part 1: Oblate Radial Functions of the First and Second Type, Naval Research Laboratory, AD-646793 (Dec. 1966).

27. S. Hanish, R. V. Baier, W. H. Buckler, and Berthel K. Carmichael, Tables of Spheroidal Wave Functions, Part 2: Prolate Radial Functions of the First and Second Type, Naval Research Laboratory, AD-650575 (January 1967).

28. S. Hanish, R. B. Baier, W. H. Buckler, and Berthel K. Carmichael, Tables of Spheroidal Wave Functions, Part 3: Second Volume of Oblate Radial Functions of the First and Second Type, Naval Research Laboratory, AD-656898 (April 1967).

29. R. J. Lytle and F. V. Schultz, Linear Antennas in Plasma Media, Purdue University, School of Electrical Engineering, TR-EE68-10 (August 1968).

30. Tomohiro Oguchi, "Eigenvalues of Spheroidal Wave Functions and Their Branch Points for Complex Values of Propagation Constants," Radio Sci. 5, 1207 (1970).

31. D. B. Hodge, "Eigenvalues and Eigenfunctions of the Spheroidal Wave Equation," J. Math. Phys. 11, 2308 (1970).

32. B. Dean, "On the Interaction Between an Electromagnetic Plane Wave and a Circular Aperture," URSI 1973 Meeting, Boulder, Colorado, August 21-24, 1973.

33. J. A. Stratton, P. M. Morse, L. J. Chu, J. D. C. Little, and F. J. Corbato, Spheroidal Wave Functions (John Wiley, New York, 1956).

Appendix

Computed Eigenvalues and Expansion Coefficients

Table A1. Sample computational results for eigenvalues $\lambda_{mn}(c)$.

m n	$Re(\lambda_{mn})$	$Im(\lambda_{mn})$	m n	$Re(\lambda_{mn})$	$Im(\lambda_{mn})$
		c = 1 + i1			
-0- 0 =	5.9472769759E-02	6.6282512219E-01	-1- 1 =	2.0182518507E+00	3.9902411977E-01
-0- 2 =	5.9591777572E+00	1.0515071690E+00	-1- 3 =	1.1994575348E+01	9.3428005583E-01
-0- 4 =	1.9992907873E+01	1.0129440721E+00	-1- 5 =	2.9996441617E+01	9.7438383167E-01
-0- 6 =	4.1996850466E+01	1.0060576853E+00	-1- 7 =	5.5997930118E+01	9.8642854494E-01
-0- 8 =	7.1998206975E+01	1.0035082692E+00	-1- 9 =	8.9998671995E+01	9.9159735763E-01
-0-10 =	1.0999883966E+02	1.0022881975E+00	-1-11 =	1.3199908088E+02	9.9428593405E-01
-0-12 =	1.5599918691E+02	1.0016102613E+00	-1-13 =	1.8199932770E+02	9.9586215106E-01
-0-14 =	2.0999939828E+02	1.0011947252E+00	-1-15 =	2.3999948746E+02	9.9686523909E-01
-0-16 =	2.7199953659E+02	1.0009216509E+00	-1-17 =	3.0599959658E+02	9.9754301444E-01
-0-18 =	3.4199963207E+02	1.0007325967E+00	-1-19 =	3.7999967434E+02	9.9802242142E-01
-0- 1 =	2.0274701052E+00	1.2004904294E+00	-1- 2 =	6.0155581788E+00	8.5702722043E-01
-0- 3 =	1.1986780725E+01	1.0217425726E+00	-1- 4 =	1.9995229952E+01	9.6114441476E-01
-0- 5 =	2.9995481339E+01	1.0085377339E+00	-1- 6 =	4.1997320447E+01	9.8182629592E-01
-0- 7 =	5.5997672435E+01	1.0045237572E+00	-1- 8 =	7.1998360250E+01	9.8947512986E-01
-0- 9 =	8.9998575041E+01	1.0028008727E+00	-1-10 =	1.0999890402E+02	9.9313539711E-01
-0-11 =	1.3199903647E+02	1.0019046870E+00	-1-12 =	1.5599921855E+02	9.9516921370E-01
-0-13 =	1.8199930454E+02	1.0013792825E+00	-1-14 =	2.0999941562E+02	9.9641582366E-01
-0-15 =	2.3999947421E+02	1.0010449202E+00	-1-16 =	2.7199954688E+02	9.9723504719E-01
-0-17 =	3.0599958847E+02	1.0008189951E+00	-1-18 =	3.4199963856E+02	9.9780220986E-01
-0-19 =	3.7999966909E+02	1.0006591928E+00	-1-20 =	4.1999970507E+02	9.9821109771E-01
		c = 10 + i10			
-0- 0 =	9.2407662147E+00	1.0010651404E+01	-1- 1 =	1.0265301312E+01	9.9815516889E+00
-0- 2 =	1.8987601872E+01	1.7998616512E+02	-1- 3 =	3.7950930369E+01	1.5994207876E+02
-0- 4 =	4.6119261872E+01	5.0177257845E+01	-1- 5 =	4.7222730385E+01	4.9993835496E+01
-0- 6 =	5.4830901942E+01	1.3978133496E+02	-1- 7 =	7.2045586613E+01	1.1944733949E+02
-0- 8 =	7.6916379087E+01	9.2203147064E+01	-1- 9 =	8.0538704537E+01	8.8781029028E+01
-0-10 =	9.7963735098E+01	1 0183151873E+02	-1-11 =	1.2323787014E+02	9.9719264018E+01
-0-12 =	1.4814435603E+02	1.0011600352E+02	-1-13 =	1.7540335039E+02	9.9652764469E+01
-0-14 =	2.0409262870E+02	1.0010417371E+02	-1-15 =	2.3493397717E+02	9.9718003847E+01
-0-16 =	2.6741654006E+02	1.0008475178E+02	-1-17 =	3.0199576960E+02	9.9770029876E+01
-0-18 =	3.3834613313E+02	1.0006943725E+02	-1-19 =	3.7675947259E+02	9.9810610888E+01
-0- 1 =	1.8987613039E+01	1.7998615939E+02	-1- 2 =	2.9275534076E+01	2.9959747372E+01
-0- 3 =	2.8204580479E+01	3.0055992717E+01	-1- 4 =	3.7951078021E+01	1.5994143598E+02
-0- 5 =	5.4809267969E+01	1.3975629422E+02	-1- 6 =	6.4040222784E+01	7.0065058175E+01
-0- 7 =	6.2979062488E+01	7.0514457665E+01	-1- 8 =	7.1237652655E+01	1.1954292415E+02
-0- 9 =	8.4191971288E+01	1.0372243082E+02	-1-10 =	1.0086164462E+02	9.8543823498E+01
-0-11 =	1.2266463242E+02	1.0000946926E+02	-1-12 =	1.4836616375E+02	9.9629380585E+01
-0-13 =	1.7521425578E+02	1.0011607141E+02	-1-14 =	2.0424229624E+02	9.9687203421E+01
-0-15 =	2.3481585042E+02	1.0009391115E+02	-1-16 =	2.6751062022E+02	9.9745571708E+01
-0-17 =	3.0192016932E+02	1.0007661795E+02	-1-18 =	3.3840742593E+02	9.9791609407E+01
-0-19 =	3.7670934673E+02	1.0006311171E+02	-1-20 =	4.1706272493E+02	9.9827344963E+01
		c = 0 + i5			
-0- 0 =	-1.6079042745E+01	0.	-1- 1 =	-7.4933882840E+00	0.
-0- 2 =	-2.4485989032E+00	0.	-1- 3 =	2.7503672149E+00	0.
-0- 4 =	8.6303959354E+00	0.	-1- 5 =	1.8439315767E+01	0.
-0- 6 =	2.9916882305E+01	0.	-1- 7 =	4.3999910388E+01	0.
-0- 8 =	5.9736180509E+01	0.	-1- 9 =	7.7814155174E+01	0.
-0-10 =	9.7652658847E+01	0.	-1-11 =	1.1971554968E+02	0.
-0-12 =	1.4360690187E+02	0.	-1-13 =	1.6965696488E+02	0.
-0-14 =	1.9757907790E+02	0.	-1-15 =	2.2761935342E+02	0.
-0-16 =	2.5956088370E+02	0.	-1-17 =	2.9359378679E+02	0.
-0-18 =	3.2954832930E+02	0.	-1-19 =	3.6757562570E+02	0.
-0- 1 =	-1.6050412679E+01	0.	-1- 2 =	-7.1278375187E+00	0.
-0- 3 =	6.0929892258E-02	0.	-1- 4 =	8.6949592545E+00	0.
-0- 5 =	1.8084568016E+01	0.	-1- 6 =	3.0161309555E+01	0.
-0- 7 =	4.3806881082E+01	0.	-1- 8 =	5.9890952878E+01	0.
-0- 9 =	7.7687567015E+01	0.	-1-10 =	9.7757940102E+01	0.
-0-11 =	1.1962671823E+02	0.	-1-12 =	1.4368279543E+02	0.
-0-13 =	1.6959141507E+02	0.	-1-14 =	1.9763623735E+02	0.
-0-15 =	2.2756908795E+02	0.	-1-16 =	2.5960541939E+02	0.
-0-17 =	2.9355406248E+02	0.	-1-18 =	3.2958397591E+02	0.
-0-19 =	3.6754346399E+02	0.	-1-20 =	4.0756846028E+02	0.
		c = 5 + i0			
-0- 0 =	4.1951288727E+00	0.	-1- 1 =	5.3504222985E+00	0.
-0- 2 =	2.0176914721E+01	0.	-1- 3 =	2.3397613125E+01	0.
-0- 4 =	3.3897096095E+01	0.	-1- 5 =	4.2658182155E+01	0.
-0- 6 =	5.5080982238E+01	0.	-1- 7 =	6.8647260378E+01	0.
-0- 8 =	8.4825930681E+01	0.	-1- 9 =	1.0260121526E+02	0.
-0-10 =	1.2271038894E+02	0.	-1-11 =	1.4457182805E+02	0.
-0-12 =	1.6864733498E+02	0.	-1-13 =	1.9455319479E+02	0.
-0-14 =	2.2260901670E+02	0.	-1-15 =	2.5254084524E+02	0.
-0-16 =	2.8458395701E+02	0.	-1-17 =	3.1853229566E+02	0.
-0-18 =	3.5458666017E+02	0.	-1-19 =	3.9252615175E+02	0.
-0- 1 =	1.2911703245E+01	0.	-1- 2 =	1.4642956245E+01	0.
-0- 3 =	2.6587359607E+01	0.	-1- 4 =	3.2421943588E+01	0.
-0- 5 =	4.3358995921E+01	0.	-1- 6 =	5.4671796708E+01	0.
-0- 7 =	6.8924772951E+01	0.	-1- 8 =	8.4621983476E+01	0.
-0- 9 =	1.0275858131E+02	0.	-1-10 =	1.2258479293E+02	0.
-0-11 =	1.4467463207E+02	0.	-1-12 =	1.6856150600E+02	0.
-0-13 =	1.9462600685E+02	0.	-1-14 =	2.2254642384E+02	0.
-0-15 =	2.5259525790E+02	0.	-1-16 =	2.8453620048E+02	0.
-0-17 =	3.1857455973E+02	0.	-1-18 =	3.5452898370E+02	0.
-0-19 =	3.9255995539E+02	0.	-1-20 =	4.3252371224E+02	0.

