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Complex Resonance Identification From Data At Multiple Spatial Locations

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Abstract

Single input, single output system identification techniques are extended to the single input, multiple output case. The extension makes possible the investigation of complex resonances of scatterers from output data at multiple spatial locations. An example is included involving acoustic scattering from a hard sphere. A model is proposed using a state-space formalism that takes into account the constraint that pole position is invariant with observation point.

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I. INTRODUCTION

Identification of a system involves the extraction of information from input-output data obtained in an experiment. The techniques are rooted in early studies in economics (Ref. 1) and have matured in modern control theory (Ref. 2). Parametric identification is a subclass of system identification where the information concerning the system is obtained in the form of a set of parameters. Such a set, for example, could be the poles and zeros describing the system in the frequency domain.

Identification of an electromagnetic system is a concept that has only recently begun to surface in the electromagnetic journals. The subject received its impetus from two investigations. The first was the research of Marin (Ref. 3) who, although not concerned with system identification, showed that the electromagnetic scattering from a large class of conducting scatterers is a meromorphic function of frequency. The second was the modal study by Baum (Ref. 4), who formulated the Singularity Expansion Method (SEM) for the formal study of body resonances. These two papers gave rise to the resurrection of Prony's method (Refs. 5,6), a method for fitting data with a complex exponential series. Subsequently, Dudley (Ref. 7) showed that Prony's method is a special case of pole-zero parametric identification. In addition, he showed that there are serious bias problems in the identification of parameters caused by the ill-posed nature of the problem.

Recently, Dudley (Ref. 8) has shown that the electromagnetic modeling problem can be fitted into the larger framework of identification proposed by Ljung (Ref. 9). Ljung uses a generalized linear model with generalized error norm minimizations. Subsequently, Dudley and Goodman (Ref. 10) have reported results using a nonlinear least squares algorithm (NLS) constructed by Goodman (Ref. 11) and based on the output error model (Ref. 8).

To date, emphasis in electromagnetic identification has been on a single aspect angle, primarily backscatter. Baum (Ref. 4), however, has shown that the complex resonances of a body are aspect angle independent. It has been conjectured that identification using multiple aspect angles can improve the ability to identify body resonances in the presence of noise.

In this paper, we consider multiple aspect angle identification. We begin with a review of the difference equation model for single-input, single-output (SISO) systems and show its extension to the single-input, multiple output (SIMO) case. We next use the NLS algorithm to study the SIMO case for a canonical structure. For the structure, we choose scalar scattering (acoustic case) from a sphere with Neumann boundary conditions. Finally, we develop a state-space formulation for electromagnetic scattering, incorporating the aspect-independent characteristics of the poles. For future work, we propose critical testing of the state-space formulation with both synthetic data and data obtained from a transient electromagnetic range.

II. THE DIFFERENCE EQUATION MODEL

Consider a SISO system with data sampled at a uniform sample rate T. Such a system can be modeled (Ref. 8) by the difference equation

$$A\tilde{y}(t) = Bu(t) \tag{1}$$

where $\tilde{y}(t)$ and $u(t)$ are the output and input data, respectively, from the model at the t^{th} time step and where A and B are time-stepping polynomials with, typically,

$$A(q^{-1}) = \sum_{n=1}^{N} a_n q^{-n} \tag{2}$$

where

$$q^{-1}y(t) = y(t-1) \tag{3}$$

For convenience, the time steps have been normalized to unity. The utility of the model described by Eq. 1 is that the delta function response of the model h(t) is the complex exponential series (Ref. 8)

$$h(t) = \sum_{n=1}^{N} R_n e^{s_n t} \tag{4}$$

where s_n and R_n are, respectively, the poles and residues of the model. In Eq. 1, if we replace the output from the model $\tilde{y}(t)$ with the output from the electromagnetic process $y(t)$, we obtain

$$Ay(t) = Bu(t) + v(t) \tag{5}$$

where $v(t)$ is an error term arising from noise in the process output and from any inadequacy in the model.

The formulation in Eq. 5, called "equation error model," is useful in obtaining initial values of the parameters in A and B. Indeed, a least-squares minimization of the error v(t) leads to the so-called "normal equations" of least squares, the solution of which follows standard procedures (Ref. 7). The algorithm NLS (Ref. 11) uses the values of the parameters so obtained as initial values for identification in the output error model

$$y(t) = \frac{B}{A} u(t) + e(t) \tag{6}$$

where e(t) is the output error. The minimization of the Euclidean norm of this error results in a NLS problem solved by descent methods.

The generalization of Eq. 6 to the SIMO case is immediate. Let $u^i(t)$ be the input at the i^{th} spatial location. Let $y^j(t)$ be the output at the j^{th} spatial location. (Note that for backscatter, $i = j$). Then, Eq. 6 becomes

$$y^i(t) = \frac{B^{ij}}{A^{ij}} u^i(t) + e^{ij}(t) \tag{7}$$

Note that, if there are I transmit locations and J receive locations, there will be IJ difference equations describing the experiment.

There remains, however, a serious limitation to the formulation in Eq. 7. Nowhere in the formulation is there contained the constraint that the poles are aspect-independent. Indeed, the solution to Eq. 7 is

$$h^{ij}(t) = \sum_{n=1}^{N} R_n^{ij} e^{s_n^{ij} t} \tag{8}$$

whereas what is required is

$$h^{ij}(t) = \sum_{n=1}^{N} R_n^{ij} e^{s_n t} \tag{9}$$

We shall next examine the use of Eq. 7 for the case of the acoustic sphere. We first briefly describe the forward problem for the sphere and then follow with some identification results using the NLS algorithm.

III. THE ACOUSTIC SPHERE, THEORY

In this section, we give the well-known Mie-series formulation (Ref. 12) for the scattering from an acoustic sphere with Neumann boundary conditions. The formulation will be used to provide data for the identification algorithm.

Consider a sphere of radius a located at the origin of a spherical coordinate system (Fig. 1). We excite the sphere with a plane wave traveling in the negative z-direction. The problem is rotationally symmetric about z and, therefore, has no ϕ - dependence. For this case, the velocity potential $V(r,\theta,\omega)$ can be decomposed into its incident V_i and scattered V_s components, viz:

$$V = V_i + V_s \tag{10}$$

where

$$V_i(r,\theta,\omega) = e^{ikr\cos\theta} = \sum_{n=o}^{\infty} (2n+1)i^n j_n(kr)\, P_n(\cos\theta) \tag{11}$$

and

$$V_s(r,\theta,\omega) = \sum_{n=o}^{\infty} -(2n+1)i^n \frac{j_n'(ka)}{h_n^{(2)\prime}(ka)} h_n^{(2)}(kr)\, P_n(\cos\theta) \tag{12}$$

In Eqs. 11 and 12, j_n is the spherical Bessel function of order n, P_n is the Legendre function of order n, $h_n^{(2)}$ is the spherical Hankel function of second kind and order n, and k is the wave number. To produce the formulation in Eqs. 10 to 12, we have assumed the boundary condition

$$\frac{d}{dr}(V_i + V_s)\Big|_{r=a} = 0 \tag{13}$$

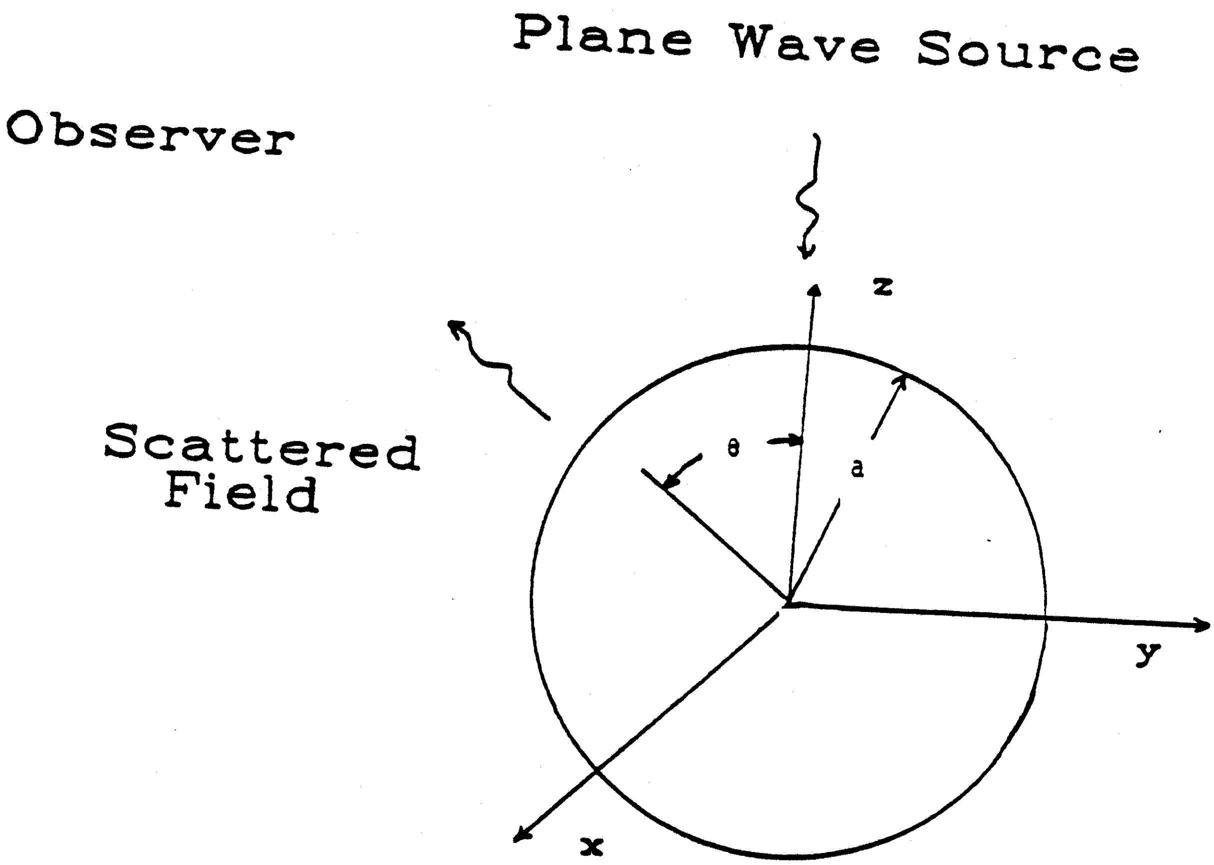

Figure 1. Plane wave interaction with a hard acoustic sphere.

The identification procedure, to be discussed in the next section, requires input and output data in the time domain. The input data is obtained by selecting a pulse $u(t)$ with Fourier transform $U(\omega)$. The output data $y(t)$ is obtained by multiplying the scattered field in Eq. 12 by $U(\omega)$ and taking the inverse transform, viz:

$$y^j(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} U(r_i,\theta_i,\omega)V_s(r_j,\theta_j,\omega)\, e^{i\omega t}\, d\omega \tag{14}$$

where we have explicitly shown the dependence of both input and output on spatial location. We have performed the inverse transform indicated in Eq.14 numerically using Filon's method (Ref. 13).

IV. THE ACOUSTIC SPHERE, IDENTIFICATION

In the identification process to be described in this section, we consider the SIMO case. In all cases, the input will be a plane wave incident in the negative z-direction and the outputs will be the scattered field at different locations around the sphere. We select an input-output pair and identify the coefficients in the A and B polynomials in the output error model (Eq. 6) using the algorithm NLS. We next transform the coefficients (Refs. 7,8) into poles and residues. We then repeat the procedure for another input-output pair. We shall be comparing the poles so obtained with theoretical poles, produced by solving the equation

$$h_n^{(2)\prime}(ka) = 0, \qquad n = 0,1, \ldots \tag{15}$$

These pole locations are well-known and have been included in Fig. 2. In all, we shall include identification of a sphere of unit radius where the output is measured at $r = 10$ m and $\theta = 0$, 15, 30, 45, 60, and 90 deg.

