

Mathematics Notes

Note 88

January 1988

SIGNAL PARAMETER ESTIMATION FROM
SPECTRAL MAGNITUDE DATA*

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ABSTRACT

An efficient algorithm to estimate the natural frequencies and the residues of a system from spectral magnitude data is presented. The transformation of the spectral magnitude response into the autocorrelated sequence has made it possible to use existing time-domain, maximum likelihood, pole estimators and to suppress the noise which originally contaminates the magnitude data. By the condition of stability the poles are uniquely identified and the residues are estimated by the imposition of the minimum-phase condition. The estimation of the residue magnitudes using an approximation technique is also discussed.

* Research for this paper was sponsored by the office of Energy Storage and Distribution of the Department of Energy (DOE), under Contract DE-AC-5-84OR21400 with Martin Marietta Energy Systems, Inc. as operator of the Oak Ridge National Laboratory.

Section 1 Introduction

Experimental techniques used to obtain the response at the terminals of important equipment within a complex facility illuminated by electromagnetic fields can be both difficult and costly. If transient fields are used, signal-to-noise levels are often a problem. If continuous wave (CW) fields are used, both amplitude and phase data are normally required. Amplitude data can be easily measured, but accurate phase information is significantly more difficult to obtain correctly. Also, the requirement for phase information greatly complicates the data acquisition system and subsequent data analysis.

In this paper, existing techniques to reconstruct the signal without phase information are examined in section 2. A new method for signal reconstruction is formulated and the effects of noise in the data and methods of using noisy data are discussed in sections 3 and 4. Example cases are considered in section 5 and conclusions are presented in section 6.

Section 2 Signal Reconstruction Algorithms

The system parameters can be directly estimated by using the least squares method to fit an appropriate curve for the complete frequency response data [1]. In many practical situations, however, the complete information about the spectral data may either be difficult to obtain or unavailable. For instance, in optical image processing one must reconstruct the images from the intensity measurements of the spectrum without the knowledge of phase spectrum; some of the measurements in electromagnetic scattering experiments are given in the form of magnitude spectral density over a certain frequency interval; a digital filter may be specified by only the magnitude characteristic, ... etc. The desire to reconstruct the signal from insufficient information about the Fourier spectrum has resulted in the development of many techniques. Among those techniques two main approaches are eminently identified. One is the Gerchberg-Saxton algorithm (GSA) and the other is a nonlinear optimization technique known as the Steiglitz-McBride algorithm (SMA).

2.1 Gerchberg-Saxton algorithm [2]

Basically the GSA is an iterative algorithm that fully utilizes the imposed constraints existing in both the time and frequency domains. Fig.1 illustrates a common approach for the iterative scheme. The GSA algorithm begins with an initial guess of the desired phase. At the k th