Table A2. Sample calculational results for expansion coefficients $d_r^{mn}(c)$ and $d_{\rho/r}^{mn}(c)$.

c = 1 - i1

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
m = 0, n = 0				
0	9.83521067E-01	1.09398482E-01		
2	-3.65667649E-02	2.17771735E-01	-3.58409372E-01	9.03894076E-01
4	-7.46068204E-03	-1.45676680E-03	-5.97407861E-02	-2.57921047E-02
6	2.19301742E-05	-1.07624838E-04	5.61414368E-04	-1.26019425E-03
8	8.58379841E-07	1.79145374E-07	1.31957151E-05	5.97552420E-06
10	-9.21478780E-10	4.34811712E-09	-3.78301043E-08	8.27066461E-08
12	-1.52332882E-11	-3.26292540E-12	-3.45218270E-10	-1.58977238E-10
14	8.44424952E-15	-3.91148793E-14	4.76035734E-13	-1.02866690E-12
16	7.67770080E-17	1.66748043E-17	2.29812131E-15	1.06743632E-15
m = 0, n = 4				
0	-3.62976778E-04	-5.48668126E-07		
2	-4.30204446E-04	-2.72143173E-02	3.56848243E-07	4.02658989E-05
4	1.00092909E+00	-1.33229876E-02	4.04307357E-06	-4.05453135E-08
6	3.75442250E-04	2.75587257E-02	2.68318946E-09	2.56339981E-07
8	-3.04337576E-04	4.20131751E-06	-4.18145173E-09	4.47462012E-11
10	-2.62348317E-08	-1.88425297E-06	-3.65816059E-13	-3.37281693E-11
12	7.57233346E-09	-1.06062247E-10	1.66031527E-13	-1.81714504E-15
14	3.02342668E-13	2.14909137E-11	6.12579763E-18	5.56083346E-16
16	-4.55325559E-14	6.42742616E-16	-1.35577851E-18	1.50085519E-20
m = 0, n = 8				
0	2.52554545E-09	5.28504710E-13		
2	3.04746519E-09	6.81880956E-07	-1.87431972E-13	-7.21550517E-11
4	-1.18141859E-04	4.49001615E-07	-9.61997333E-13	2.79892624E-15
6	-5.29803850E-05	-1.46423340E-02	4.40408416E-17	1.45378014E-14
8	1.00022726E+00	-3.53406437E-03	5.34257109E-16	-1.65247756E-18
10	5.22918178E-05	1.46666319E-02	5.24831523E-20	1.67554401E-17
12	-9.54309409E-05	3.42401771E-07	-1.53966011E-19	4.86382946E-22
14	-1.34421703E-09	-3.72904925E-07	-2.41385939E-24	-7.59461788E-22
16	9.95505676E-10	-3.60139194E-12	2.42480463E-24	-7.74250829E-27
m = 0, n = 12				
0	-2.76616839E-15	-1.79317672E-19		
2	-3.36951365E-15	-1.61820030E-12	4.39496795E-20	3.59239535E-17
4	6.37173003E-10	-1.13130317E-12	1.99575786E-19	-2.72729417E-22
6	3.49419697E-10	2.06530450E-07	-1.42964292E-24	-1.00565911E-21
8	-5.36062163E-05	8.86013784E-08	-5.90719068E-24	8.56981375E-27
10	-1.62722673E-05	-9.98164417E-03	7.42254786E-29	5.05354132E-26
12	1.00010229E+00	-1.61564590E-03	1.16151165E-27	-1.72027022E-30
14	1.62102476E-05	9.98675420E-03	3.61112882E-32	2.42367060E-29
16	-4.59663298E-05	7.48842473E-08	-1.54763675E-31	2.31630424E-34

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
m = 0, n = 2				
0	5.24160424E-04	-4.48764005E-02		
2	1.00360562E+00	-5.30500926E-02	-2.29091826E-02	6.02615838E-04
4	2.72379475E-03	4.09811555E-02	-9.43094948E-05	-3.02811108E-03
6	-8.23518918E-04	4.60366011E-05	7.99512107E-05	-2.63302426E-06
8	-4.12529127E-07	-7.16160641E-06	3.17873383E-08	9.38434679E-07
10	3.83581579E-08	-2.22774662E-09	-6.30546471E-09	2.17241727E-10
12	8.16186142E-12	1.39731111E-10	-9.61190325E-13	-2.75832091E-11
14	-3.69268150E-13	2.16619332E-14	8.50180554E-14	-2.96625712E-15
16	-4.36148844E-17	-7.41126871E-16	6.88756944E-18	1.94850813E-16
m = 0, n = 6				
0	-6.11619806E-10	1.28155194E-06		
2	-2.01830073E-04	1.53472817E-06	6.40666490E-08	-2.80000072E-10
4	-1.22997704E-04	-1.90759261E-02	-8.38128959E-12	-1.70739987E-09
6	1.00040308E+00	-6.13572757E-03	-9.04984998E-11	4.63061288E-13
8	1.19069976E-04	1.91466362E-02	-1.98326404E-14	-3.79486243E-12
10	-1.56915048E-04	9.84426078E-07	4.46399003E-14	-2.36324416E-16
12	-4.74908020E-09	-7.52313318E-07	1.46036277E-18	2.73468018E-16
14	2.41477150E-09	-1.53134726E-11	-1.05986727E-18	5.69527723E-21
16	3.56735584E-14	5.60560080E-12	-1.55013488E-23	-2.87120877E-21
m = 0, n = 10				
0	3.49071059E-16	-3.17455843E-12		
2	1.30949198E-09	-3.85144781E-12	-5.87870421E-14	1.01153248E-16
4	6.95611289E-10	3.57498891E-07	9.23332737E-19	4.79804014E-16
6	-7.66846064E-05	1.82996631E-07	3.80843490E-10	-7.62564583E-21
8	-2.76739100E-05	-1.18727250E-02	-8.46492839E-23	-4.14194214E-20
10	1.00014637E+00	-2.29910457E-03	-1.17022518E+21	2.42164328E-24
12	2.74914718E-05	1.18830491E-02	-6.11909027E-26	-2.93227617E-23
14	-6.40787065E-05	1.48946939E-07	2.20973648E-25	-4.63918325E-28
16	-4.92680403E-10	-2.11190440E-07	1.92287638E-30	9.11781548E-28
m = 0, n = 14				
0	-7.32264237E-23	1.76099727E-18		
2	-1.39308145E-15	2.16150644E-18	1.70095102E-20	-1.85534289E-23
4	-9.87716762E-16	-7.45909370E-13	-7.01403764E-26	-6.87250003E-23
6	3.37814616E-10	-4.26306913E-13	-2.42414814E-25	2.57308879E-28
8	1.59320671E-10	1.29214662E-07	1.00145574E-30	9.24650170E-28
10	-3.95270236E-05	4.80737207E-08	4.35119993E-30	-4.77087566E-33
12	-1.03765829E-05	-8.60921067E-03	-3.40014009E-35	-3.07551380E-32
14	1.00007558E+00	-1.19763387E-03	-5.96190841E-34	6.63052344E-37
16	1.03516288E-05	8.61202284E-03	-1.19072842E-38	-1.06587210E-35

Table A2. (continued).

c = 1 - i1

m = 0, n = 16

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	8.65571339E-22	2.42634994E-26		
2	1.06020703E-21	8.82861292E-19	-4.54772189E-27	-6.41162215E-24
4	-6.16473948E-16	6.32064004E-19	-1.97280216E-26	1.56091757E-29
6	-3.61891487E-16	-3.70255033E-13	4.25836275E-32	5.17590530E-29
8	1.93868479E-10	-1.85176750E-13	1.40758286E-31	-1.18153359E-34
10	6.09833875E-11	8.59574992E-08	-3.71112730E-37	-4.36736739E-34
12	-3.03294710E-05	2.83165282E-08	-1.72003093E-36	1.47365067E-39
14	-7.02138873E-06	-7.56810711E-03	8.93661173E-42	1.03691924E-38
16	1.00005814E+00	-9.23409516E-04	1.73855627E-40	-1.50506256E-43

m = 0, n = 1

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	9.91130535E-01	1.19654758E-01	1.25061895E-01	-3.06553645E-01
3	-8.16502215E-03	7.95823059E-02	1.03909069E-01	3.78390465E-02
5	-1.80724368E-03	-1.72994070E-04	-1.40166718E-03	3.08113685E-03
7	1.81125361E-08	-1.96747553E-05	-6.04581798E-05	-2.09560279E-05
9	1.28317252E-07	1.13425844E-08	1.70610406E-07	-4.96725616E-07
11	-4.72963316E-11	5.35938296E-10	2.56530479E-09	8.75844400E-10
13	-1.61597504E-12	-1.40815413E-13	-3.09734986E-12	9.11045098E-12
15	3.14348516E-16	-3.64271552E-15	-2.36303765E-14	-8.00854683E-15

m = 0, n = 5

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-2.74919276E-04	-2.20640082E-07	-7.44689256E-08	6.10637555E-06
3	1.71539646E-04	-2.24479944E-02	3.38942865E-07	3.58216261E-09
5	1.00034563E+00	8.40090321E-03	2.35862183E-10	-2.33017661E-08
7	-1.86250764E-04	2.25930570E-02	1.17592809E-09	1.16593537E-11
9	-2.12615965E-04	-1.73241911E-06	-1.57490531E-13	1.60767867E-11
11	9.26955131E-09	-1.14926307E-06	-1.11980073E-13	-1.09823418E-15
13	4.1024[illegible]064E-09	3.29674842E-11	4.63379785E-18	-4.85747241E-16
15	-8.38320917E-14	1.04766080E-11	1.45519643E-18	1.40022818E-20

m = 0, n = 9

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	1.89393281E-09	2.83223389E-13	5.46581946E-14	-1.40733776E-11
3	-1.16550052E-09	4.87656760E-07	-2.23381211E-13	-7.48366305E-16
5	-9.41441237E-05	-2.48436933E-07	-7.01234977E-18	2.18810412E-15
7	3.58604231E-05	-1.31133355E-02	2.76203323E-17	8.66416153E-20
9	1.00015596E+00	2.78577809E-03	2.73205287E-21	-8.82019261E-19
11	-3.62381678E-05	1.31288089E-02	2.45699407E-20	7.54760127E-23
13	-7.74300702E-05	-2.12821157E-07	-6.21356832E-25	2.03466948E-22
15	7.57791963E-10	-2.76884378E-07	-9.14323989E-25	-2.77996773E-27

m = 0, n = 18

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	6.64220349E-30	-3.34466082E-25		
2	4.28977947E-22	-4.10507898E-25	-1.96756185E-27	1.11407621E-30
4	3.03270256E-22	3.72359356E-19	3.01139873E-33	4.76984015E-30
6	-2.[illegible]0305900E-16	2.26234204E-19	9.70661810E-33	-6.37130545E-36
8	-1.51044908E-16	-1.90330002E-13	-1.33502010E-38	-1.99362836E-35
10	1.18701074E-10	-8.91800445E-14	-4.46749980E-38	3.02832087E-41
12	4.46608479E-11	5.99801565E-08	8.01662139E-44	1.17298525E-40
14	-2.39984173E-05	1.77558262E-08	3.97789802E-43	-2.73472292E-46
16	-4.97155797E-06	-6.75140487E-03	-1.44444996E-48	-2.09176523E-45

m = 0, n = 3

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	1.13790754E-04	-3.43283829E-02	-1.89793454E-03	-6.22807355E-05
3	9.99861767E-01	2.11624366E-02	-1.20967819E-05	4.25058957E-04
5	-7.19888184E-04	3.52318937E-02	-3.63361928E-05	-9.94422942E-07
7	-4.70183602E-04	-9.42321126E-06	1.96944441E-08	-7.33648726E-07
9	6.73826542E-08	-3.40339388E-06	7.02117173E-09	1.86345339E-10
11	1.56361196E-08	3.06968738E-10	-1.05156495E-12	3.99199027E-11
13	-9.73791693E-13	4.99035281E-11	-1.51510135E-13	-3.96386591E-15
15	-1.17397242E-13	-2.28016910E-15	1.07684643E-17	-4.12614203E-16

m = 0, n = 7

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-2.96612061E-10	9.66046856E-07	1.14991359E-08	7.29022812E-11
3	-1.52145653E-04	-5.97495416E-07	1.67963269E-12	-3.06614867E-10
5	7.17209387E-05	-1.65696717E-02	-5.83771660E-12	-3.06074522E-14
7	1.00023209E+00	4.48460250E-03	-1.29888029E-15	2.53042153E-13
9	-7.36310425E-05	1.36101037E-02	-9.07960485E-15	-4.60305134E-17
11	-1.20499135E-04	-5.30009217E-07	4.71110460E-19	-9.36923995E-17
13	2.26942526E-09	-5.18904533E-07	5.12087345E-19	2.55999320E-21
15	1.51268795E-09	6.58713014E-12	-8.92532207E-24	1.79312739E-21

m = 0, n = 11

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	1.99081769E-16	-2.38553175E-12	-1.20494054E-14	-3.16067696E-17
3	9.04621052E-10	1.45396570E-12	-2.88190643E-19	1.27505655E-16
5	-4.77314423E-10	2.66874946E-07	7.80634056E-19	1.68721725E-21
7	-6.36191140E-05	-1.17165192E-07	1.11186049E-23	-5.25660626E-21
9	2.03476921E-05	-1.08455076E-02	-5.03108734E-23	-1.05071293E-25
11	1.00011028E+00	1.89768922E-03	-2.64065807E-27	1.27507178E-24
13	-2.04786778E-05	1.08526583E-02	-2.90338489E-26	-5.97727166E-29
15	-5.39001832E-05	-1.01279144E-07	4.11402056E-31	-2.00720764E-28