1. INPUT SELECTION

We choose for the input signal $u(t)$ the time-limited pulse

$$u(t) = \left\{ \begin{array}{ll} & t < 0 \\ \cos 2\pi\ (150)t \sin^2 350\, t & 0 < t < \frac{\pi}{350} \\ & t > \frac{\pi}{350} \end{array} \right\} \tag{16}$$

We display the pulse in Fig. 3 and its Fourier transform in Fig. 4. We have chosen the pulse form so that its frequency spectrum is that of a

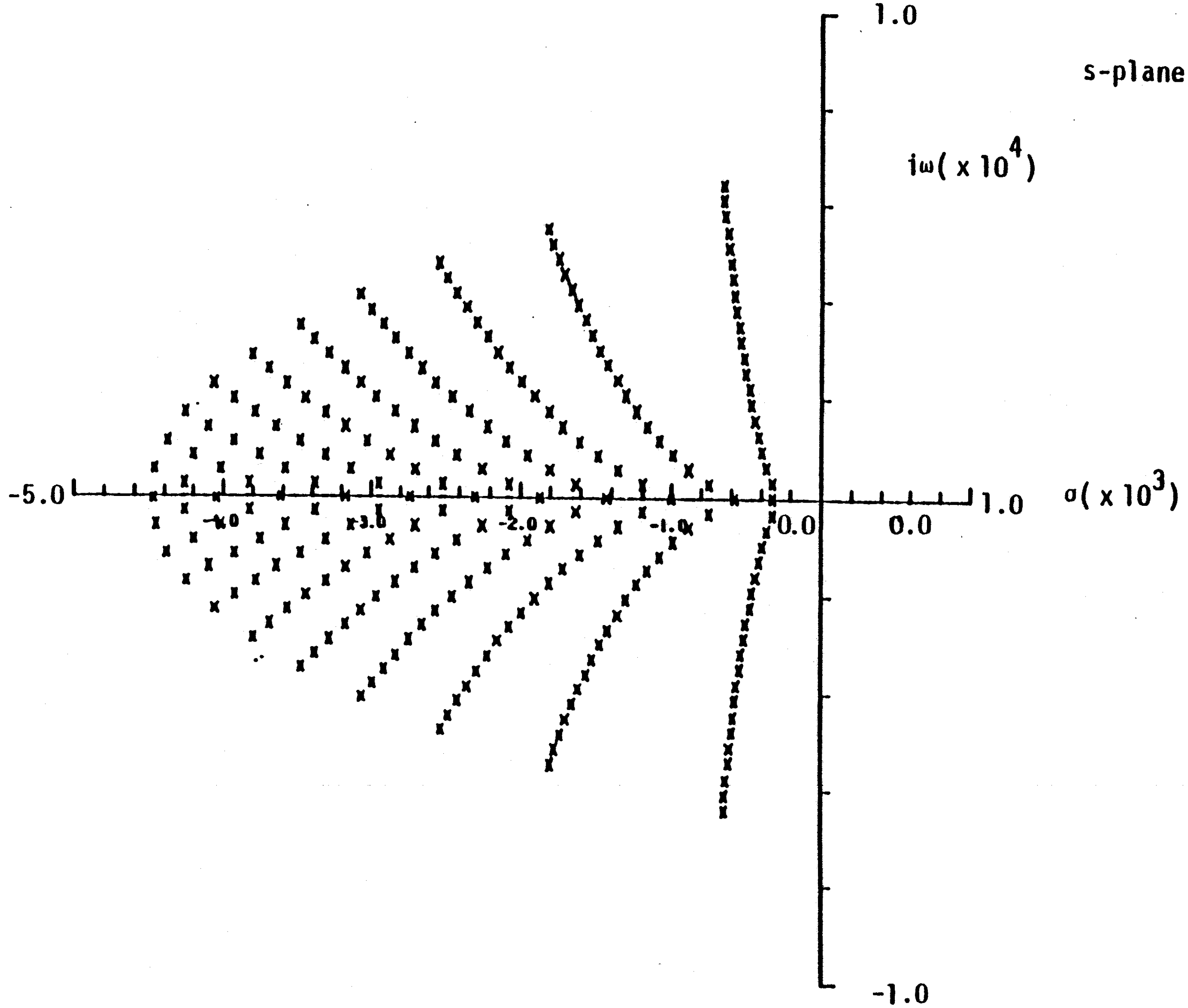

Figure 2. Theoretical poles for hard acoustic sphere (s-plane).

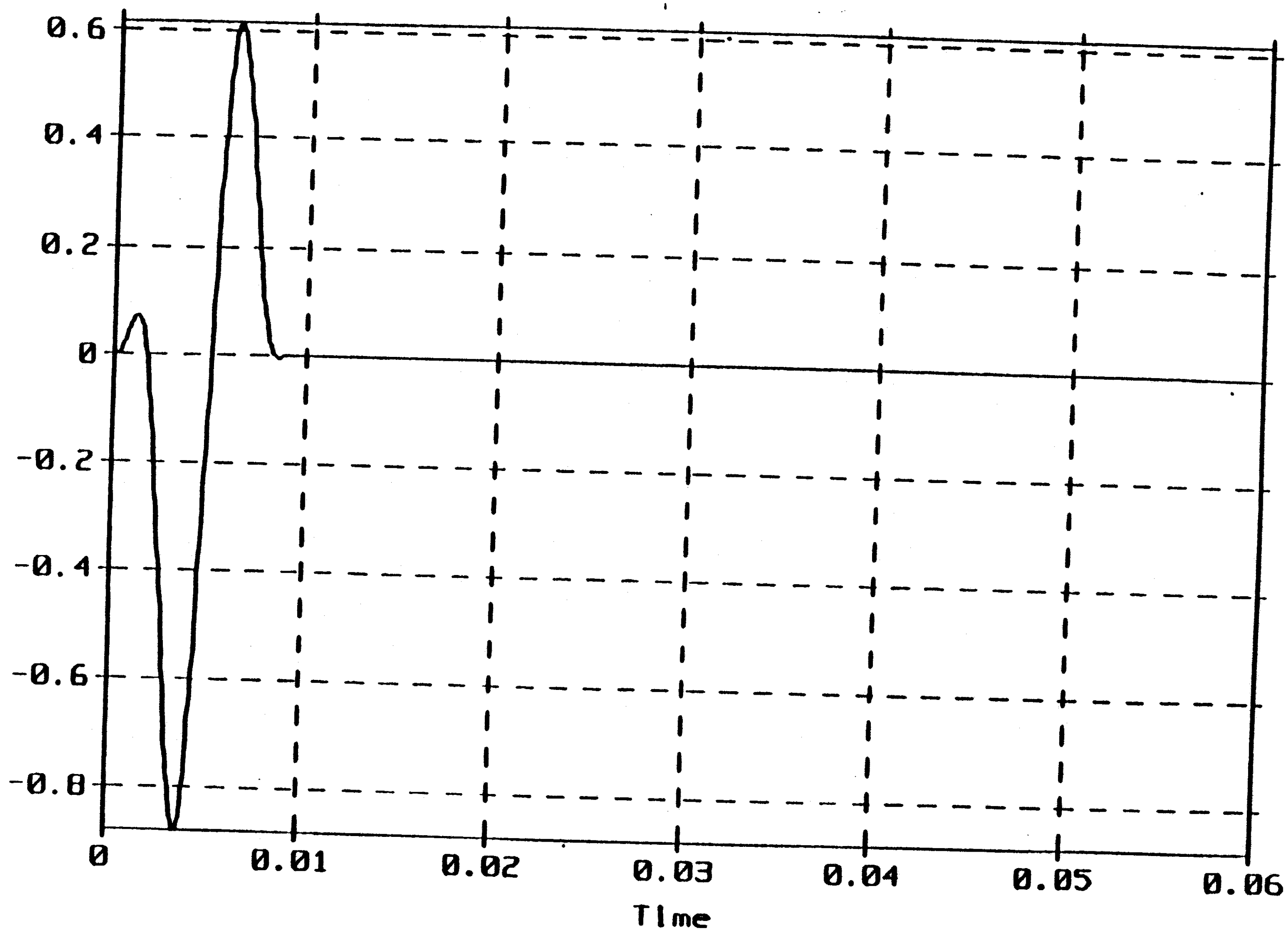

Figure 3. Input u(t), 1000 points

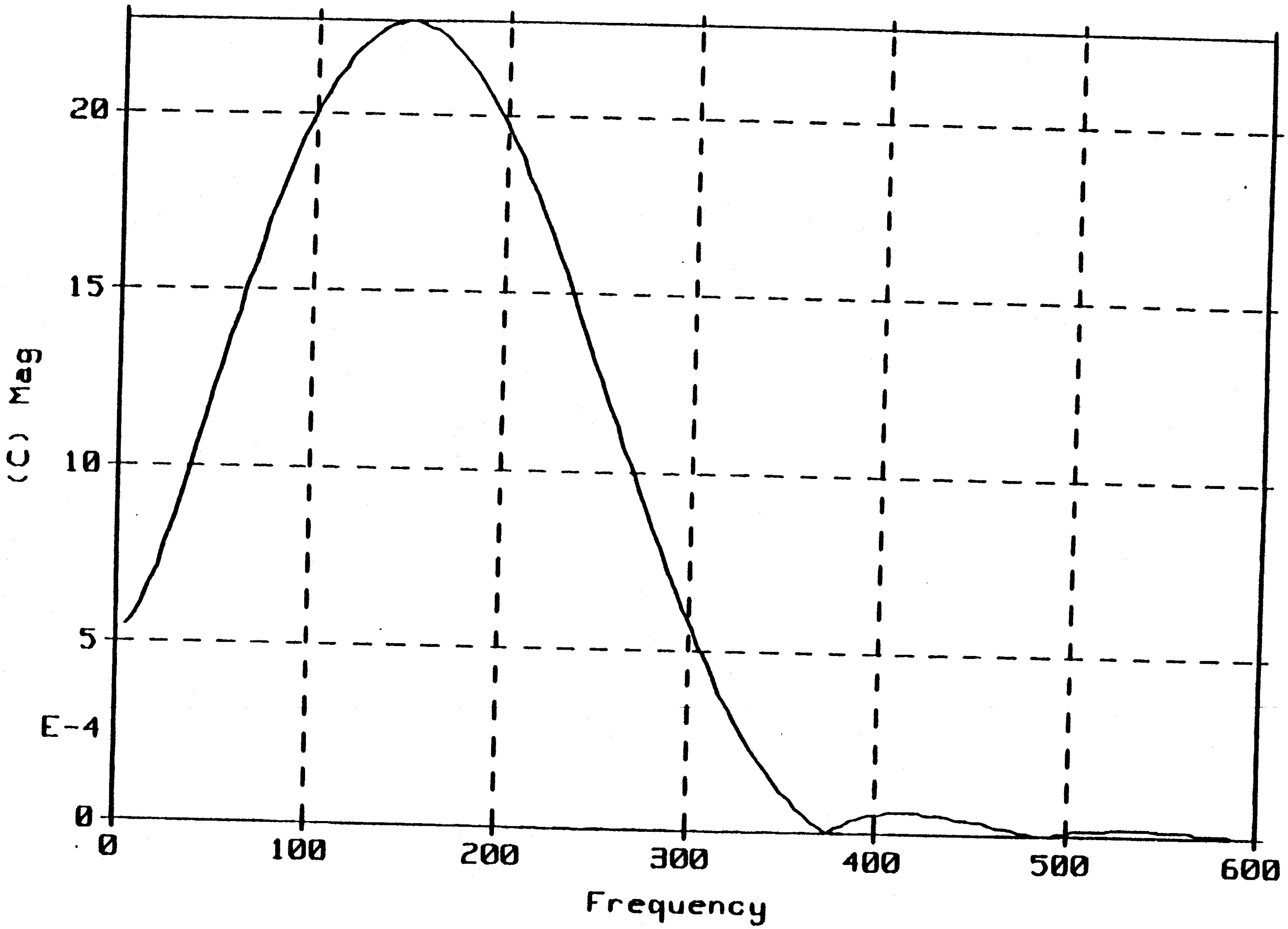

Figure 4. FFT of u(t).

bandpass filter. The center frequency of the passband has been adjusted to coincide with the frequency of the n = 3 pole (152.5 Hz) in the first layer (Fig. 2). The pulse energy falls off either side of the n = 3 pole so that a relatively small number of poles on the first layer are in the passband. (The relatively high attenuation of the poles on successive higher layers makes them a small contributor to the pulse waveform.) The principle here is that the smaller the number of poles in the passband, the smaller the model order necessary in the identification. In usual cases, the smaller the model order, the better the results of the identification.