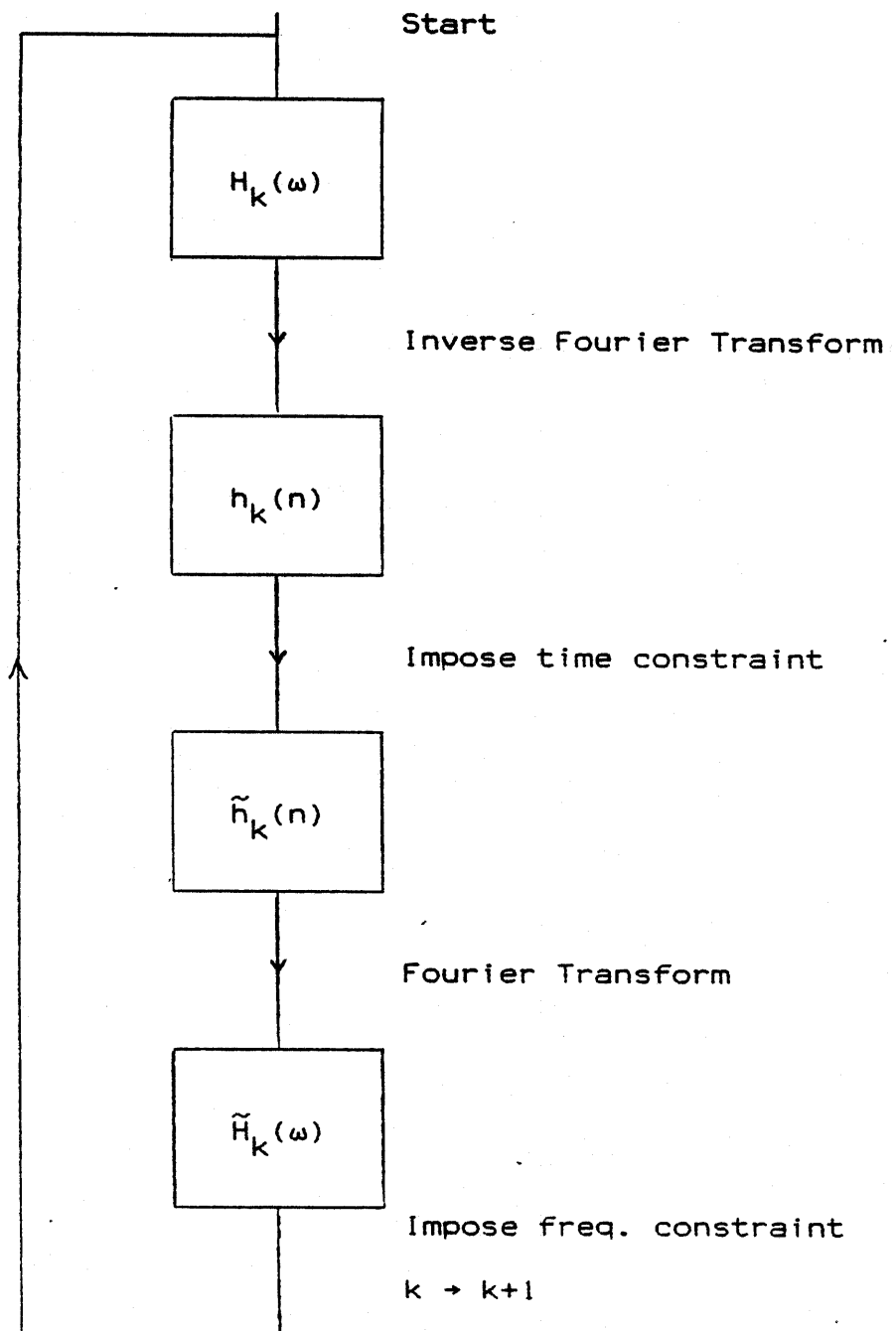


Fig.1 Iterative Reconstruction Scheme

iteration, the time constraint is imposed on the signal estimate $h_k(n)$ and this step yields $\tilde{h}_k(n)$. The algorithm again imposes the frequency constraint on the $\tilde{H}_k(\omega)$, the Fourier transform of $\tilde{h}_k(n)$. Thus $\tilde{H}_k(\omega)$ is replaced by $H_k(\omega)$. With the increase of an index by 1, the whole procedure is repeated until convergence occurs. Reference [2] originally proposed the phase reconstruction scheme when the constraints in both the time and frequency domains were given by the known magnitudes. Several modified versions of the Gerchberg-Saxton algorithm have been studied by giving a different set of constraints [3,4].

Despite the fact that there are always ambiguities when the phase is retrieved from the spectral magnitude data [2,5,6], some recent works have revealed that the signal can be uniquely specified by the additional information to the spectral magnitude [7,8]. The uniqueness problem may not be severe in multi-dimensional signal reconstruction [3,9,10]. GSA has been attractive because of its simplicity in structure of the algorithm, but both the less reliable capability against noise and the slow convergence are problems in reconstructing the signal [2,3,5,6,11].

2.2 Steiglitz-McBride algorithm

The Steiglitz-McBride algorithm (SMA) [12,13,14] is a nonlinear, least-squares, parameter identification technique which estimates the system parameters in the time domain from noisy data. Likewise SMA can be applied to estimate the parameters in the frequency domain when the data set is

the magnitude spectrum. The procedure described in [16] uses a least square approximation based on an iterative solution to fit an appropriate curve for the given magnitude response data. To determine the parameters of the transfer function

$$F(s) = \frac{\sum_{i=0}^M c_i s^i}{\sum_{j=0}^N d_j s^j},$$

Jong & Shanmugam [16] used the magnitude squared function

$$G(\omega^2) = |F(j\omega)|^2$$

$$= \frac{\sum_{i=0}^M a_i \omega^{2i}}{\sum_{j=0}^N b_j \omega^{2j}}.$$

After the parameters a_i 's and b_j 's are estimated using the SMA, the original parameter set can be found under the assumption that the system is stable and is minimum phase. It has been reported [16] that when the technique is applied to spectra having resonant peaks with large Q, the iterative solution becomes highly oscillatory. Unlike the time-domain SMA, the frequency-domain SMA has difficulty in analyzing the noise statistics. Because the magnitude squared data is no longer Gaussian even if the original magnitude data was

Gaussian. Note that the SMA was applied to estimate the parameters in the frequency domain directly while the GSA utilized both the time and frequency domain constraints repeatedly until the convergence was obtained.