Table A2. (continued).

c = 1 - i1

m = 0, n = 13

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-2.07769261E-15	-1.06723513E-19	-1.43871921E-20	7.61051377E-18
3	1.25965862E-15	-1.09078434E-12	5.76551316E-20	9.38459299E-23
5	4.56949683E-10	5.86268903E-13	3.79742711E-25	-2.44041224E-22
7	-2.14556713E-10	1.61944531E-07	-1.05644351E-24	-1.60856443E-27
9	-4.57699236E-05	-6.17680401E-08	-8.29203789E-30	5.51602189E-27
11	1.26190672E-05	-9.24482309E-03	4.26883562E-29	6.36330234E-32
13	1.00008163E+00	1.37560804E-03	1.33023704E-33	-8.97747212E-31
15	-1.26680598E-05	9.24857151E-03	1.72877985E-32	2.55022815E-35

m = 0, n = 15

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-4.49576248E-23	1.32822119E-18	3.68948070E-21	5.26520727E-24
3	-9.22004613E-16	-8.01955712E-19	2.56168807E-26	-2.10226277E-23
5	5.00668450E-16	-5.20286704E-13	-6.55248517E-26	-7.69111085E-29
7	2.53641459E-10	2.53235061E-13	-2.30336376E-31	2.00564832E-28
9	-1.06495902E-10	1.04683351E-07	6.85685999E-31	7.77412227E-34
11	-3.44744826E-05	-3.54748728E-08	3.30438457E-36	-2.93929850E-33
13	8.35305779E-06	-8.05518246E-03	-1.91240870E-35	-2.13707638E-38
15	1.00006269E+00	1.04280957E-03	-3.82534995E-40	3.43857120E-37

m = 0, n = 17

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	6.49739811E-22	1.52536380E-26	1.57901113E-27	-1.41554984E-24
3	-3.90987707E-22	5.76101844E-19	-6.29132304E-27	-6.03037049E-30
5	-4.19261185E-16	-3.15019672E-19	-1.38310314E-32	1.50840627E-29
7	2.09358362E-16	-2.66586631E-13	3.46281563E-32	3.10640746E-35
9	1.50615108E-10	1.19633480E-13	7.52702850E-38	-8.49953782E-35
11	-5.73879494E-11	7.14246190E-08	-2.41660923E-37	-2.12198687E-40
13	-2.68873644E-05	-2.17711212E-08	-7.68572576E-43	8.80573477E-40
15	5.81076485E-06	-7.13647789E-03	4.94607277E-42	4.29762090E-45

m = 0, n = 19

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	4.25433322E-30	-2.51038225E-25	-4.40416874E-28	-3.94557828E-31
3	2.76769399E-22	1.50646705E-25	-1.20947706E-33	1.57011213E-30
5	-1.51979671E-22	2.52081831E-19	2.99068634E-33	2.20135049E-36
7	-2.04216699E-16	-1.28009459E-19	3.86149330E-39	-5.36252634E-36
9	9.48723457E-17	-1.48050715E-13	-1.00000334E-38	-7.10838713E-42
11	9.47706892E-11	6.11035586E-14	-1.45470140E-44	2.06390436E-41
13	-3.30398612E-11	5.08482293E-08	5.03831591E-44	3.52822400E-47
15	-2.15502800E-05	-1.40811899E-08	1.11414686E-49	-1.59755778E-46

m = 1, n = 1

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	9.96958064E-01	3.97905228E-02		
2	-2.47794508E-03	2.64796859E-02	-3.04539375E-01	-4.17052621E-01
4	-3.59600024E-04	-4.11221088E-05	-7.44641328E-02	4.23004000E-02
6	3.49979294E-07	-2.79108770E-06	-7.61191277E-04	-1.44311502E-03
8	1.39207539E-08	1.84085725E-09	1.46912873E-05	-7.47752390E-06
10	-6.61011431E-12	4.82839961E-11	4.52598920E-08	9.08104057E-08
12	-1.23113684E-13	-1.72704470E-14	-3.75936836E-10	1.84791806E-10
14	3.43402741E-17	-2.40405829E-16	-5.42306828E-13	-1.11413291E-12
16	3.70969584E-19	5.37359078E-20	2.47949855E-16	-1.19010483E-15
18	-6.79202364E-23	4.63711333E-22	2.06371960E-18	4.29817114E-18

m = 1, n = 3

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	2.34752535E-03	-6.86416534E-02		
2	1.00231019E+00	-1.92665336E-02	-7.59353945E-03	4.13732972E-04
4	3.60340739E-04	2.12006772E-02	-3.52366770E-05	-8.48790489E-04
6	-2.02134292E-04	3.19601520E-06	3.63208353E-05	-1.38640127E-06
8	-1.71587172E-08	-1.13811533E-06	1.78915309E-08	4.89043336E-07
10	4.27841622E-09	-6.23634666E-11	-3.51073137E-09	1.25206106E-10
12	1.64237367E-13	1.15546233E-11	-6.59954407E-13	-1.59701500E-11
14	-2.35586435E-14	3.28394619E-16	5.05135946E-14	-1.74975215E-16
16	-5.16999216E-18	-3.76976609E-17	4.04758352E-18	1.17918818E-18
18	4.82823761E-20	-6.54274227E-22	-2.11882028E-19	7.22148918E-21

m = 1, n = 5

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	-8.24783134E-04	-1.21093984E-05		
2	-1.96469630E-04	-3.30845079E-02	7.14404207E-07	3.66360603E-05
4	1.00058171E+00	-8.11022996E-03	-1.01699186E-06	1.48684410E-08
6	1.23368900E-04	1.61424113E-02	4.64105267E-10	3.49504207E-08
8	-1.18155152E-04	8.63110605E-07	-1.17587483E-09	1.48865567E-11
10	-3.74177341E-09	-5.22553335E-07	-1.48327052E-13	-1.20584052E-11
12	1.57837431E-09	-1.10790066E-11	6.71940966E-14	-8.10785103E-16
14	2.41511281E-14	3.49398818E-12	2.89078434E-18	2.42895777E-16
16	-5.93979537E-15	4.05586015E-17	-6.23717378E-19	7.34671908E-21
18	-5.42218707E-20	-8.02010696E-18	-1.40690360E-23	-1.20407833E-21

m = 1, n = 7

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	-3.14090569E-08	3.66423575E-06		
2	-3.04323762E-04	8.31350040E-07	9.19024123E-08	-9.17214085E-10
4	-8.70008570E-05	-2.20044932E-02	9.05756181E-12	1.22640148E-08
6	1.00029076E+00	-4.40287358E-03	1.16760919E-11	-7.79072671E-14
8	5.49044906E-05	1.29196534E-02	-2.14246999E-15	-3.37389181E-13
10	-7.66868209E-05	3.17962240E-07	8.07980780E-15	-5.59309780E-17
12	-1.13766866E-09	-2.79431023E-07	4.52415635E-19	7.49560460E-17
14	7.05974939E-10	-2.83424583E-12	-3.41402317E-19	2.03077756E-21
16	5.26044029E-15	1.32498215E-12	-6.02868486E-24	-1.02467856E-21
18	-1.93260516E-15	7.60366755E-18	2.21624472E-24	-1.29283704E-26

Table A2. (continued).

c = 1 - i1

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
m = 1, n = 9				
0	9.50001622E-09	4.89719422E-11		
2	1.91146464E-09	1.21919260E-06	-8.51347331E-13	-1.40733519E-10
4	-1.56911412E-04	3.63187624E-07	1.11691695E-12	-4.96864571E-15
6	-4.26499663E-05	-1.63920736E-02	-2.19480794E-17	-5.47034069E-15
8	1.00017730E+00	-2.75442805E-03	-4.60349773E-17	1.75326290E-19
10	2.88553792E-05	1.07420169E-02	4.06878464E-21	1.10252622E-18
12	-5.30068218E-05	1.41714251E-07	-2.45702112E-20	8.87060731E-23
14	-4.33357933E-10	-1.66135057E-07	-6.03533525E-25	-1.69557988E-22
16	3.50884951E-10	-9.31423051E-13	6.53097684E-25	-2.29844626E-27
18	1.50481637E-15	5.88097673E-13	5.84800328E-30	1.67645105E-27
m = 1, n = 13				
0	-1.45438701E-14	-3.76796463E-17		
2	-2.69475333E-15	-3.81778456E-12	3.10341044E-19	1.06547180E-16
4	1.07389046E-09	-1.14974922E-12	-4.03586941E-19	8.57433380E-22
6	3.44365963E-10	2.83404746E-07	1.63105867E-24	8.54147212E-22
8	-6.40782715E-05	8.26607236E-08	2.46504467E-24	-4.45991461E-27
10	-1.44135413E-05	-1.07856864E-02	-1.69043682E-29	-9.65308219E-27
12	1.00008659E+00	-1.36800176E-03	-5.97640754E-29	1.02394973E-31
14	1.08256230E-05	8.01547376E-03	1.76654904E-33	1.04737349E-30
16	-3.03295779E-05	4.05517260E-08	-1.72878562E-32	2.88167550E-35
18	-9.59963958E-11	-7.23901386E-08	-1.48213890E-37	-8.97382290E-35
m = 1, n = 17				
0	5.84758158E-21	9.16661660E-24		
2	1.03188372E-21	2.59246785E-18	-4.35305995E-26	-2.54798979E-23
4	-1.25760859E-15	7.75226512E-19	5.66219606E-26	-7.02354929E-29
6	-4.23287556E-16	-6.04325831E-13	-7.55345144E-32	-6.78783663E-29
8	2.71107783E-10	-2.01948105E-13	-1.03864614E-31	1.09414249E-34
10	8.27558381E-11	1.07137150E-07	1.94824588E-37	1.91239890E-34
12	-3.45635370E-05	2.73456489E-08	4.34990364E-37	-4.33361304E-40
14	-6.46061280E-06	-8.02855222E-03	-1.29501214E-42	-1.32086255E-39
16	1.00005130E+00	-8.15015543E-04	-6.35925122E-42	6.16013385E-45
18	5.16342853E-06	6.38646414E-03	8.38711026E-47	8.73997303E-44
m = 1, n = 2				
1	9.97001536E-01	6.11237113E-02	-1.64691146E-02	-1.29460551E-01
3	-1.67930519E-03	2.44195245E-02	6.50645028E-02	-7.97161035E-04
5	-2.74030477E-04	-1.98046803E-05	-1.36188850E-06	2.89460118E-03
7	1.32877952E-07	-1.78902652E-06	-4.59654100E-05	-2.46420379E-07
9	7.66934958E-09	5.79727086E-10	3.06707331E-09	-3.85813347E-07
11	-1.78230422E-12	2.32891559E-11	2.01762050E-09	1.92723160E-11
13	-5.27694570E-14	-4.07400471E-15	-7.68608397E-14	7.22457477E-12
15	7.20458086E-18	-9.26823391E-17	-1.88490994E-14	-2.15444550E-16
17	1.29921114E-19	1.01540711E-20	4.50421757E-19	-3.74468718E-17