2. TYPICAL IDENTIFICATION SEQUENCE

In the identification sequence, all signal processing operations are carried out using the interactive signal processing algorithm SIG (Ref. 14). The identification of the coefficients in the output error model is then performed using NLS. Finally, the partial fraction expansion and identification of the poles and residues are obtained through use of the algorithm PARTIA, written by D. M. Goodman at Lawrence Livermore National Laboratory, but as yet unpublished.

Figure 5 is the scattered field measured at $r = 10$ m and $\theta = 0$ deg. The Fourier transform (Fig. 6) clearly indicates the presence of poles in the passband. The first step in the identification sequence is to lowpass filter the input and output data and then decimate the data down near to, but higher than, the Nyquist rate. We filter at 300 Hz with an 8^{th} order Butterworth filter and then decimate the 1000 points in the input and output to 200 points. Figures 7 and 8 show, respectively, the input pulse after filtering and decimating. Figures 9 and 10 show the same operations on the scattered field.

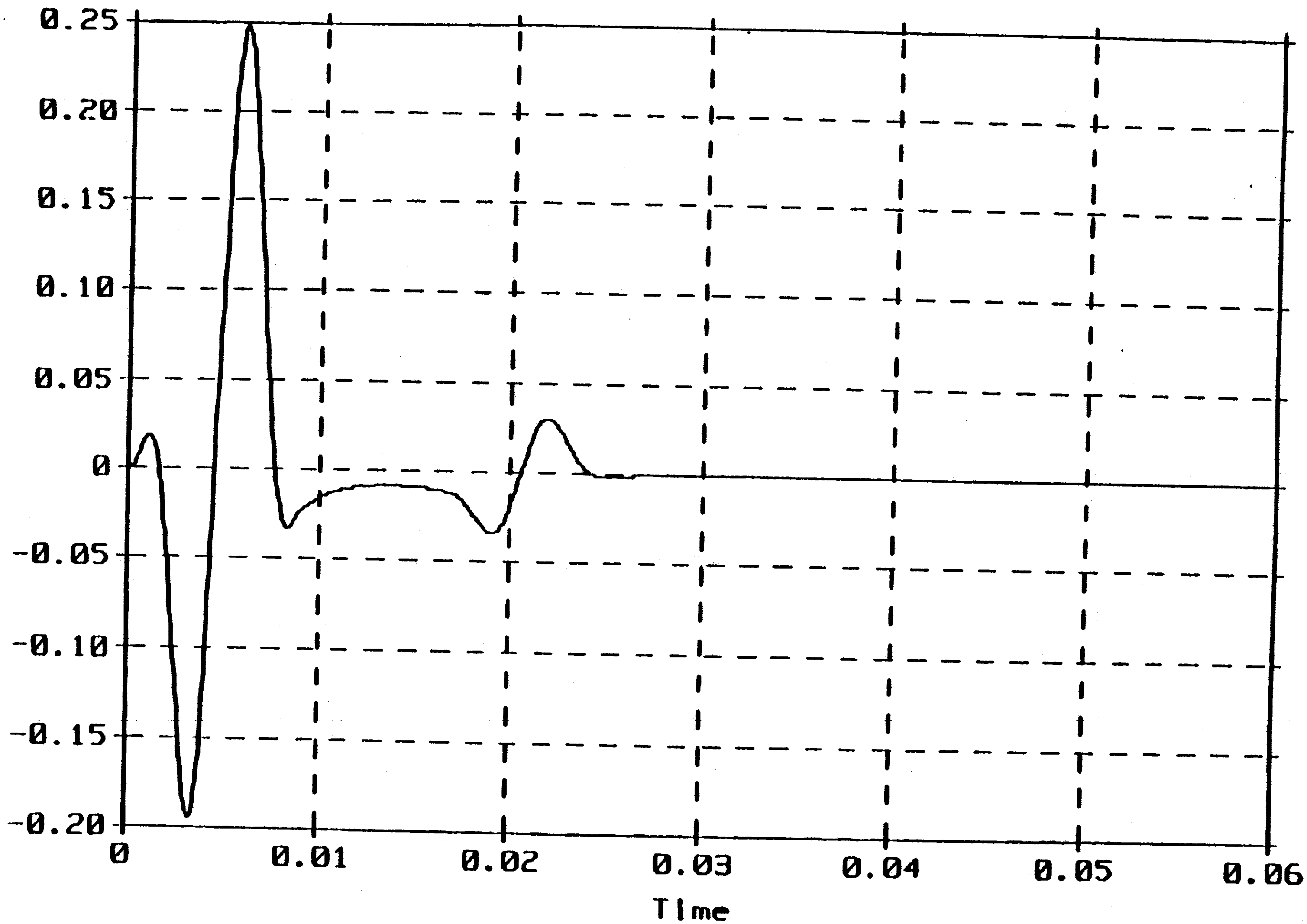

Figure 5. Output y(t), a = 1 m, r = 10m, θ = 0 deg, 1000 points.

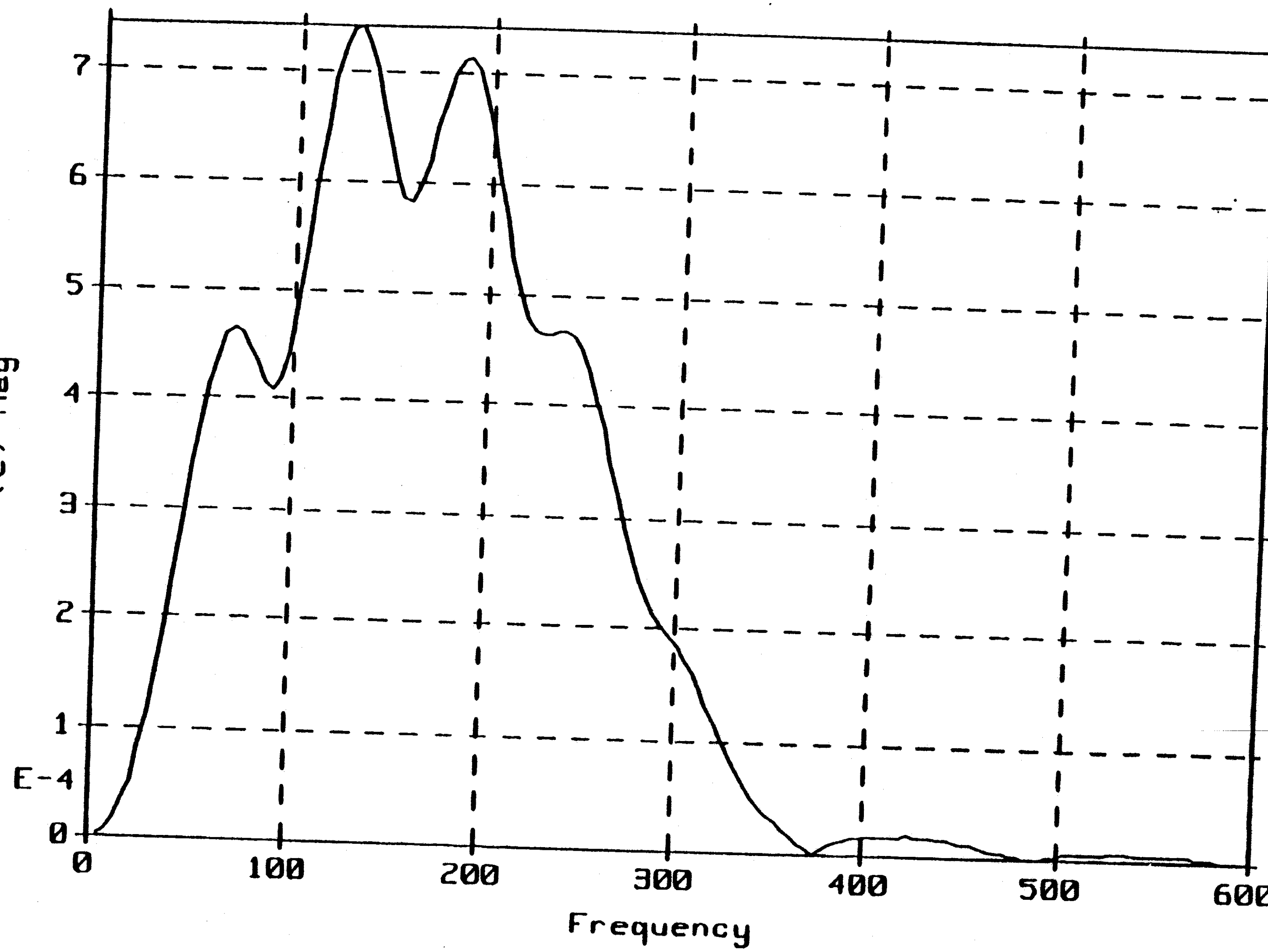

Figure 6. FFT of y(t).