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
m = 1, n = 11				
0	5.09494881E-14	-1.43150297E-11		
2	2.71391601E-09	-2.74725812E-12	-1.44592793E-13	5.87021931E-16
4	8.16873918E-10	5.37357270E-07	-2.27322596E-18	-7.65037489E-16
6	-9.54298056E-05	1.63711764E-07	-2.34191743E-18	6.26357311E-21
8	-2.36730699E-05	-1.30147498E-02	2.65619920E-23	1.05132961E-20
10	1.00011989E+00	-1.88318698E-03	7.54671842E-23	-1.85335691E-25
12	1.70020085E-05	9.18311924E-03	-3.67754948E-27	-1.53008938E-24
14	-3.95272561E-05	7.22384362E-08	2.90340183E-26	-6.87147462E-29
16	-1.92600773E-10	-1.06587982E-07	4.02486819E-31	1.72047511E-28
18	2.03111864E-10	-3.64337029E-13	-5.74121028E-31	1.33100135E-33
m = 1, n = 15				
0	-2.11273961E-20	1.06257790E-17		
2	-3.68804124E-15	1.91602933E-18	5.90316909E-20	-1.29362275E-22
4	-1.10740739E-15	-1.36744247E-12	2.68131316E-25	1.68181273E-22
6	5.07284918E-10	-4.59263203E-13	2.62099929E-25	-3.75183810E-28
8	1.61142324E-10	1.67494743E-07	-7.24954519E-31	-5.34840767E-28
10	-4.59651360E-05	4.58373049E-08	-1.37137541E-30	1.79850654E-33
12	-9.40302204E-06	-9.20595147E-03	6.03272105E-36	4.70289059E-33
14	1.00006551E+00	-1.03844359E-03	2.54988774E-35	-3.21943651E-38
16	7.30925976E-06	7.10543439E-03	-4.90278030E-40	-3.92980003E-37
18	-2.39584726E-05	2.44746428E-08	5.73680415E-39	-7.09089803E-42
m = 1, n = 19				
0	3.16203485E-27	-2.51038316E-24		
2	1.38385031E-21	-4.35250223E-25	-8.80833780E-27	1.20606521E-29
4	4.11990326E-22	8.40274225E-19	-1.55841252E-32	-1.57011311E-29
6	-5.12042930E-16	2.85675749E-19	-1.49534439E-32	1.33058168E-35
8	-1.76747792E-16	-2.97701855E-13	1.50486048E-38	1.78751106E-35
10	1.57951364E-10	-9.72767400E-14	2.50001081E-38	-2.03530664E-41
12	4.58311974E-11	7.26404230E-08	-3.28593993E-44	-4.12797299E-41
14	-2.69378842E-05	1.72898371E-08	-8.39386900E-44	6.57474791E-47
16	-4.62712301E-06	-7.11745870E-03	1.76605451E-49	2.28222809E-46
18	1.00004127E+00	-6.56614344E-04	9.87188008E-49	-7.56718249E-52
m = 1, n = 4				
1	9.71829510E-04	-4.53737014E-02	-1.81250860E-03	5.52867396E-05
3	1.00030842E+00	1.39892159E-02	1.27816473E-07	-2.01182815E-04
5	-2.74054764E-04	1.83620941E-02	-6.72940844E-06	-2.77767952E-08
7	-1.52098256E-04	-2.35279499E-06	1.39482053E-09	-2.56003058E-07
9	1.19216178E-08	-7.53358608E-07	2.98371992E-09	1.83476216E-11
11	2.52300001E-09	4.05568655E-11	-1.23253443E-13	1.87194619E-11
13	-9.98010073E-14	6.13762011E-12	-7.53962319E-14	-5.18736554E-16
15	-1.13783291E-14	-1.86647718E-16	1.51590780E-18	-2.13675258E-16
17	2.74903351E-19	-1.66429935E-17	4.51501850E-19	3.27646672E-21

Table A2. (continued).

$c = 1 - i1$

m = 1, n = 6

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-4.70979630E-04	-5.03387701E-06	1.21544862E-07	8.93866624E-06
3	1.92910291E-04	-2.67035459E-02	-4.48388592E-07	-4.44715521E-10
5	1.00027129E+00	6.23202376E-03	-1.03846738E-11	3.98345643E-09
7	-9.38439484E-05	1.43598330E-02	1.26668564E-10	4.09232787E-13
9	-9.41388752E-05	-6.30868493E-07	-1.35033249E-14	3.79414825E-12
11	2.56456017E-09	-3.76117488E-07	-3.47137507E-14	-1.30610758E-16
13	1.03479682E-09	7.14563632E-12	6.78744153E-19	-1.73995070E-16
15	-1.46551108E-14	2.10189016E-12	5.70601112E-19	2.28308494E-21
17	-3.29989930E-15	-2.31837838E-17	-5.46204478E-24	1.33967350E-21

m = 1, n = 8

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-1.30711032E-08	2.04570849E-06	2.27281732E-08	-1.73760926E-10
3	-2.12646036E-04	-9.32334042E-07	-5.03665922E-13	6.49359077E-10
5	7.22446294E-05	-1.88251308E-02	2.80595027E-12	4.92038310E-15
7	1.00018354E+00	3.58319992E-03	5.41912295E-17	-2.61671219E-14
9	-4.31632757E-05	1.17328511E-02	-6.86854399E-16	-1.55356918E-18
11	-6.35182835E-05	-2.38353641E-07	3.98701119E-20	-1.67544899E-17
13	8.09050187E-10	-2.13080903E-07	1.25965416E-19	3.09854366E-22
15	4.97735373E-10	1.90917099E-12	-1.32379424E-24	5.25753715E-22
17	-3.35716065E-15	8.60256048E-13	-1.45480792E-24	-3.72705190E-27

m = 1, n = 10

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	4.80153798E-09	2.03908023E-11	-1.71330086E-13	-3.49190559E-11
3	-2.31154520E-09	7.86466001E-07	6.46642495E-13	3.78568449E-16
5	-1.20502659E-04	-3.11195346E-07	2.10378196E-18	-1.75953799E-15
7	3.50591742E-05	-1.45106636E-02	-8.37839703E-18	-1.19839012E-20
9	1.00012787E+00	2.32002111E-03	-1.01180679E-22	6.50867580E-20
11	-2.34085263E-05	9.90237680E-03	1.43023581E-21	2.33352366E-24
13	-4.57697790E-05	-1.09697928E-07	-4.93736111E-26	2.93220974E-23
15	3.19689703E-10	-1.31991985E-07	-1.86973674E-25	-3.21913412E-28
17	2.67961817E-10	6.54404069E-13	1.17032043E-30	-6.68603754E-28

m = 1, n = 12

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	2.12198870E-14	-7.01225094E-12	-3.59596504E-14	1.22804809E-16
3	1.65664027E-09	3.48876835E-12	2.09985136E-19	-4.67006154E-16
5	-7.11465423E-10	3.83553757E-07	-8.64819572E-19	-7.60221217E-22
7	-7.74305861E-05	-1.34564480E-07	-2.73028080E-24	2.61466858E-21
9	1.97006235E-05	-1.17963891E-02	1.09703992E-23	1.24145762E-26
11	1.00009316E+00	1.62600629E-03	8.65546591E-29	-7.29951189E-26
13	-1.41149280E-05	8.56000447E-03	-1.37267791E-27	-1.67823389E-30
15	-3.44744745E-05	-5.74602472E-08	3.02771553E-32	-2.42364473E-29
17	1.46737228E-10	-8.72966522E-08	1.34127143E-31	1.70265039E-34

m = 1, n = 14

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-6.96543474E-15	-1.57787684E-17	6.65994002E-20	2.65347446E-17
3	3.54180136E-15	-2.23799123E-12	-2.55141100E-19	-9.02465533E-23
5	7.23825645E-10	1.01082698E-12	-2.31020871E-25	3.43623091E-22
7	-2.82010076E-10	2.15356858E-07	7.27240606E-25	5.78044149E-28
9	-5.39002467E-05	-6.78710314E-08	1.70343080E-30	-1.96138323E-27
11	1.21825409E-05	-9.93365836E-03	-7.25196567E-30	-6.52587211E-33
13	1.00007055E+00	1.20338953E-03	-3.88847162E-35	4.19306785E-32
15	-9.16704413E-06	7.53548567E-03	6.87907724E-34	6.51426565E-37
17	-2.68873749E-05	-3.29913287E-08	-1.02530413E-38	1.06586598E-35

m = 1, n = 16

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-8.60138376E-21	5.00300586E-18	1.47146431E-20	-2.83611552E-23
3	-2.09600768E-15	-2.50344422E-18	-3.10170476E-26	1.08997187E-22
5	9.79145712E-16	-8.99188756E-13	1.11791756E-25	5.92879144E-29
7	3.66232092E-10	3.74250470E-13	1.09964984E-31	-1.75980234E-28
9	-1.30596109E-10	1.32843065E-07	-3.41840523E-31	-2.30713566E-34
11	-3.96615096E-05	-3.79668886E-08	-5.82327593E-37	8.24949558E-34
13	8.06225654E-06	-8.57716425E-03	2.65822236E-36	1.93242577E-39
15	1.00005516E+00	9.26805544E-04	1.00641989E-41	-1.35596857E-38
17	-6.29034382E-06	6.72867539E-03	-1.97035581E-40	-1.48520938E-43

m = 1, n = 18

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	2.71685443E-21	3.81889516E-24	-9.69026774E-27	-6.35521676E-24
3	-1.41906713E-21	1.43776396E-18	3.73835915E-26	8.72780113E-30
5	-7.88135066E-16	-6.87904117E-19	1.29543714E-32	-3.02069256E-29
7	3.43141435E-16	-4.18697297E-13	-3.68850768E-32	-1.85887627E-35
9	2.05028800E-10	1.61706046E-13	-2.94234071E-38	5.41126666E-35
11	-6.73763385E-11	8.76631638E-08	9.43137104E-38	5.36049483E-41
13	-3.03979436E-05	-2.29329686E-08	1.18547632E-43	-2.02606180E-40
15	5.61339743E-06	-7.54567455E-03	-5.81384115E-43	-3.47225379E-46
17	1.00004424E+00	7.35852685E-04	-1.60676160E-48	2.64956598E-45

m = 1, n = 20

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	1.32571188E-27	-1.15382771E-24	-2.18785261E-27	2.71741866E-30
3	7.52416324E-22	6.03047297E-25	2.04046021E-33	-1.06166120E-28
5	-3.66437635E-22	5.12361304E-19	-6.07353244E-33	-2.43184027E-38
7	-3.42962183E-16	-2.30411169E-19	-2.78605731E-39	6.71406872E-36
9	1.39988376E-16	-2.16459243E-13	7.73925118E-39	3.46205048E-42
11	1.23666364E-10	7.79524706E-14	4.81429360E-45	-1.03005900E-41
13	-3.76547237E-11	6.08651770E-08	-1.61349676E-44	-7.76108250E-48
15	-2.40362245E-05	-1.46686616E-08	-1.52791953E-50	3.11261382E-47
17	4.06583432E-06	-6.73510301E-03	8.03873675E-50	4.00616018E-53

Table A2. (continued).

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary	r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
				c = 5 + i0					
m = 0, n = 0					m = 0, n = 2				
0	5.60317604E-01	0.			0	2.50650596E-01	0.		
2	-6.95612643E-01	0.	-1.32031393E+00	0.	2	8.90580219E-01	0.	3.33548756E-01	0.
4	2.15476299E-01	0.	6.76318914E-01	0.	4	-6.74979326E-01	0.	-1.21487542E+00	0.
6	-3.27212079E-02	0.	-1.42024497E-01	0.	6	1.51363615E-01	0.	4.40269599E-01	0.
8	2.53592772E-03	0.	1.63044509E-02	0.	8	-1.69846275E-02	0.	-6.84121066E-02	0.
10	-1.73179046E-04	0.	-1.17367364E-03	0.	10	1.15895218E-03	0.	5.86612299E-03	0.
12	7.20830668E-06	0.	5.77296530E-05	0.	12	-5.34460894E-05	0.	-3.26844160E-04	0.
14	-2.22702627E-07	0.	-2.05904865E-06	0.	14	1.78188807E-06	0.	1.27279922E-05	0.
16	5.30446898E-09	0.	5.56255025E-08	0.	16	-4.50101362E-08	0.	-3.67524265E-07	0.
m = 0, n = 4					m = 0, n = 6				
0	4.97464016E-02	0.			0	2.43528834E-03	0.		
2	3.81511563E-01	0.	8.92997781E-02	0.	2	3.41531866E-02	0.	1.66637692E-03	0.
4	1.01177439E+00	0.	6.18909410E-02	0.	4	2.49572193E-01	0.	6.00839805E-04	0.
6	-3.65299610E-01	0.	-6.50617286E-02	0.	6	1.01097639E+00	0.	3.05631411E-04	0.
8	5.51023505E-02	0.	1.40163947E-02	0.	8	-2.51347753E-01	0.	-1.77558832E-04	0.
10	-4.34503983E-03	0.	-1.44897130E-03	0.	10	2.60930164E-02	0.	2.66592625E-05	0.
12	2.20857535E-04	0.	9.04731093E-05	0.	12	-1.57480053E-03	0.	-2.06077729E-06	0.
14	-7.85945340E-06	0.	-3.82473300E-06	0.	14	6.34647567E-05	0.	1.00385424E-07	0.
16	2.10471905E-07	0.	1.17378927E-07	0.	16	-1.84722765E-06	0.	-3.41175226E-09	0.
m = 0, n = 8					m = 0, n = 10				
0	6.09546538E-05	0.			0	9.63154059E-07	0.		
2	1.39900112E-03	0.	2.27140212E-05	0.	2	3.30486176E-05	0.	2.28733132E-07	0.
4	1.90976597E-02	0.	3.64668259E-06	0.	4	7.15744127E-04	0.	2.34772766E-08	0.
6	1.87050251E-01	0.	7.61502518E-07	0.	6	1.22347165E-02	0.	2.34821666E-09	0.
8	1.00760006E+00	0.	3.00990633E-07	0.	8	1.50846091E-01	0.	3.29253142E-10	0.
10	-1.88515820E-01	0.	-1.24628510E-07	0.	10	1.00539294E+00	0.	1.05543421E-10	0.
12	1.54416776E-02	0.	1.44754048E-08	0.	12	-1.51248387E-01	0.	-3.42072434E-11	0.
14	-7.57055419E-04	0.	-8.96977754E-10	0.	14	1.02400758E-02	0.	3.24474125E-12	0.
16	2.53214337E-05	0.	3.59015680E-11	0.	16	-4.22830908E-04	0.	-1.67866376E-13	0.
m = 0, n = 12					m = 0, n = 14				
0	1.05161152E-08	0.			0	8.41634618E-11	0.		
2	5.05704154E-07	0.	1.73708635E-09	0.	2	5.41064069E-09	0.	1.02449713E-11	0.
4	1.58471924E-05	0.	1.21007309E-10	0.	4	2.30944792E-07	0.	5.18448593E-13	0.
6	4.10056370E-04	0.	7.64368078E-12	0.	6	8.35516557E-06	0.	2.28917178E-14	0.
8	8.49800560E-03	0.	5.64173469E-13	0.	8	2.55400130E-04	0.	1.09343137E-15	0.
10	1.26220173E-01	0.	6.16629122E-14	0.	10	6.24246552E-03	0.	6.45547220E-17	0.
12	1.00397048E+00	0.	1.65584956E-14	0.	12	1.06547702E-01	0.	5.79684438E-18	0.
14	-1.26426558E-01	0.	-4.42137425E-15	0.	14	1.00302715E+00	0.	1.33645945E-18	0.
16	7.29576907E-03	0.	3.54589831E-16	0.	16	-1.08663862E-01	0.	-3.03812099E-19	0.

Table A2. (continued).

c = 5 + i0

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
m = 0, n = 16				
0	5.15165140E-13	0.		
2	4.26944073E-11	0.	4.81608772E-14	0.
4	2.37852246E-09	0.	1.85490688E-15	0.
6	1.14177926E-07	0.	6.08844235E-17	0.
8	4.78012802E-06	0.	2.07150468E-18	0.
10	1.69423830E-04	0.	8.04453779E-20	0.
12	4.77820477E-03	0.	3.97114286E-21	0.
14	9.52367517E-02	0.	3.03016347E-22	0.
16	1.00237691E+00	0.	6.11284200E-23	0.
m = 0, n = 1				
1	3.32896487E-01	0.	1.50519048E-01	0.
3	-3.17561913E-01	0.	-6.25501655E-01	0.
5	8.69711951E-02	0.	2.85423702E-01	0.
7	-1.15678972E-02	0.	-5.25966269E-02	0.
9	9.14145593E-04	0.	5.29817674E-03	0.
11	-4.79504422E-05	0.	-3.37465694E-04	0.
13	1.79245830E-06	0.	1.48349324E-05	0.
15	-5.01870888E-08	0.	-4.77412817E-07	0.
m = 0, n = 5				
1	4.07778567E-02	0.	1.04760907E-02	0.
3	2.50801450E-01	0.	8.13496397E-03	0.
5	6.41976742E-01	0.	4.78448957E-03	0.
7	-2.52046901E-01	0.	-3.53690723E-03	0.
9	3.02645980E-02	0.	6.24114727E-04	0.
11	-2.06719292E-03	0.	-5.51329131E-05	0.
13	9.28719376E-05	0.	3.01486775E-06	0.
15	-2.97927976E-06	0.	-1.13537297E-07	0.
m = 0, n = 9				
1	4.60179482E-05	0.	4.09020075E-06	0.
3	9.20834588E-04	0.	8.20595552E-07	0.
5	1.41267538E-02	0.	1.01588516E-07	0.
7	1.56453265E-01	0.	1.66446647E-08	0.
9	9.41597016E-01	0.	5.89723512E-09	0.
11	-1.56995319E-01	0.	-2.14283578E-09	0.
13	1.16379692E-02	0.	2.23744858E-10	0.
15	-5.21700916E-04	0.	-1.26166639E-11	0.

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
m = 0, n = 18				
0	2.48953451E-15	0.		
2	2.58587945E-13	0.	1.84444581E-16	0.
4	1.82085464E-11	0.	5.59498856E-18	0.
6	1.11715296E-09	0.	1.42404131E-19	0.
8	6.10174542E-08	0.	3.65789911E-21	0.
10	2.92034010E-06	0.	1.02525383E-22	0.
12	1.17997840E-04	0.	3.36814741E-24	0.
14	3.77442762E-03	0.	1.43082664E-25	0.
16	8.48449413E-02	0.	9.49880467E-27	0.
m = 0, n = 3				
1	2.95678761E-01	0.	1.65909441E-01	0.
3	6.61440341E-01	0.	1.69357607E-01	0.
5	-3.48446056E-01	0.	-3.14224806E-01	0.
7	6.17274855E-02	0.	8.71865226E-02	0.
9	-5.76245046E-03	0.	-1.08774860E-02	0.
11	3.37427159E-04	0.	7.92058327E-04	0.
13	-1.36426121E-05	0.	-3.81815982E-05	0.
15	4.05123373E-07	0.	1.31462793E-06	0.
m = 0, n = 7				
1	1.85512762E-03	0.	2.60044946E-04	0.
3	2.24763188E-02	0.	8.91302394E-05	0.
5	1.32794711E-01	0.	2.25195473E-05	0.
7	9.09178260E-01	0.	1.00367215E-05	0.
9	-1.93953474E-01	0.	-4.84236984E-06	0.
11	1.77558058E-02	0.	6.33945530E-07	0.
13	-9.60533089E-04	0.	-4.36008042E-08	0.
15	3.51104306E-05	0.	1.91581147E-09	0.
m = 0, n = 11				
1	7.24892580E-07	0.	4.44509024E-08	0.
3	2.15950918E-05	0.	5.91667887E-09	0.
5	5.11223061E-04	0.	4.54560453E-10	0.
7	9.65863241E-03	0.	3.85067452E-11	0.
9	1.31253335E-01	0.	4.72748267E-12	0.
11	9.59464010E-01	0.	1.38216123E-12	0.
13	-1.31524519E-01	0.	-4.04590825E-13	0.
15	8.19481429E-03	0.	3.51629942E-14	0.

Table A2. (continued).

c = 5 + i0

m = 0, n = 13

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	7.90440421E-09	0.	3.54186213E-10	0.
3	3.27606476E-07	0.	3.36784588E-11	0.
5	1.10017385E-05	0.	1.78546433E-12	0.
7	3.10119660E-04	0.	9.63301920E-14	0.
9	7.00117424E-03	0.	6.34725137E-15	0.
11	1.12656201E-01	0.	6.25620214E-16	0.
13	9.70284794E-01	0.	1.55248811E-16	0.
15	-1.13004466E-01	0.	-3.81204478E-17	0.

m = 0, n = 17

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	3.86800552E-13	0.	1.03959987E-14	0.
3	2.72181461E-11	0.	5.78835698E-16	0.
5	1.58256179E-09	0.	1.73621944E-17	0.
7	8.10679917E-08	0.	4.98547021E-19	0.
9	3.63497683E-06	0.	1.53073215E-20	0.
11	1.37826509E-04	0.	5.44630777E-22	0.
13	4.14757283E-03	0.	2.48665672E-23	0.
15	8.79482179E-02	0.	1.76540773E-24	0.

m = 1, n = 1

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	7.45681954E-01	0.		
2	-1.43507038E-01	0.	-6.32483443E-01	0.
4	1.86632566E-02	0.	-5.24424136E-01	0.
6	-1.58078901E-03	0.	8.67874006E-02	0.
8	8.96879347E-05	0.	-9.04865923E-03	0.
10	-3.64137123E-06	0.	6.19203229E-04	0.
12	1.10822656E-07	0.	-2.95366349E-05	0.
14	-2.60268528E-09	0.	1.03227909E-06	0.
16	4.87671560E-11	0.	-2.74895099E-08	0.
18	-7.44322484E-13	0.	5.75382603E-10	0.

m = 1, n = 5

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	1.00942948E-01	0.		
2	4.19934905E-01	0.	9.52749530E-02	0.
4	1.00631154E+00	0.	-3.37368135E-02	0.
6	-2.11298707E-01	0.	-1.02446134E-02	0.
8	1.55438872E-02	0.	4.83890668E-03	0.
10	-1.00564901E-03	0.	-6.32007683E-04	0.
12	4.10969940E-05	0.	4.43309322E-05	0.
14	-1.13907455E-06	0.	-2.01011613E-06	0.
16	2.42321057E-08	0.	6.46528451E-08	0.
18	-4.09311292E-10	0.	-1.56221912E-09	0.

m = 0, n = 15

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	6.32223904E-11	0.	2.15902396E-12	0.
3	3.47547559E-09	0.	1.54228601E-13	0.
5	1.56640090E-07	0.	6.01623459E-15	0.
7	6.10373804E-06	0.	2.30440349E-16	0.
9	2.01367653E-04	0.	9.06265550E-18	0.
11	5.29969097E-03	0.	5.30141202E-19	0.
13	9.88687818E-02	0.	4.37319944E-20	0.
15	9.77310361E-01	0.	9.41154316E-21	0.

m = 0, n = 19

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	1.86872851E-15	0.	4.05457873E-17	0.
3	1.63757536E-13	0.	1.80999283E-18	0.
5	1.19212933E-11	0.	4.31214177E-20	0.
7	7.74538901E-10	0.	9.66912664E-22	0.
9	4.50164560E-08	0.	2.25482350E-23	0.
11	2.23212966E-06	0.	5.82049911E-25	0.
13	9.83458978E-05	0.	1.77684495E-26	0.
15	3.33248887E-03	0.	7.05875611E-28	0.

m = 1, n = 3

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	5.06084982E-01	0.		
2	9.68168337E-01	0.	-5.26386594E+00	0.
4	-2.57061956E-01	0.	3.88659165E+00	0.
6	3.03330740E-02	0.	-2.25416314E+00	0.
8	-2.11668650E-03	0.	3.76360914E-01	0.
10	9.88058668E-05	0.	-3.34533555E-02	0.
12	-3.31644400E-06	0.	1.88787271E-03	0.
14	8.42322823E-08	0.	-7.42075429E-05	0.
16	-1.67487276E-09	0.	2.15553375E-06	0.
18	2.68137331E-11	0.	-4.82388006E-08	0.

m = 1, n = 7

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	6.67657955E-03	0.		
2	4.80191645E-02	0.	2.54944583E-03	0.
4	2.82173135E-01	0.	-4.17022703E-04	0.
6	1.00840515E+00	0.	-5.18927914E-05	0.
8	-1.66570588E-01	0.	-1.55004582E-05	0.
10	1.24455684E-02	0.	5.56366877E-06	0.
12	-5.68730217E-04	0.	-5.80646076E-07	0.
14	1.79947629E-05	0.	3.32237136E-08	0.
16	-4.22668527E-07	0.	-1.24950715E-09	0.
18	7.71268905E-09	0.	3.38317289E-11	0.