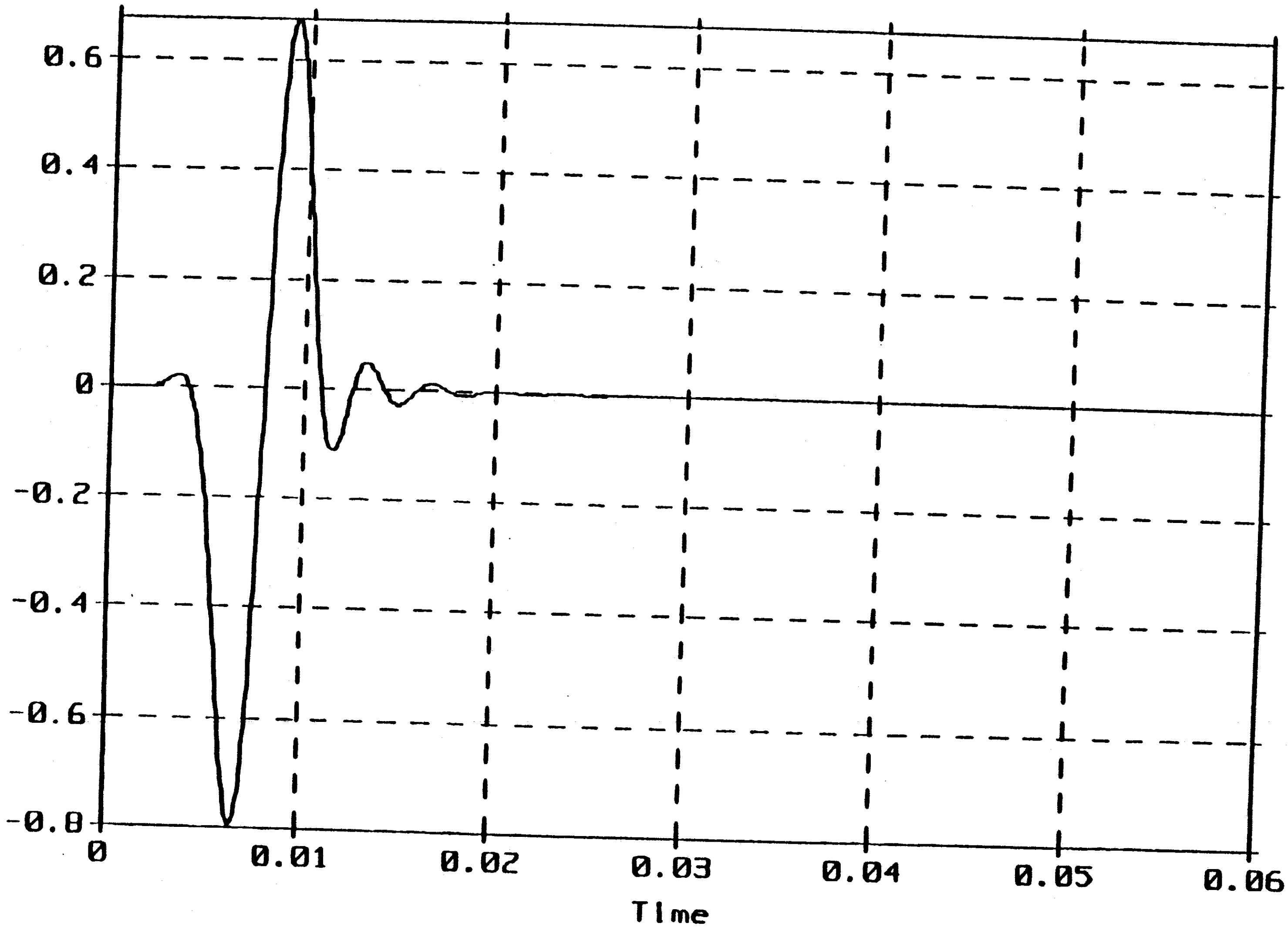

Figure 7. Input u(t), 8^{th} order Butterworth filter at 300 Hz.

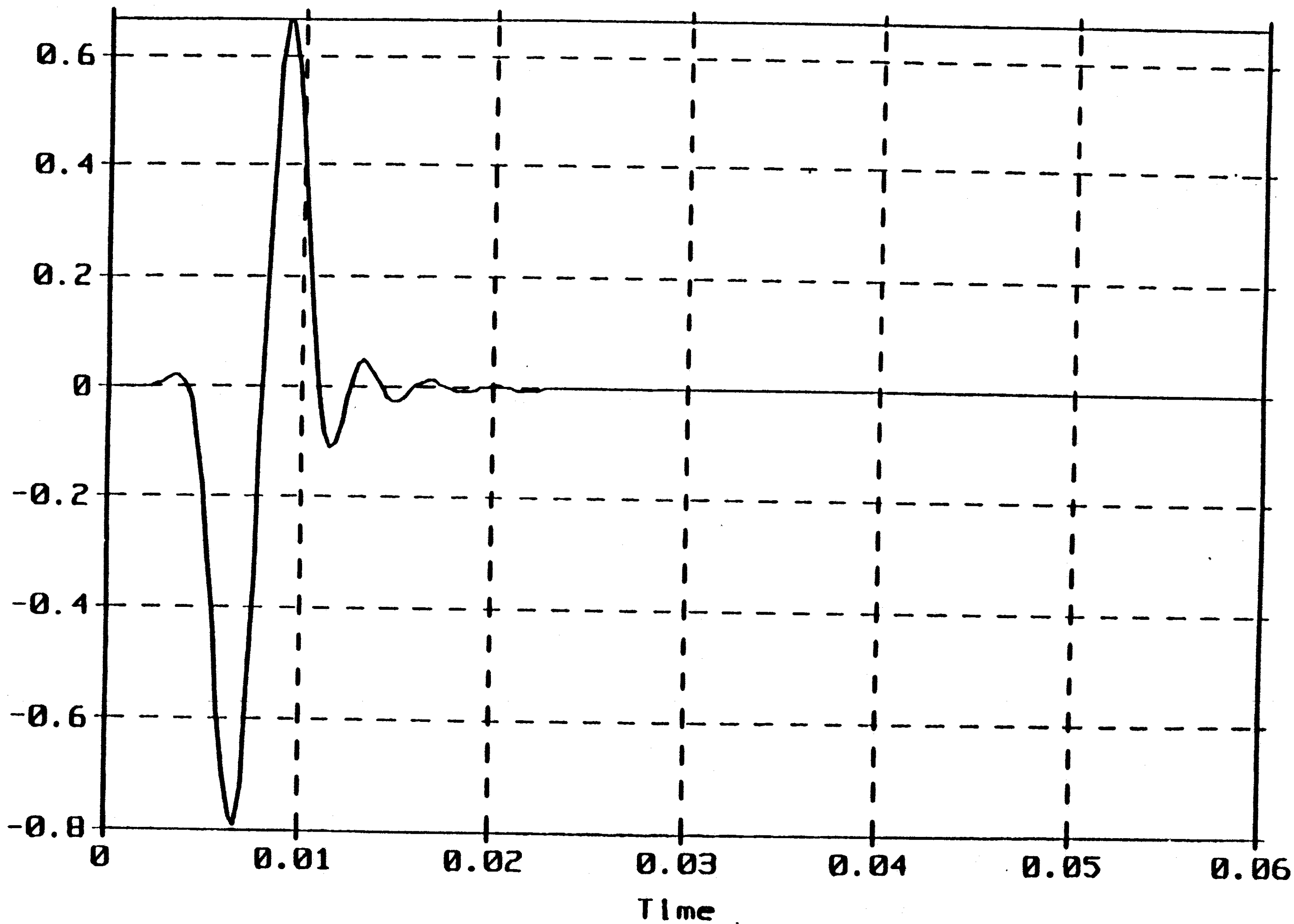

Figure 8. Input u(t), filtered and decimated to 200 points.

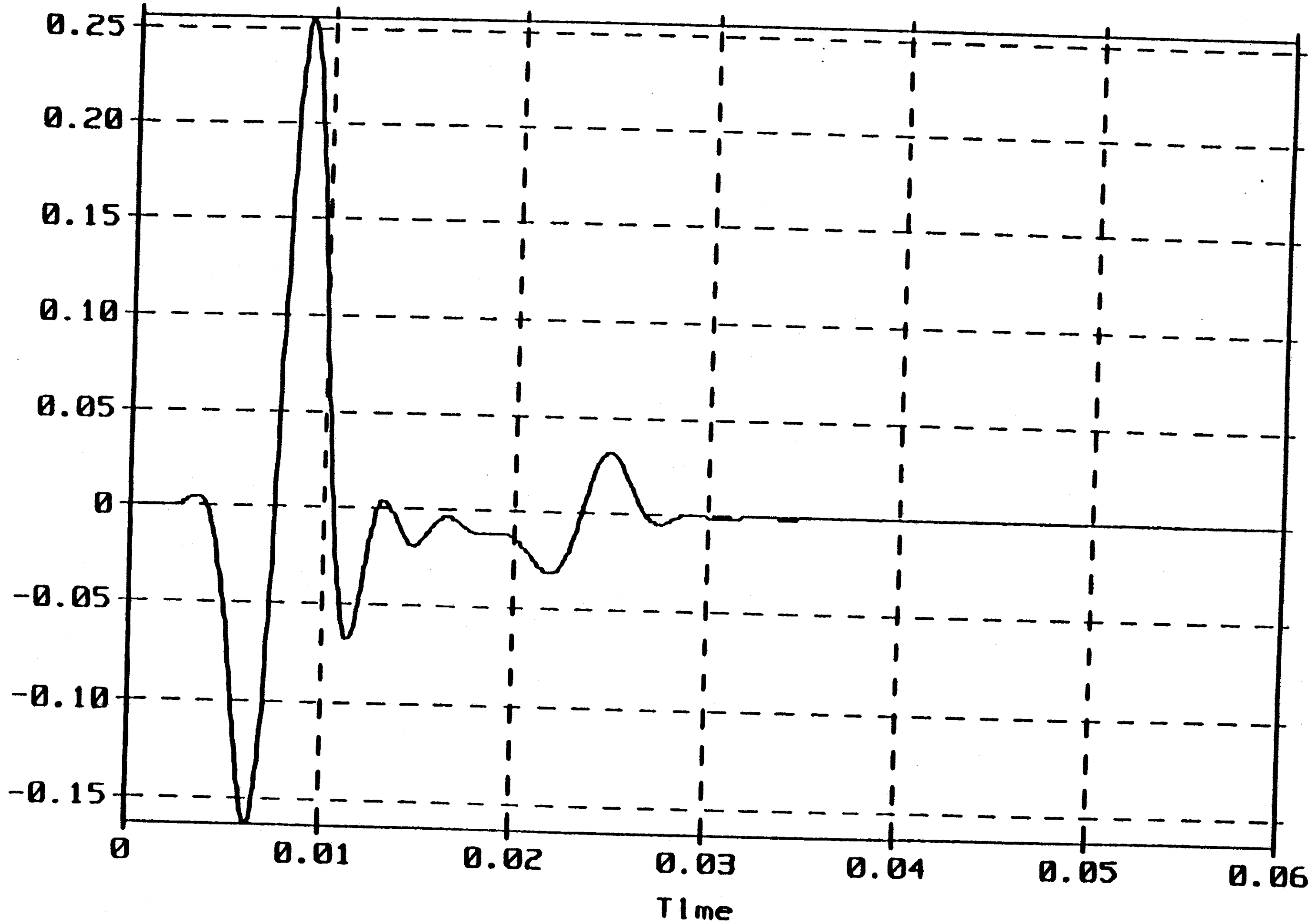

Figure 9. Output y(t), filtered at 300 Hz.