Table A2. (continued).

	d_r^{mn}		d_{-r}^{mn} or $d_{\rho/r}^{mn}$			d_r^{mn}		d_{-r}^{mn} or $d_{\rho/r}^{mn}$	
r	Real	Imag-inary	Real	Imag-inary	r	Real	Imag-inary	Real	Imag-inary
				c = 5 + i0					
m = 1, n = 9					m = 1, n = 11				
0	2.15454030E-04	0.			0	4.15824361E-06	0.		
2	2.40306110E-03	0.	4.62737663E-05	0.	2	6.67400069E-05	0.	5.79601939E-07	0.
4	2.49435605E-02	0.	-4.51541822E-06	0.	4	1.08330626E-03	0.	-3.78663701E-08	0.
6	2.08328234E-01	0.	-2.77236604E-07	0.	6	1.51203647E-02	0.	-1.44709234E-09	0.
8	1.00614823E+00	0.	-3.01495358E-08	0.	8	1.64690407E-01	0.	-8.15227736E-11	0.
10	-1.37031117E-01	0.	-8.02457910E-09	0.	10	1.00451003E+00	0.	-7.49217094E-12	0.
12	8.58671279E-03	0.	2.32626504E-09	0.	12	-1.16426757E-01	0.	-1.75353920E-12	0.
14	-3.33360513E-04	0.	-2.02165337E-10	0.	14	6.28385700E-03	0.	4.27236579E-13	0.
16	9.06298916E-06	0.	9.76314710E-12	0.	16	-2.12129384E-04	0.	-3.18080215E-14	0.
18	-1.64761144E-07	0.	-3.13769728E-13	0.	18	5.05727672E-06	0.	1.32957870E-15	0.
m = 1, n = 13					m = 1, n = 15				
0	5.35503854E-08	0.			0	4.93344871E-10	0.		
2	1.17174702E-06	0.	5.26387264E-09	0.	2	1.41325703E-08	0.	3.61359935E-11	0.
4	2.64075607E-05	0.	-2.46950273E-10	0.	4	4.26747543E-07	0.	-1.27776331E-12	0.
6	5.59683176E-04	0.	-6.52199684E-12	0.	6	1.24944710E-05	0.	-2.48530156E-14	0.
8	1.01215740E-02	0.	-2.35392575E-13	0.	8	3.30088045E-04	0.	-6.33807308E-16	0.
10	1.36076269E-01	0.	-1.15603596E-14	0.	10	7.24394110E-03	0.	-2.03293177E-17	0.
12	1.00340533E+00	0.	-9.10753985E-16	0.	12	1.15996779E-01	0.	-8.73744935E-19	0.
14	-1.01239873E-01	0.	-1.88358240E-16	0.	14	1.00264701E+00	0.	-6.00348454E-20	0.
16	4.79942488E-03	0.	3.96271885E-17	0.	16	-8.95752072E-02	0.	-1.10762740E-20	0.
18	-1.43348098E-04	0.	-2.58077943E-18	0.	18	3.78616755E-03	0.	2.05068060E-21	0.
m = 1, n = 17					m = 1, n = 19				
0	3.41368189E-12	0.			0	1.83939919E-14	0.		
2	1.24071751E-10	0.	1.93827729E-13	0.	2	6.27325908E-13	0.	8.34135287E-16	0.
4	4.82765132E-09	0.	-5.35377908E-15	0.	4	4.02739517E-11	0.	-1.85009418E-17	0.
6	1.85743554E-07	0.	-8.01203171E-17	0.	6	1.96475106E-09	0.	-2.20002726E-19	0.
8	6.65786423E-06	0.	-1.53219420E-18	0.	8	9.14094137E-08	0.	-3.29609759E-21	0.
10	2.10786905E-04	0.	-3.52624483E-20	0.	10	3.82009055E-06	0.	-5.74462032E-23	0.
12	5.43874712E-03	0.	-1.00331299E-21	0.	12	1.42732920E-04	0.	-1.18594608E-24	0.
14	1.00935505E-01	0.	-3.81631629E-23	0.	14	4.23279547E-03	0.	-3.01631973E-26	0.
16	1.00211028E+00	0.	-2.32169377E-24	0.	16	8.93660152E-02	0.	-1.02691594E-27	0.
18	-8.03304349E-02	0.	-3.85629841E-25	0.	18	1.00171879E+00	0.	-5.60128015E-29	0.
m = 1, n = 2					m = 1, n = 4				
1	5.49906390E-01	0.	-5.30949181E-01	0.	1	3.92263676E-01	0.	5.28518805E-01	0.
3	-1.43518701E-01	0.	1.16808892E+00	0.	3	7.76354495E-01	0.	-5.69327698E-01	0.
5	1.65467710E-02	0.	-5.07042317E-01	0.	5	-1.85714068E-01	0.	-1.47052092E-01	0.
7	-1.43966789E-03	0.	8.99940752E-02	0.	7	1.94105464E-02	0.	8.05565916E-02	0.
9	7.46180200E-05	0.	-8.86378236E-03	0.	9	-1.20530486E-03	0.	-1.19488197E-02	0.
11	-2.76572810E-06	0.	5.56447294E-04	0.	11	5.05497709E-05	0.	9.42165239E-04	0.
13	7.69484857E-08	0.	-2.42174444E-05	0.	13	-1.53816423E-06	0.	-4.75357946E-05	0.
15	-1.66611799E-09	0.	7.73688522E-07	0.	15	3.55561863E-08	0.	1.68559888E-06	0.
17	2.88729099E-11	0.	-1.89149395E-08	0.	17	-6.52032063E-10	0.	-4.45423049E-08	0.

Table A2. (continued).

$c = 5 + i0$

m = 1, n = 6

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	6.21933344E-02	0.	2.09604208E-02	0.
3	2.97448726E-01	0.	-1.12267687E-02	0.
5	8.81600612E-01	0.	-1.30546683E-03	0.
7	-1.62979911E-01	0.	-4.03076081E-04	0.
9	1.34750257E-02	0.	1.64518376E-04	0.
11	-6.75738913E-04	0.	-1.90989387E-05	0.
13	2.32931642E-05	0.	1.20363484E-06	0.
15	-5.92251210E-07	0.	-4.94899337E-08	0.
17	1.16353318E-08	0.	1.45508358E-09	0.

m = 1, n = 10

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	1.10433353E-04	0.	1.13166560E-05	0.
3	1.47314603E-03	0.	-2.45385592E-06	0.
5	1.81068248E-02	0.	-8.30699689E-08	0.
7	1.74175551E-01	0.	-4.96398240E-09	0.
9	9.51630862E-01	0.	-4.95751562E-10	0.
11	-1.19170149E-01	0.	-1.23726027E-10	0.
13	6.91093184E-03	0.	3.27510923E-11	0.
15	-2.49569486E-04	0.	-2.62648416E-12	0.
17	6.33980528E-06	0.	1.17695426E-13	0.

m = 1, n = 14

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	2.57710538E-08	0.	1.30455684E-09	0.
3	6.68368400E-07	0.	-1.51409121E-10	0.
5	1.73283425E-05	0.	-2.54025771E-12	0.
7	4.12008224E-04	0.	-6.71539598E-14	0.
9	8.25856527E-03	0.	-2.28869361E-15	0.
11	1.21563197E-01	0.	-1.05017936E-16	0.
13	9.74120311E-01	0.	-7.71076392E-18	0.
15	-9.23129474E-02	0.	-1.50456970E-18	0.
17	4.12543496E-03	0.	2.96289091E-19	0.

m = 1, n = 18

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	1.58955366E-12	0.	4.82371427E-14	0.
3	6.76587983E-11	0.	-3.47087136E-15	0.
5	2.97089070E-09	0.	-3.49801484E-17	0.
7	1.26322296E-07	0.	-5.33512517E-19	0.
9	4.94924039E-06	0.	-9.78211205E-21	0.
11	1.69269379E-04	0.	-2.13159835E-22	0.
13	4.59368279E-03	0.	-5.72781689E-24	0.
15	9.31132280E-02	0.	-2.05844380E-25	0.
17	9.83962607E-01	0.	-1.18407010E-26	0.

m = 1, n = 8

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	3.63906655E-03	0.	6.10356840E-04	0.
3	3.11372166E-02	0.	-1.97792028E-04	0.
5	2.20959281E-01	0.	-1.09769955E-05	0.
7	9.27774241E-01	0.	-1.29353140E-06	0.
9	-1.38501737E-01	0.	-3.66223122E-07	0.
11	9.43987458E-03	0.	1.17409858E-07	0.
13	-3.95265334E-04	0.	-1.11351750E-08	0.
15	1.15820149E-05	0.	5.83164984E-10	0.
17	-2.52925589E-07	0.	-2.02105866E-11	0.

m = 1, n = 12

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	2.05296456E-06	0.	1.43002439E-07	0.
3	3.92788571E-05	0.	-2.21552674E-08	0.
5	7.29255241E-04	0.	-5.10695233E-10	0.
7	1.17792026E-02	0.	-1.93000216E-11	0.
9	1.43320894E-01	0.	-1.01589353E-12	0.
11	9.05444509E-01	0.	-8.62522760E-14	0.
13	-1.04150754E-01	0.	-1.89589624E-14	0.
15	5.25712895E-03	0.	4.27988895E-15	0.
17	-1.66615383E-04	0.	-2.97350576E-16	0.

m = 1, n = 16

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	2.33028874E-10	0.	8.97866555E-12	0.
3	7.80369015E-09	0.	-8.09102759E-13	0.
5	2.70306174E-07	0.	-1.03444432E-14	0.
7	8.81193906E-06	0.	-2.03397344E-16	0.
9	2.55709782E-04	0.	-4.93884745E-18	0.
11	6.10488968E-03	0.	-1.49095107E-19	0.
13	1.05488885E-01	0.	-6.01952045E-21	0.
15	9.79911105E-01	0.	-3.88511712E-22	0.
17	-8.28035368E-02	0.	-6.79345540E-23	0.

m = 1, n = 20

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	6.46865352E-15	0.	2.07807626E-16	0.
3	4.43689591E-13	0.	-1.22109391E-17	0.
5	2.41964370E-11	0.	-9.95759668E-20	0.
7	1.29626206E-09	0.	-1.21507354E-21	0.
9	6.54606964E-08	0.	-1.75038647E-23	0.
11	2.99188153E-06	0.	-2.91215519E-25	0.
13	1.17783428E-04	0.	-5.70339501E-27	0.
15	3.71970375E-03	0.	-1.37612787E-28	0.
17	8.33109221E-02	0.	-4.44952209E-30	0.

Table A2. (continued).

c = 0 + i5

m = 0, n = 0

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	7.60844027E+00	0.		
2	1.63304698E+01	0.	5.62384678E+03	0.
4	6.97270465E+00	0.	3.99439918E+03	0.
6	1.17440267E+00	0.	9.66416699E+02	0.
8	1.12139639E-01	0.	1.20067480E+02	0.
10	6.89433448E-03	0.	9.08645024E+00	0.
12	2.95554739E-04	0.	4.62818880E-01	0.
14	9.33482894E-06	0.	1.69315165E-02	0.
16	2.26176988E-07	0.	4.66339616E-04	0.

m = 0, n = 4

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	5.24972465E-02	0.		
2	-2.67164723E-01	0.	-7.01447516E-02	0.
4	6.98539916E-01	0.	4.77931039E-02	0.
6	2.64446529E-01	0.	4.98567350E-02	0.
8	3.70338044E-02	0.	1.06920500E-02	0.
10	2.96447156E-03	0.	1.10212507E-03	0.
12	1.50429803E-04	0.	6.86796568E-05	0.
14	5.37369410E-06	0.	2.89921925E-06	0.
16	1.43036748E-07	0.	8.88775092E-08	0.

m = 0, n = 8

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	6.12022276E-05	0.		
2	-1.25163964E-03	0.	-2.12772652E-05	0.
4	1.73517363E-02	0.	3.57497256E-06	0.
6	-1.71309601E-01	0.	-7.05491544E-07	0.
6	9.21874612E-01	0.	2.78542290E-07	0.
10	1.72324220E-01	0.	1.15197072E-07	0.
12	1.41071844E-02	0.	1.33704244E-08	0.
14	6.91319168E-04	0.	8.28088145E-10	0.
16	2.31150038E-05	0.	3.31316834E-11	0.

m = 0, n = 12

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	1.05332484E-08	0.		
2	-4.80127272E-07	0.	-1.68467909E-09	0.
4	1.51579372E-05	0.	1.16937931E-10	0.
6	-3.93048884E-04	0.	-7.37644187E-12	0.
8	8.15354012E-03	0.	5.44049744E-13	0.
10	-1.21172704E-01	0.	-5.94358217E-14	0.
12	9.64182111E-01	0.	1.59562098E-14	0.
14	1.21392071E-01	0.	4.25954334E-15	0.
16	7.00418631E-03	0.	3.41552549E-16	0.

m = 0, n = 2

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	-3.73689117E-01	0.		
2	6.59718364E-01	0.	-1.08134190E-01	0.
4	6.64162537E-01	0.	2.44607887E+00	0.
6	1.61101751E-01	0.	1.02332382E+00	0.
8	1.86037340E-02	0.	1.65512789E-01	0.
10	1.31444596E-03	0.	1.46790923E-02	0.
12	6.16245000E-05	0.	8.31343373E-04	0.
14	2.07921762E-06	0.	3.28286271E-05	0.
16	5.30017370E-08	0.	9.57842478E-07	0.

m = 0, n = 6

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	-2.46742200E-03	0.		
2	2.83138271E-02	0.	1.49022558E-03	0.
4	-2.11434526E-01	0.	-5.29733350E-04	0.
6	8.63974669E-01	0.	2.63830405E-04	0.
8	2.14227000E-01	0.	1.55531302E-04	0.
10	2.22039321E-02	0.	2.33050633E-05	0.
12	1.33661494E-03	0.	1.79914036E-06	0.
14	5.39031477E-05	0.	8.75588754E-08	0.
16	1.56795922E-06	0.	2.97374167E-09	0.

m = 0, n = 10

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	-9.65830591E-07	0.		
2	3.07093540E-05	0.	2.19078730E-07	0.
4	-6.72135046E-04	0.	-2.23718022E-08	0.
6	1.15240175E-02	0.	2.23323435E-09	0.
8	-1.42265179E-01	0.	-3.12794613E-10	0.
10	9.49098965E-01	0.	1.00215667E-10	0.
12	1.42724432E-01	0.	3.24647783E-11	0.
14	9.66026415E-03	0.	3.07845502E-12	0.
16	3.98803786E-04	0.	1.59224696E-13	0.

m = 0, n = 14

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	-8.42568201E-11	0.		
2	5.20465750E-09	0.	1.00132759E-11	0.
4	-2.23418237E-07	0.	-5.05378773E-13	0.
6	8.09547570E-06	0.	2.22918253E-14	0.
8	-2.47641578E-04	0.	-1.06420065E-15	0.
10	6.05537667E-03	0.	6.28078579E-17	0.
12	-1.05323389E-01	0.	-5.63866716E-18	0.
14	9.73422095E-01	0.	1.29979869E-18	0.
16	1.05444721E-01	0.	2.95439525E-19	0.

Table A2. (continued).

c = 0 + i5

m = 0, n = 16

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	5.15526800E-13	0.		
2	-4.14319945E-11	0.	-4.73140667E-14	0.
4	2.31629560E-09	0.	1.81856163E-15	0.
6	-1.11420424E-07	0.	-5.96444120E-17	0.
8	4.66729402E-06	0.	2.02846816E-18	0.
10	-1.65478197E-04	0.	-7.87537205E-20	0.
12	4.66791345E-03	0.	3.88635017E-21	0.
14	-9.30523263E-02	0.	-2.96553646E-22	0.
16	9.79494210E-01	0.	5.98193538E-23	0.

m = 0, n = 1

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	4.70723837E+00	0.	-6.30339561E+02	0.
3	3.35043791E+00	0.	-1.47108222E+03	0.
5	6.11330983E-01	0.	-5.60309135E+02	0.
7	1.00848727E-01	0.	-9.44326110E+01	0.
9	7.63604536E-03	0.	-9.02048640E+00	0.
11	3.88939199E-04	0.	-5.54719041E-01	0.
13	1.42309411E-05	0.	-2.37846487E-02	0.
15	3.92004609E-07	0.	-7.51317444E-04	0.

m = 0, n = 5

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	4.19778294E-02	0.	-1.46149202E-02	0.
3	-3.04467961E-01	0.	1.00840829E-02	0.
5	1.05399393E+00	0.	-6.40018970E-03	0.
7	3.13425366E-01	0.	-4.67322176E-03	0.
9	3.75033812E-02	0.	-8.20831821E-04	0.
11	2.55587527E-03	0.	-7.23107361E-05	0.
13	1.14644393E-04	0.	-3.94680679E-06	0.
15	3.67336170E-06	0.	-1.48430370E-07	0.

m = 0, n = 9

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	4.61923051E-05	0.	-4.50950205E-06	0.
3	-9.77449144E-04	0.	8.92665397E-07	0.
5	1.50943243E-02	0.	-1.10104850E-07	0.
7	-1.67578459E-01	0.	1.80080943E-08	0.
9	1.00926796E+00	0.	-6.37551832E-09	0.
11	1.68262401E-01	0.	-2.31492485E-09	0.
13	1.24696844E-02	0.	-2.41599613E-10	0.
15	5.58816741E-04	0.	-1.36187749E-11	0.

m = 0, n = 18

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	-2.49077183E-15	0.		
2	2.52475638E-13	0.	1.81851416E-16	0.
4	-1.78400993E-11	0.	-5.50735053E-18	0.
6	1.09559092E-09	0.	1.40085781E-19	0.
8	-5.98653631E-08	0.	-3.59715500E-21	0.
10	2.86598531E-06	0.	1.00802115E-22	0.
12	-1.15821040E-04	0.	-3.31107225E-24	0.
14	3.70523534E-03	0.	1.40643793E-25	0.
16	-8.32368803E-02	0.	-9.33616994E-27	0.

m = 0, n = 3

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-4.53649736E-01	0.	8.49619932E-01	0.
3	1.30252039E+00	0.	-1.00545567E+00	0.
5	6.68039446E-01	0.	-1.46267574E+00	0.
7	1.14517052E-01	0.	-3.86158030E-01	0.
9	1.04879366E-02	0.	-4.69975262E-02	0.
11	6.06502956E-04	0.	-3.37032460E-03	0.
13	2.43053647E-05	0.	-1.60780624E-04	0.
15	7.16992157E-07	0.	-5.49380561E-06	0.

m = 0, n = 7

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-1.87008909E-03	0.	3.05435119E-04	0.
3	2.47879166E-02	0.	-1.02413064E-04	0.
5	-2.15119063E-01	0.	2.57158984E-05	0.
7	1.01871063E+00	0.	-1.14423759E-05	0.
9	2.17002736E-01	0.	-5.50906613E-06	0.
11	1.99473098E-02	0.	-7.20401215E-07	0.
13	1.07296215E-03	0.	-4.95081940E-08	0.
15	3.92004331E-05	0.	-2.17413250E-09	0.

m = 0, n = 11

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-7.26416049E-07	0.	4.74705852E-08	0.
3	2.24796412E-05	0.	-6.26143617E-09	0.
5	-5.34483139E-04	0.	4.79854534E-10	0.
7	1.01146872E-02	0.	-4.06043173E-11	0.
9	-1.37570278E-01	0.	4.98130126E-12	0.
11	1.00610621E+00	0.	-1.45583575E-12	0.
13	1.37891619E-01	0.	-4.26015180E-13	0.
15	8.58978295E-03	0.	-3.70163491E-14	0.

Table A2. (continued).

c = 0 + i5

m = 0, n = 13

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	7.91459421E-09	0.	-3.71347336E-10	0.
3	-3.37198623E-07	0.	3.50788360E-11	0.
5	1.13590840E-05	0.	-1.85637194E-12	0.
7	-3.20571933E-04	0.	1.00591366E-13	0.
9	7.24167700E-03	0.	-6.59058987E-15	0.
11	-1.16778332E-01	0.	6.49394612E-16	0.
13	1.00427210E+00	0.	-1.61116770E-16	0.
15	1.16945530E-01	0.	-3.95544044E-17	0.

m = 0, n = 17

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	3.87027838E-13	0.	-1.06901075E-14	0.
3	-2.76838266E-11	0.	5.92893623E-16	0.
5	1.61253355E-09	0.	-1.77649479E-17	0.
7	-8.26638079E-08	0.	5.09858601E-19	0.
9	3.70790647E-06	0.	-1.56501291E-20	0.
11	-1.40624058E-04	0.	5.56723300E-22	0.
13	4.23241980E-03	0.	-2.54153205E-23	0.
15	-8.97575377E-02	0.	1.80418703E-24	0.

m = 1, n = 1

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	2.61261584E+00	0.		
2	1.36960603E+00	0.	-2.48726585E+00	0.
4	2.65426414E-01	0.	4.84004140E+01	0.
6	2.74512097E-02	0.	1.43365949E+01	0.
8	1.77738158E-03	0.	1.97556500E+00	0.
10	7.90156589E-05	0.	1.59345240E-01	0.
12	2.56377853E-06	0.	8.47132905E-03	0.
14	6.34221104E-08	0.	3.19705037E-04	0.
16	1.23613162E-09	0.	9.01537310E-06	0.
18	1.94761360E-11	0.	1.97248755E-07	0.

m = 1, n = 5

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	1.46413149E-01	0.		
2	-3.66216401E-01	0.	-5.61753586E-02	0.
4	8.10831634E-01	0.	-2.25822379E-02	0.
6	1.73012722E-01	0.	6.69625072E-03	0.
8	1.61587701E-02	0.	3.36381202E-03	0.
10	9.03297849E-04	0.	4.44977305E-04	0.
12	3.43487302E-05	0.	3.14540072E-05	0.
14	9.55212673E-07	0.	1.43378219E-06	0.
16	2.03739224E-08	0.	4.62941285E-08	0.
18	3.44863308E-10	0.	1.12192051E-09	0.

m = 0, n = 15

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-6.32760094E-11	0.	2.23749383E-12	0.
3	3.55185877E-09	0.	-1.59040221E-13	0.
5	-1.60463765E-07	0.	6.19550875E-15	0.
7	6.25809434E-06	0.	-2.37156701E-16	0.
9	-2.06552043E-04	0.	1.01463826E-17	0.
11	5.43768355E-03	0.	-5.45261271E-19	0.
13	-1.01487625E-01	0.	4.49715406E-20	0.
15	1.00314026E+00	0.	-9.67720106E-21	0.

m = 0, n = 19

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-1.86952097E-15	0.	4.14643633E-17	0.
3	1.66001263E-13	0.	-1.84519611E-18	0.
5	-1.21023731E-11	0.	4.39224742E-20	0.
7	7.86747391E-10	0.	-9.64401241E-22	0.
9	-4.57396665E-08	0.	2.29526193E-23	0.
11	2.32936428E-06	0.	-5.92399079E-25	0.
13	-9.99569008E-05	0.	1.80824607E-26	0.
15	3.38738662E-03	0.	-7.18293405E-28	0.

m = 1, n = 3

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	-9.13655456E-01	0.		
2	6.12949678E-01	0.	5.48734754E-01	0.
4	2.06447742E-01	0.	2.79631884E-01	0.
6	2.67971157E-02	0.	3.13164767E-01	0.
8	1.97414250E-03	0.	6.01295376E-02	0.
10	9.54840748E-05	0.	5.73305448E-03	0.
12	3.28872207E-06	0.	3.37886310E-04	0.
14	8.50668560E-08	0.	1.36807804E-05	0.
16	1.71644643E-09	0.	4.06089819E-07	0.
18	2.78030223E-11	0.	9.23886030E-09	0.

m = 1, n = 7

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
0	-8.19559133E-03	0.		
2	4.49330734E-02	0.	1.97961400E-03	0.
4	-2.55422227E-01	0.	3.45765608E-04	0.
6	9.01569192E-01	0.	-4.38334533E-05	0.
8	1.49571174E-01	0.	1.31602679E-05	0.
10	1.12066742E-02	0.	4.75282341E-06	0.
12	5.13135443E-04	0.	4.97890932E-07	0.
14	1.62600300E-05	0.	2.85453968E-08	0.
16	3.82368519E-07	0.	1.07540381E-09	0.
18	6.98385308E-09	0.	2.91560430E-11	0.