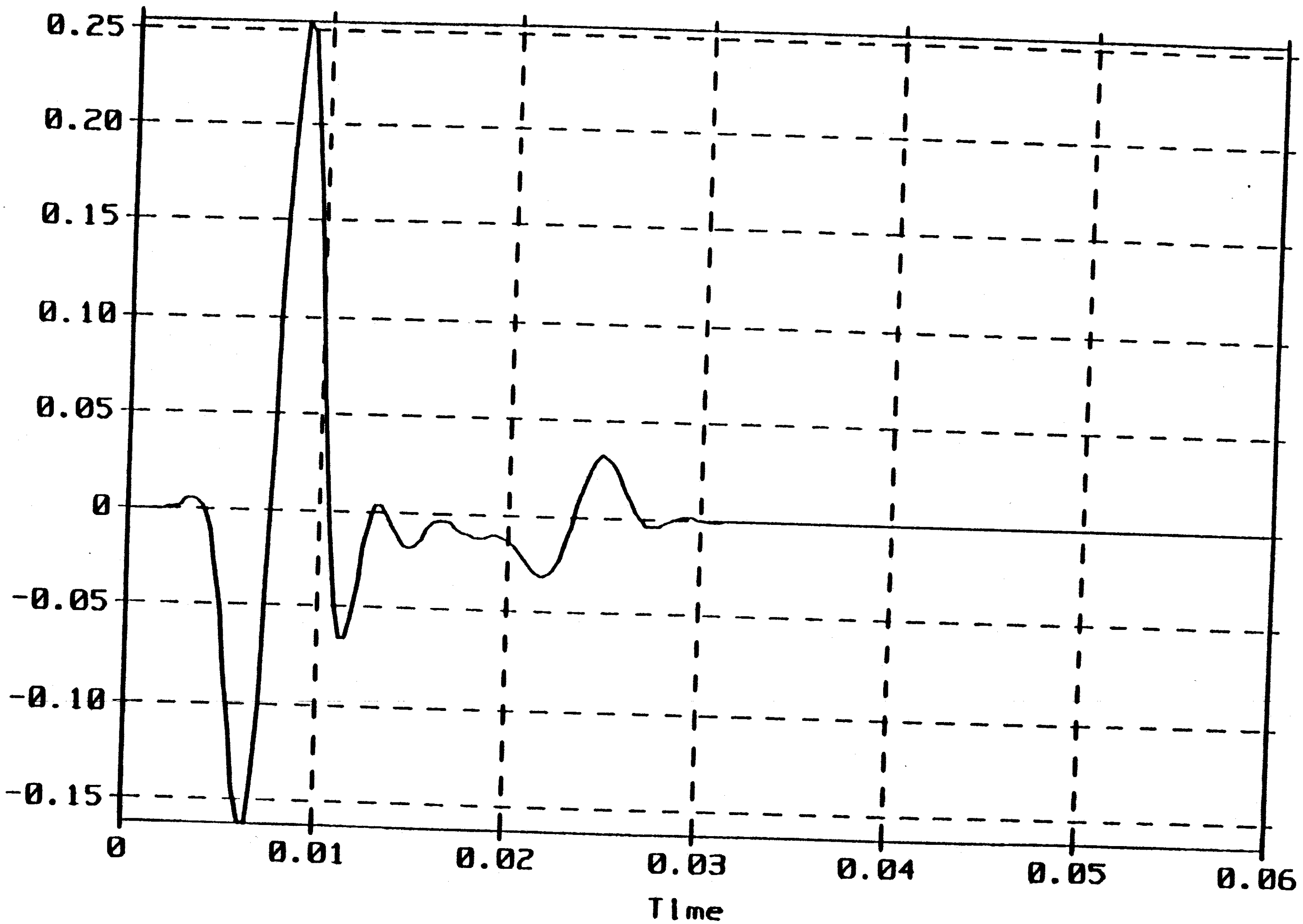

Figure 10. Output y(t), filtered at 300 Hz, decimated to 200 points.

The filtered and decimated input and output are used as drivers of the identification algorithm NLS. We select a 40^{th} order model, run NLS and PARTIA and obtain the results for the poles in Fig. 11, where we compare the identified poles (x) with the first layer theoretical poles (+). The numbers on the plots close to the identified poles are indicative of the identified residues.

3. IDENTIFICATION RESULTS

In Figs. 11 through 16, we give, respectively, the results of the identification procedure at $r = 10$ m and for $\theta = 0, 15, 30, 45, 60,$ and 90 deg. In all cases, we have used a 40^{th} order model except at 90 deg (Fig. 16). Note that at 90 deg, symmetry considerations eliminate half the theoretical poles so that we are able to cut our model order in half.

The results displayed are the product of many multiple runs of NLS at each angle. Even so, the match of the identified poles to the theoretical poles is quite uneven from angle to angle. Perhaps the best results are at $\theta = 60$ deg, where the d.c. pole and the first four poles on the first layer have been identified quite closely. It is emphasized that in these cases, we knew the theoretical pole positions and, therefore, were selective in our choice of the "best" in a series of multiple runs at each angle. In addition, we emphasize that closeness of the theoretical scattered field waveform to the waveform produced by running the input through the identified model cannot be correlated with the closeness of the identified poles to their theoretical counterpart. From another point of view, the size of the final error in the error minimization procedure in NLS is not an indicator of the closeness of the results in the pole positions.

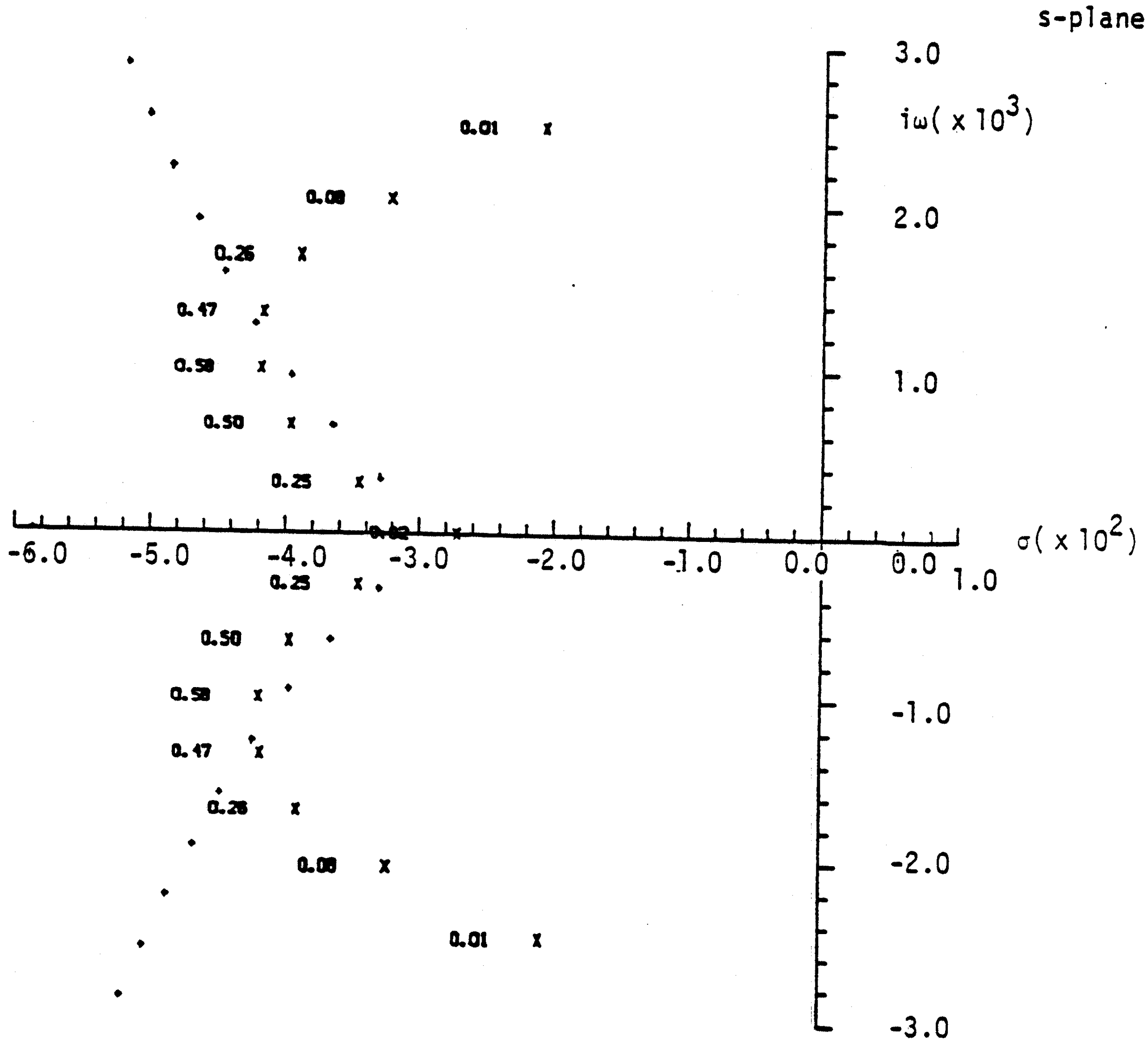

Figure 11. Theoretical (+) and identified (x) poles (s-plane); a = 1 m, r = 10 m, θ = 0 deg.

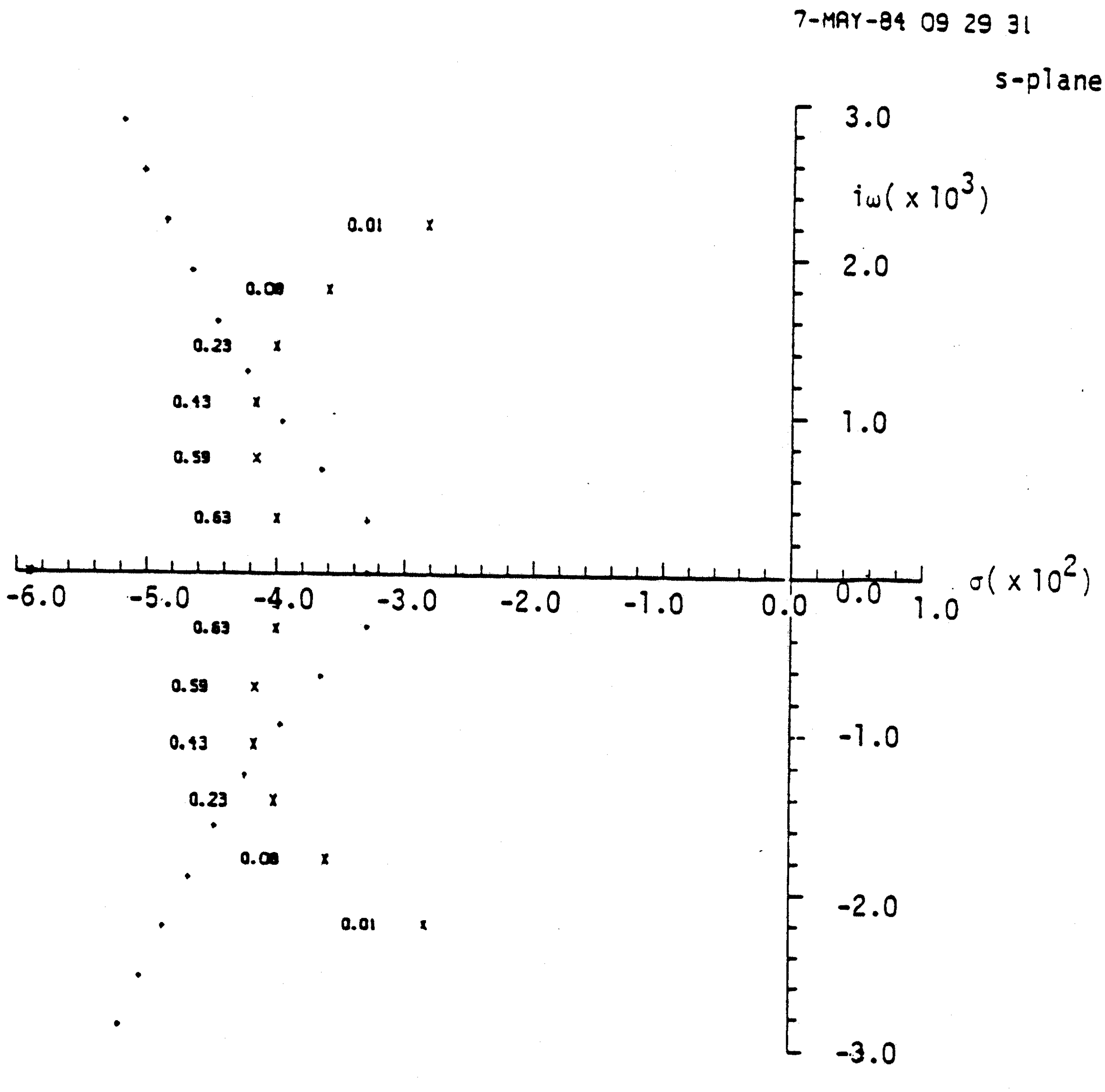

Figure 12. Theoretical (+) and identified (x) poles (s-plane); $a = 1$ m, $r = 10$ m, $\theta = 15$ deg.

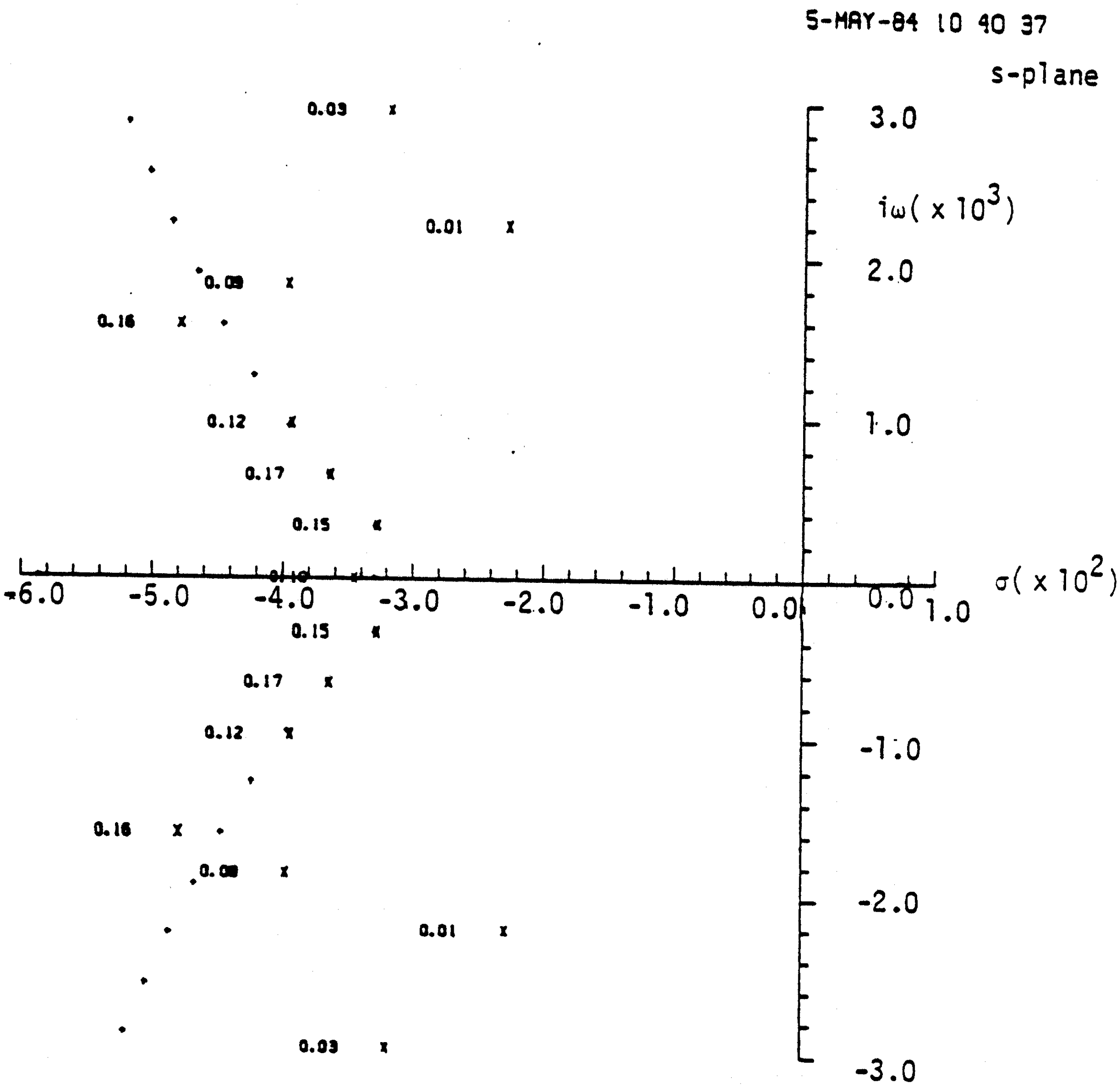

Figure 13. Theoretical (+) and identified (x) poles (s-plane); a = 1 m, r = 10 m, θ = 30 deg.

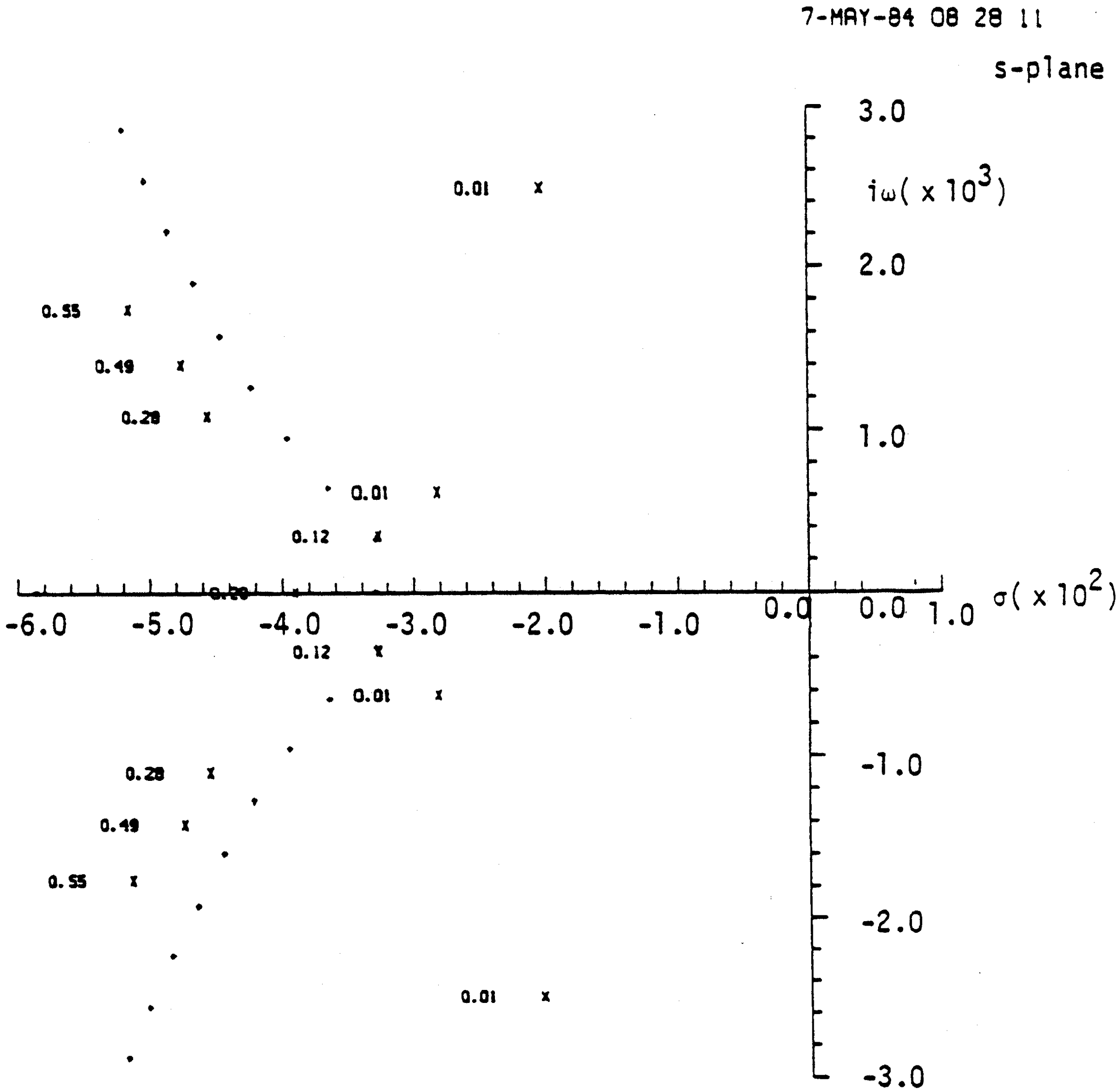

Figure 14. Theoretical (+) and identified (x) poles (s-plane); $a = 1$ m, $r = 10$ m, $\theta = 45$ deg.

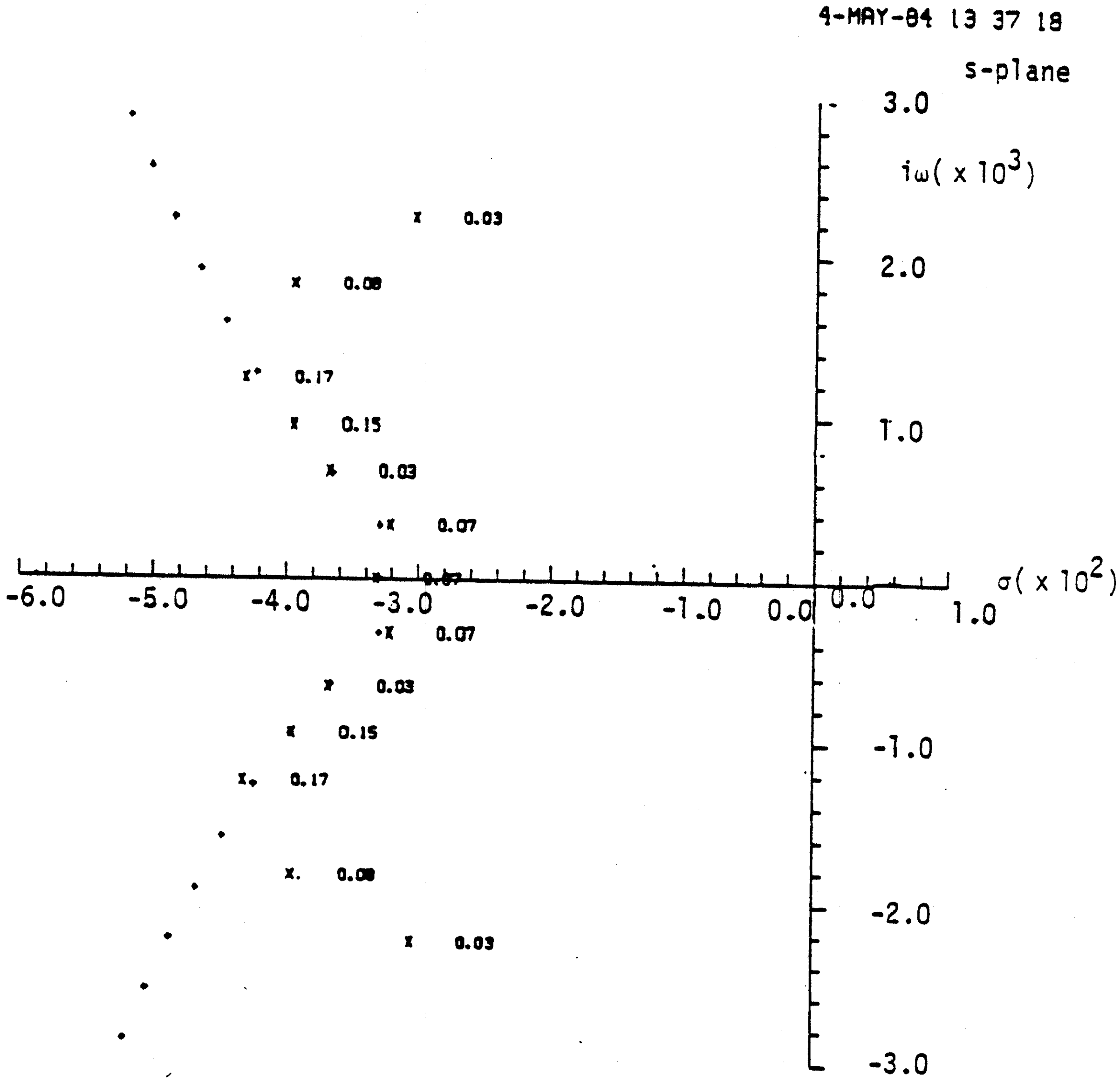

Figure 15. Theoretical (+) and identified (x) poles (s-plane); $a = 1$ m, $r = 10$ m, $\theta = 60$ deg .