Table A2. (continued).

$c = 0 + i5$

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
m = 1, n = 9				
0	2.45201408E-04	0.		
2	-2.31163087E-03	0.	-3.97483218E-05	0.
4	2.35363501E-02	0.	-4.03770744E-06	0.
6	-1.95142437E-01	0.	2.50700184E-07	0.
8	9.32794916E-01	0.	-2.74038349E-08	0.
10	1.28076360E-01	0.	7.31034494E-09	0.
12	8.03521170E-03	0.	2.12387064E-09	0.
14	3.12230655E-04	0.	1.84837117E-10	0.
16	8.49450585E-06	0.	8.93554143E-12	0.
18	1.73269647E-07	0.	2.87399610E-13	0.
m = 1, n = 13				
0	5.71565705E-08	0.		
2	-1.15132266E-06	0.	-4.89376589E-09	0.
4	2.57077409E-05	0.	-2.34160705E-10	0.
6	-5.42920346E-04	0.	6.21759178E-12	0.
8	9.79399628E-03	0.	-2.24970710E-13	0.
10	-1.31599609E-01	0.	1.10646698E-14	0.
12	9.69622495E-01	0.	-8.72549542E-16	0.
14	9.76746582E-02	0.	1.80602019E-16	0.
16	4.64148259E-03	0.	3.80071822E-17	0.
18	1.33668829E-04	0.	2.47623293E-18	0.
m = 1, n = 17				
0	3.55016256E-12	0.		
2	-1.22844885E-10	0.	-1.85720433E-13	0.
4	4.75381009E-09	0.	-5.19023398E-15	0.
6	-1.82518439E-07	0.	7.79211835E-17	0.
8	6.54431618E-06	0.	-1.49236057E-18	0.
10	-2.00753942E-04	0.	3.43753490E-20	0.
12	5.33218670E-03	0.	-9.78623393E-22	0.
14	-9.60243264E-02	0.	3.72388196E-23	0.
15	9.81889164E-01	0.	-2.26613684E-24	0.
18	7.87223938E-02	0.	3.76481489E-25	0.
m = 1, n = 2				
1	2.47540059E+00	0.	-1.36505936E+00	0.
3	7.52787952E-01	0.	-5.26556959E+01	0.
5	1.04787362E-01	0.	-2.93977415E+01	0.
7	8.49634533E-03	0.	-5.76458127E+00	0.
9	4.53400400E-04	0.	-6.02621716E-01	0.
11	1.71527750E-05	0.	-3.91872494E-02	0.
13	4.84556652E-07	0.	-1.74715809E-03	0.
15	1.06162351E-08	0.	-5.68065802E-05	0.
17	1.85709052E-10	0.	-1.40745813E-06	0.
m = 1, n = 11				
0	-4.54586351E-06	0.		
2	6.50825693E-05	0.	5.23542393E-07	0.
4	-1.02360039E-03	0.	3.51490752E-08	0.
6	1.44843051E-02	0.	-1.35328578E-09	0.
6	-1.57351675E-01	0.	7.65045213E-11	0.
10	9.58189630E-01	0.	-7.04556164E-12	0.
12	1.11148929E-01	0.	1.65082944E-12	0.
14	6.03264826E-03	0.	4.02613519E-13	0.
16	2.02732486E-04	0.	2.99957939E-14	0.
18	4.83509016E-06	0.	1.25449793E-15	0.
m = 1, n = 15				
0	-5.18501199E-10	0.		
2	1.39505813E-08	0.	3.42083318E-11	0.
4	-4.18313777E-07	0.	1.22700214E-12	0.
6	1.22147045E-05	0.	-2.39787448E-14	0.
8	-3.22238711E-04	0.	6.12675558E-16	0.
10	7.06546411E-03	0.	-1.96731824E-17	0.
12	-1.12983369E-01	0.	8.46153773E-19	0.
14	9.76632329E-01	0.	-5.81690287E-20	0.
16	8.73006844E-02	0.	1.07357133E-20	0.
18	3.69079640E-03	0.	1.98822568E-21	0.
m = 1, n = 19				
0	-1.89663354E-14	0.		
2	8.20851654E-13	0.	8.06058509E-16	0.
4	-3.97832467E-11	0.	1.60473917E-17	0.
6	1.93753186E-09	0.	-2.15161403E-19	0.
8	-9.00623666E-08	0.	3.21763026E-21	0.
10	3.82079340E-06	0.	-5.62885184E-23	0.
12	-1.40498643E-04	0.	1.16257261E-24	0.
14	4.16539087E-03	0.	-2.95781473E-26	0.
16	-8.79445801E-02	0.	1.00723631E-27	0.
18	9.85404703E-01	0.	-5.49495376E-29	0.
m = 1, n = 4				
1	-6.84307771E-01	0.	2.03089063E-01	0.
3	1.15618237E+00	0.	5.36020354E-01	0.
5	2.87938344E-01	0.	-1.39009455E-01	0.
7	3.06998231E-02	0.	-8.24194075E-02	0.
9	1.93086607E-03	0.	-1.25598014E-02	0.
11	8.16391938E-05	0.	-1.00574564E-03	0.
13	2.49976458E-06	0.	-5.12628180E-05	0.
15	5.82254022E-08	0.	-1.83093969E-06	0.
17	1.06879191E-09	0.	-4.86489702E-08	0.

Table A2. (continued)

c = 0 + i5

m = 1, n = 6

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	6.15650278E-02	0.	-1.47866373E-02	0.
3	-3.58423241E-01	0.	-1.15084508E-02	0.
5	1.03480504E+00	0.	1.39642627E-03	0.
7	1.92818943E-01	0.	-4.34420804E-04	0.
9	1.60178974E-02	0.	-1.79476614E-04	0.
11	8.05878813E-04	0.	-2.09600271E-05	0.
13	2.78455539E-05	0.	-1.32607133E-06	0.
15	7.09349640E-07	0.	-5.46755501E-08	0.
17	1.39551403E-08	0.	-1.61090205E-09	0.

m = 1, n = 8

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-4.27310874E-03	0.	5.03364445E-04	0.
3	3.47642164E-02	0.	2.01655989E-04	0.
5	-2.43329344E-01	0.	-1.14597243E-05	0.
7	1.01516073E+00	0.	1.36319949E-06	0.
9	1.51946712E-01	0.	-3.67254206E-07	0.
11	1.03746957E-02	0.	-1.24590830E-07	0.
13	4.36075974E-04	0.	-1.18425077E-08	0.
15	1.27649816E-05	0.	-6.21092841E-10	0.
17	2.78835953E-07	0.	-2.15495570E-11	0.

m = 1, n = 10

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	1.22834232E-04	0.	-1.00062197E-05	0.
3	-1.58597502E-03	0.	-2.49001878E-06	0.
5	1.93183487E-02	0.	8.55948183E-08	0.
7	-1.85049276E-01	0.	-5.14521774E-09	0.
9	1.00856081E+00	0.	5.15509800E-10	0.
11	1.26445899E-01	0.	-1.28858158E-10	0.
13	7.33903343E-03	0.	-3.41590176E-11	0.
15	2.65199941E-04	0.	-2.74207215E-12	0.
17	6.74030437E-06	0.	-1.22965021E-13	0.

m = 1, n = 12

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-2.21450725E-06	0.	1.31282343E-07	0.
3	4.14069280E-05	0.	2.24057176E-08	0.
5	-7.63927075E-04	0.	-5.22051551E-10	0.
7	1.23029826E-02	0.	1.98110423E-11	0.
9	-1.49438878E-01	0.	-1.04509321E-12	0.
11	1.00553050E+00	0.	8.88546366E-14	0.
13	1.08539669E-01	0.	-1.95465901E-14	0.
15	5.48114040E-03	0.	-4.41568315E-15	0.
17	1.73777132E-04	0.	-3.06942934E-16	0.

m = 1, n = 14

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	2.72736985E-08	0.	-1.22503411E-09	0.
3	-6.95374742E-07	0.	-1.52754039E-10	0.
5	1.79445107E-05	0.	2.58333304E-12	0.
7	-4.26350397E-04	0.	-6.65055482E-14	0.
9	8.52288907E-03	0.	2.33844969E-15	0.
11	-1.25371043E-01	0.	-1.07409301E-16	0.
13	1.00388356E+00	0.	7.89185671E-18	0.
15	9.51655461E-02	0.	-1.54059278E-18	0.
17	4.25404308E-03	0.	-3.03501910E-19	0.

m = 1, n = 16

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-2.43510689E-10	0.	8.55608050E-12	0.
3	8.10994882E-09	0.	8.14879843E-13	0.
5	-2.77769844E-07	0.	-1.04825450E-14	0.
7	9.04006981E-06	0.	2.06600749E-16	0.
9	-2.62075066E-04	0.	-5.02291778E-18	0.
11	6.25289225E-03	0.	1.51750703E-19	0.
13	-1.07998063E-01	0.	-6.12997596E-21	0.
15	1.00288579E+00	0.	3.95794475E-22	0.
17	8.47623409E-02	0.	-6.92265044E-23	0.

m = 1, n = 18

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	1.64641972E-12	0.	-4.64324344E-14	0.
3	-6.93458016E-11	0.	-3.49119022E-15	0.
5	3.03645240E-09	0.	3.53572392E-17	0.
7	-1.28337967E-07	0.	-5.40278165E-19	0.
9	5.04782344E-06	0.	9.91602389E-21	0.
11	-1.72554143E-04	0.	-2.16211036E-22	0.
13	4.78307120E-03	0.	5.81224125E-24	0.
15	-9.48615544E-02	0.	-2.08941910E-25	0.
17	1.00223367E+00	0.	1.20216820E-26	0.

m = 1, n = 20

r	d_r^{mn} Real	d_r^{mn} Imaginary	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Real	d_{-r}^{mn} or $d_{\rho/r}^{mn}$ Imaginary
1	-8.71545812E-15	0.	2.01481590E-16	0.
3	4.52747339E-13	0.	1.22705501E-17	0.
5	-2.46330565E-11	0.	-1.00460721E-19	0.
7	1.31822203E-09	0.	1.22774596E-21	0.
9	-6.65278705E-08	0.	-1.77007452E-23	0.
11	3.03941157E-06	0.	2.94630724E-25	0.
13	-1.15620090E-04	0.	-5.77240544E-27	0.
15	3.77609311E-03	0.	1.39312290E-28	0.
17	-8.45779704E-02	0.	-4.50531733E-30	0.