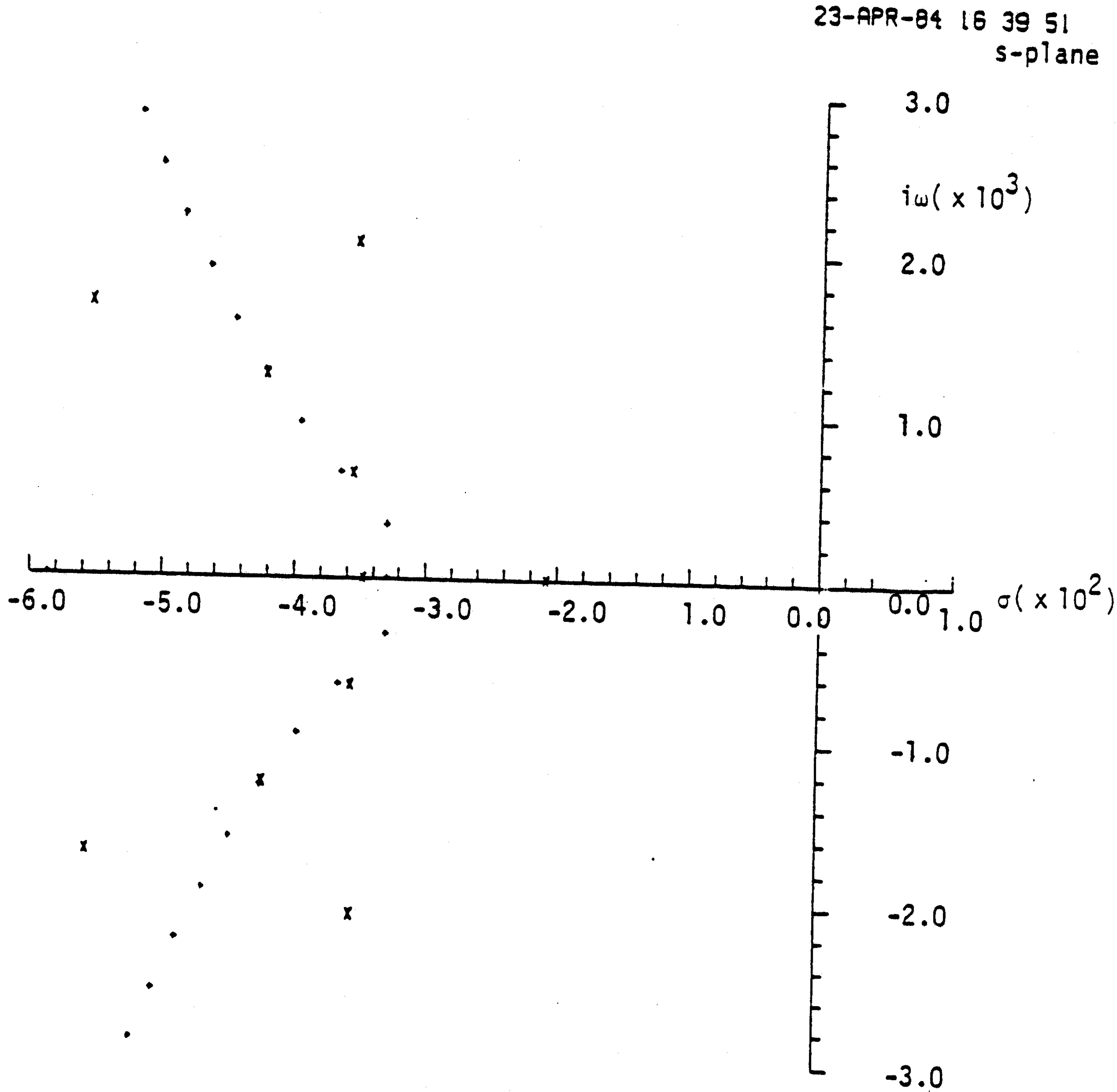

Figure 16. Theoretical (+) and identified (x) poles (s-plane); $a = 1$ m, $r = 10$ m, $\theta = 90$ deg.

There are two principal difficulties in the identification procedure described above. First, it is well known that the pole series does not completely describe the scattered field from the sphere (Ref. 15). Indeed, there is an entire function contribution that comprises a sizable portion of the scattered field waveform. The pole-residue model is, therefore, incomplete and, in effect, the entire function contribution is being modeled as a part of the pole series. This difficulty will be discussed more completely in the next section. Second, there is no constraint in the identification algorithm that introduces the fact that the poles are aspect invariant. Each angle has been treated as a separate identification problem by the NLS algorithm. This difficulty will be discussed and overcome in the next section.

V. A STATE-SPACE FORMULATION

Marin (Ref. 3) has shown that the electromagnetic scattering from a perfectly conducting obstacle of finite extent is, with certain mathematical restrictions, a meromorphic function of frequency. His work provides a formal statement of the basic principles underlying the SEM (Ref. 4). Based on Marin's result and the Mittag-Leffler theorem, the scattered fields can be expressed formally in the frequency domain by an expansion over the poles of the meromorphic function plus an entire function (Ref. 15). The pole expansion yields the complex body resonances of the scatterer, while the entire function yields, through the inverse Laplace or Fourier transform, a time-limited response that comprises a portion of the early time scattering signature.

To date, parametric descriptions of conducting scatterers have included only the complex resonance portion of the scattering, neglecting the entire function contribution. In addition, all these descriptions are for the SISO case, thereby neglecting the spatial aspect invariance of the complex resonances. Such was the case in the acoustic spere identification described in the previous section.

In this section, we give a realization of the scattering problem in state-space form. For the SISO case, the canonical companion matrix description (Ref. 16) is selected so that the aspect-independent pole contribution is separated from the aspect-dependent zeros, the former occurring in the state equation and the latter occurring in the output equation. We next generalize the description to the SIMO case by changing the vector containing the zeros to a matrix whose rows are the zeros for each output spatial location. We then discuss the entire function and conclude with some comments on the uses and limitations of the formulation. We particularly emphasize the need for a more physical parametrization of the entire function.

1. SCATTERING THEORY

Consider the classic solution to the scattering problem for a perfectly conducting obstacle with compact support, given by the magnetic field integral equation (MFIE). In this section, lower case quantities denote functions of time, while upper case quantities denote, through the Laplace transform, functions of complex frequency $s = \sigma + i\omega$. A current source $\underline{j}(\underline{r},t)$ radiates in the presence of the obstacle with surface S and produces a magnetic field $\underline{h}(\underline{r},t)$ at the point $P(\underline{r})$. Herein, in the spirit of linear system theory, it is assumed that the temporal portion of the current is given by the delta function, viz:

$$\underline{j}(\underline{r},t) = \underline{J}(\underline{r})\delta(t) \tag{17}$$

so that the resulting fields $[\underline{e}^{\delta}(\underline{r},t),\underline{h}^{\delta}(\underline{r},t)]$ at any given observation point $P(\underline{r})$ comprise the delta function response of the electromagnetic system. Let $\underline{h}_i^{\delta}(\underline{r},t)$ be the magnetic field in the absence of the scattering obstacle. Define the scattered field $\underline{h}_s^{\delta}(\underline{r},t)$ to be the difference between the total magnetic field $\underline{h}^{\delta}$ and the incident field $\underline{h}_i^{\delta}$, viz:

$$\underline{h}_s^{\delta}(\underline{r},t) = \underline{h}^{\delta}(\underline{r},t) - \underline{h}_i^{\delta}(\underline{r},t) \tag{18}$$

The Laplace transform of Eq. 18 yields

$$\underline{H}_s(\underline{r},s) = \underline{H}(\underline{r},s) - \underline{H}_i(\underline{r},s) \tag{19}$$

where it is well-known (Ref. 17) that the incident and scattered magnetic fields are given by

$$\underline{H}_i(\underline{r},s) = \int_V \underline{J}(\underline{r}') \times \nabla'\Psi(\underline{r},\underline{r}',s)\, dV' \tag{20}$$

$$\underline{H}_s(\underline{r},s) = \int_S [\underline{n}' \times \underline{H}(\underline{r}',s)] \times \nabla'\Psi(\underline{r},\underline{r}',s)\, dS' \tag{21}$$

and Ψ is the free space Green's function given by

$$\Psi(\underline{r},\underline{r}',s) = \frac{\exp(-s|r - r'|/c)}{4\pi|\underline{r} - \underline{r}'|} \tag{22}$$

In the usual manner, unprimed and primed spatial coordinates refer to observation and source (real or induced) points, respectively. The final step in the resolution of the scattering problem involves the determination of the tangential magnetic field over the surface of the scatterer. This step can be accomplished by solution to the magnetic field integral equation (Ref. 17)

$$\underline{K}_i(\underline{r},s) = \frac{1}{2}\underline{K}(\underline{r},s) - \int_S \underline{n} \times [\underline{K}(\underline{r}'s) \times \nabla'\Psi(\underline{r},\underline{r}',s)]\, dS' \, (\underline{r}\varepsilon S) \tag{23}$$

where the integral is principal value and

$$\underline{K}_i = \underline{n} \times \underline{H}_i \tag{24}$$

$$\underline{K} = \underline{n} \times \underline{H} \tag{25}$$

In operator notations, Eq. 23 can be written

$$\underline{K}_i = (\tfrac{1}{2}\underline{I} - \underline{L}) \cdot \underline{K} \tag{26}$$

where $\underline{I}$ is the identity operator and $\underline{L}$ is the surface integral operator. the solution to the integral equation, normally carried out by numerical means, yields the surface current $\underline{K}$, viz:

$$\underline{K} = (\tfrac{1}{2}\underline{I} - \underline{L})^{-1} \cdot \underline{K}_i \tag{27}$$

Once the surface current $\underline{K}$ is determined, the result is substituted into Eq. 21 to give the scattered field. The incident field is given directly by Eq. 20 and the additive combination of incident and scattered field yields the total field $\underline{H}(\underline{r},s)$ at point P in the complex frequency domain. The problem is completed by multiplying this result by the spectrum $F(s)$ of the incident temporal pulse $f(t)$, followed by the inverse Laplace transform, viz:

$$\underline{h}(\underline{r},t) = \frac{1}{2\pi i} \int_{Br} \underline{H}(\underline{r},s)\, F(s) e^{st}\, ds \tag{28}$$

2. THE MEROMORPHIC FORMALISM

Marin (Ref. 3) has shown that, with certain mild mathematical restrictions on the shape of the scattering surface, the scattering problem described above has the following properties:

1. The operator inverse $(\frac{1}{2}\underline{I} - \underline{L})^{-1}$ is an analytic operator-valued function of complex frequency s except at a countable set of points where it has poles.
2. The surface current $\underline{K}$ over S is a meromorphic function of complex frequency. This property is based on the fact that $\underline{K}_i$ is an entire function of s.
3. The scattered fields $(\underline{E}_s, \underline{H}_s)$ are meromorphic functions of frequency.

In addition to these properties, it has been speculated that the poles predicted by the meromorphic property are simple, but this item remains without proof.

The meromorphic property admits to an expansion of the fields by the Mittag-Leffler theorem (Ref. 18) in a form comprising one of the cornerstones of the Singularity Expansion Method. For the scattered magnetic field, assuming simple poles, we obtain (Ref. 15)

$$\underline{H}_s(\underline{r},s) = \sum_{k=1}^{\infty} \frac{\underline{R}_k(\underline{r})}{s - s_k} + \underline{\Phi}(\underline{r},s) \tag{29}$$

where the summation is the Heaviside expansion over simple poles and $\underline{\Phi}$ is an entire function. Note that the vector residues $\underline{R}_k$ and the entire function are functions of both position and complex frequency s, whereas the natural frequencies s_k are aspect independent. Note also that $\underline{H}_s(\underline{r},s)$ is a system transfer function in the sense that it is the Laplace transform of the scattered magnetic field responding to the temporal delta function. The inverse Laplace transform yields the delta function response

$$\underline{h}_s^{\delta}(\underline{r},t) = \sum_{k=1}^{\infty} \underline{R}_k(\underline{r}) e^{s_k t} + \underline{\phi}(\underline{r},t) \tag{30}$$

which completes the meromorphic formalism.

3. THE SISO ELECTROMAGNETIC SYSTEM

In the SISO case, consider an electromagnetic source U(s) radiating one component of linear polarization and producing a response Y(s) at a fixed spatial location $P(\underline{r})$. The response Y(s) can be any component of electric or magnetic field. From Eq. 29, the system transfer function T(s) is given by the scalar equation

$$T(s) = \frac{Y(s)}{U(s)} = \sum_{k=1}^{\infty} \frac{R_k}{s - s_k} + \Phi(s) \tag{31}$$

$$= T_n(s) + \Phi(s) + \Gamma(s) \tag{32}$$

where

$$T_n(s) = \sum_{k=1}^{n} \frac{R_k}{s - s_k} \tag{33}$$

and where $\Gamma(s)$ is the error in truncating the Heaviside expansion after n terms. In practice, the error can be made small by filtering the input and output to produce bandlimited data. Indeed, in many cases, the filtering operation is a natural one supplied by the finite bandwidth of the input and/or measuring system. The truncated Heaviside expansion can be put in rational form by gathering all n terms over a common demnominator, viz:

$$T_n(s) = \frac{\sum_{k=0}^{n-1} b_k s^k}{\sum_{k=0}^{n} a_k s^k} \tag{34}$$

Therefore, if the error $\Gamma(s)$ is made small by filtering, the system transfer function $T(s)$ is given by the sum of a rational function and an entire function.

Efforts in the parametric inverse problem in transient electromagnetic scattering have to date been concerned with the rational portion $T_n(s)$ of the system transfer function (Refs. 6,7,8), ignoring the contribution of the entire function to the system transfer function. If the entire function is neglected the SISO can be represented by an n^{th} order differential equation or, in discrete time, by an n^{th} order difference equation, as in Refs. 6, 7, and 8. With the entire function neglected, the SISO system can also be represented by the following state-space description:

$$\dot{\underline{x}} = F\underline{x} + \underline{g}u \tag{35}$$

$$y = \underline{h}^T\underline{x} \tag{36}$$

where $\dot{\underline{x}}$ is the time derivative of $\underline{x}$, $\underline{x}$ is an n x 1 state vector, F is an n x n state matrix, $\underline{g}$ is an n x 1 vector, u is a scalar input, y is a scalar output, and $\underline{h}$ is an $n \times 1$ vector. The controllable canonical realization (Ref. 16) of $T_n(s)$ is given by

$$F = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix} \tag{37}$$

$$\underline{g} = [0 \quad 0 \quad \ldots \quad 1]^T \tag{38}$$

$$\underline{h} = [b_0 \quad b_1 \quad \ldots \quad b_{n-1}]^T \tag{39}$$

Although this is only one of many possible realizations, it is an important one in the transient electromagnetic case since it separates the coefficients yielding the poles from the coefficients yielding the zeros. The former occur in the state Eq. 35, while the latter occur in the output Eq. 36. Note that the Laplace transform of Eqs. 35 and 36 yields

$$T(s) = \frac{Y(s)}{U(s)} \doteq T_n(s) = \frac{\underline{h}^T \mathrm{adj}(sI - F)\underline{g}}{|sI - F|} \tag{40}$$

where adj signifies the adjoint operation. The poles of the system are contained in the denominator determinant $|sI - F|$.

4. THE SIMO ELECTROMAGNETIC SYSTEM

In the SIMO case, an electromagnetic source $U(s)$ radiating one component of linear polarization is monitored at p spatial positions

$\underline{r}_1, \underline{r}_2, \ldots, \underline{r}_p$. Let $Y^{(j)}(s)$ be any component of electric or magnetic field monitored at the j^{th} spatial positions, $j = 1, 2, \ldots, p$. Then, from Eq. 29, the system transfer function $T^{(j)}(s)$ between $U(s)$ and $Y^{(j)}(s)$ is

$$T^{(j)}(s) = \frac{Y^{(j)}(s)}{U(s)} = \sum_{k=1}^{\infty} \frac{R_k^{(j)}}{s - s_k} + \phi^{(j)}(s) \tag{41}$$

$$= T_n^{(j)} + \phi^{(j)}(s) + \Gamma^{(j)}(s) \tag{42}$$

where

$$T_n^{(j)} = \sum_{k=1}^{n} \frac{R_k^{(j)}}{s - s_k} \qquad j = 1, 2, \cdots, p \tag{43}$$

and where $\Gamma^{(j)}$ is the error made by truncating the Heaviside expansion in Eq. 43 after n terms. As in the SISO case, the truncated Heaviside expansion can be put into rational form, viz:

$$T_n^{(j)}(s) = \frac{\sum_{k=0}^{n=1} b_k^{(j)} s^k}{\sum_{k=0}^{n} a_k s^k} \tag{44}$$

where again the error can be made small by filtering. If the entire function contribution is neglected, a state-space representation of the SIMO system is given by

$$\dot{\underline{x}} = F\underline{x} + \underline{g}u \tag{45}$$

$$\underline{y} = H\underline{x} \tag{46}$$

where F is an n x n matrix, $\underline{x}$ is an n x 1 state vector, $\underline{g}$ is an n x 1 vector, $\underline{y}$ is a p x 1 output vector, and H is a p x n matrix. The matrix F and the vector $\underline{g}$ are again given by Eqs. 37 and 35. The H matrix consists of the numerator coefficients for each output spatial position, viz:

$$H = \begin{bmatrix} b_0^{(1)} & b_1^{(1)} & \cdots & b_{n-1}^{(1)} \\ b_0^{(2)} & b_1^{(2)} & \cdots & b_{n-1}^{(1)} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ b_0^{(p)} & b_1^{(p)} & \cdots & b_{n-1}^{(p)} \end{bmatrix} \tag{47}$$

The state-space formulation in Eqs. 45 and 46 constitutes a realization of potential importance in the parametric inverse problem. The formulation allows monitoring of the scattered field at different spatial positions with the constraint that the poles are position invariant. To cast the state-space realization in a form for parametric inversion, let $\hat{\underline{y}}$ be the vector containing measured outputs at the p receiver locations. Then

$$\hat{\underline{y}} = \underline{y} + \underline{e} \tag{48}$$

where $\underline{e}$ is the measurement error vector. Substitution into Eq. 46 gives the state-space description

$$\dot{\underline{x}} = F\underline{x} + \underline{g}u \tag{49}$$

$$\underline{y} = H\underline{x} + \underline{e} \tag{50}$$

In a SIMO electromagnetic experiment, input-output data $[u,\hat{\underline{y}}]$ can be substituted into Eqs. 49 and 50 and the error $\underline{e}$ minimized in some sense by adjustment of the $[a_k, b_k^{(j)}]$ parameters. A Kalman filter could be used to accomplish this, or, any number of continuous or discrete time least squares state-space algorithms (see Ref. 19 for examples). Although this paper is concerned with the continuous time case, the extension to discrete time is straightforward.

5. ENTIRE FUNCTION

In consideration of the entire function, it is important to distinguish between the parametric inverse problem and system identification. In system identification, models are chosen whose parameters are adjusted to match input-output data. In the parametric inverse, the parameters must give physical insight concerning the scattering structure. In the system identification context, it is possible to model the entire function in many different ways. Consider, for example, the following series expansion of the entire function in the SIMO case:

$$\phi^{(j)}(s) = \sum_{k=1}^{q} c_k^{(j)} s^k \qquad . \tag{51}$$

This expansion is simply a power series about $s = 0$ truncated after q terms. The series has two disadvantages: First, the coefficients $c_k^{(j)}$ have no physical meaning; second, the powers of s act as differentiators in the time domain and, therefore, would tend to make the modeling system unstable. A better expansion might be

$$\phi^{(j)}(s) = \sum_{k=1}^{q} c_k^{(j)} s^{-k} \tag{52}$$

This expansion is stabilizing since its terms act as integrators. It has the disadvantage of having a q^{th} order pole at the origin, a fact that destroys the entire function property at low frequencies. This difficulty in practice might be taken care of by highpass filtering the data. The expansion in Eq. 52 is easily incorporated into the state-space formulation by augmenting the matrices and vectors in Eqs. 49 and 50. This procedure, however, will not be included here because the model contains the defect that the parameters in the Heaviside expansion have physical meaning whereas the parameters in the entire function series expansion do not. The utility of the expansion, therefore, is questionable in parametric inverse experiments.

6. DISCUSSION

In attempts to obtain parametric inverses by use of the complex resonances of a scattering object, efforts to date have been subject to two limitations: First, the algorithms have been confined to SISO systems and, therefore, have not accounted in any systematic manner for the pole invariance with monitoring position around the scattering object. The state-space formulation given above for SIMO systems removes this limitation and, therefore, has potential importance in the parametric inverse problem. Second, present algorithms ignore the contribution of the entire function. The state-space formulation, while able to incorporate the entire function in the context of system identification, does not improve on this limitation for the parametric inverse. As pointed out in Refs. 20 and 21, the difficulty is inherent to the form of the scattering solution in Eq. 41. Note in Eqs. 31 and 41 that the Heaviside expansion is parametrized by the poles and residues whereas the entire function is not parametrized at all. Because of this fact, parametric models continue to be based on an incomplete model of the transient scattering system. It is essential to progress in the parametric inverse to find solutions that parametrize the entire function. Indeed, the entire function often dominates the transient waveform (Refs. 22, 23). Since the entire function is important in early time (Ref. 21), object features such as radius of curvature and physical optics reflection coefficient should be considered.

VI. CONCLUSIONS AND RECOMMENDATION

In this report, we have considered multi-aspect identification. We began with a description of the difference equation model for SISO systems and have described its extension to the SIMO case. We next used a canonical problem, acoustic scattering by a hard sphere, to examine identification of SEM poles at multiple scattering locations. We find that, despite the use of modern techniques, SIG and NLS, we obtain at best uneven results as a function of observation point.

To improve identification results, we have proposed a state-space formulation of the SIMO case. We recommend for future work development of a state-space algorithm, followed by an exhaustive set of identification experiments, first with simulated data and then with data from an electromagnetic transient range. We caution, however, that dramatic progress in this regard may depend on our ability to find methods to parametrize the entire function.

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