In addition to the two techniques which have been developed to reconstruct the signal from the magnitude spectrum, the cepstral analysis method is used for reconstructing the signal where the cepstrum is defined as the sequence of inverse Fourier transform of the log-magnitude spectrum. That is, for a minimum-phase signal the log-magnitude of the spectrum is related to the phase spectrum through the Hilbert transform [5,17]. However if the spectral magnitude of certain sampling points is too small or happens to be zero, one may not be able to use those sampling points for finding the cepstrum. In that sense the application of the cepstral analysis technique may be considerably limited especially when the measurements are noisy. The other technique is rooted from the application of the Prony method to the magnitude-squared data in the frequency domain [18]. It is known that the Prony method provides the accurate solution when the data is noise-free, but it loses its reliability for estimating the system parameters when the data is noisy [19].

It seems that the noise which is interfering with the frequency data makes the estimation problem much more complicated than does the noise in the time domain. The method proposed in this paper first transforms the uniformly

sampled energy spectral data, which are obtained by squaring the magnitude data, into its auto-correlated sequence in the time domain. As the characteristic equation for this correlated sequence has roots consisting of system poles and their conjugate reciprocals, the poles are uniquely determined by the stability condition. Because of the symmetry in the characteristic equation coefficients, without actually doubling the order of the equation, any kind of existing time domain linear prediction algorithm such as the covariance method or the SVD(Singular Value Decomposition)- based methods can be applied to extract the poles. We also use the existing time domain iterative pole estimation algorithm which works well when the data contain the measurement noise.

Section 3 Mathematical Formulation

Suppose that the continuous-time system response $h(t)$ is expressed as

$$h(t) = \sum_{j=1}^K r_j \exp(s_j t), \quad (1)$$

then for the predetermined sampling period T the n -th sampled data $h_n = h(t)|_{t=nT}$ is expressed as

$$h_n = \sum_{j=1}^K r_j \exp(s_j T)^n. \quad (2)$$

The z -transform of the sequence $\{h_n, n=0,1,\dots\}$ is defined as

$$\begin{aligned} H(z) &= \sum_{n=0}^{\infty} h_n z^{-n} \\ &= \sum_{j=1}^K \frac{r_j}{1-p_j z^{-1}} \end{aligned} \quad (3)$$

where $p_j = \exp(s_j T)$ are the simple poles in the z -transform domain and r_j are the residues of the system.

The problem is formulated as follows:

Given the uniformly sampled spectral magnitude data

$$H_m = |H(z=e^{jm\pi}/M)|, \quad m=0,1,\dots,M-1, \quad (4)$$

it is desired to extract the poles and the zeros.

Instead of finding the $2K$ unknowns $\{r_j, p_j; j=1,2,\dots,K\}$, one first identifies the different set of $2K$ unknowns $\{A_0, z_1, \dots, z_{k-1}, p_1, \dots, p_k\}$ in the equivalent pole-zero model

$$H(z) = \frac{A_0 \prod_{i=1}^{K-1} (1 - z_i z^{-1})}{\prod_{i=1}^K (1 - p_i z^{-1})} \quad (5)$$

and then residues are found from the relationship

$$r_i = H(z) (1 - p_i z^{-1}) \Big|_{z=p_i} \quad (6)$$

Direct attempts to find the parameters from the spectral magnitude data $\{H_m\}$ lead to a nonlinear optimization problem. This approach begins with the approximation of $H(z)$ to the finite impulse response model

$$H(z) = \sum_{m=0}^{M-1} h_m z^{-m} \quad (7)$$

The approximation is justified only when $\sum_{m=M}^{\infty} |h_m|$ is negligibly small. Fortunately selecting a large M does not make the computations heavily burdensome; rather, it helps to suppress the noise level. The approximation enables one to take advantage of the simplicity of the discrete Fourier transform. This treatment uses (7) as an alternative expression of $H(z)$ under the assumption that (7) is a good approximation to $H(z)$. It is well known that the energy spectrum is related to the auto-correlation sequence via the discrete Fourier transform (DFT) [17]. Formally this yields

$$H(z)H(1/z) = \sum_{m=-M+1}^{M-1} R_m z^{-m} \quad \text{with } R_m = R_{-m} \quad (8)$$

A sample auto-correlated sequence $\{R_m, m=0,1,2,\dots,M-1\}$ can be obtained by taking the DFT of the known energy spectral data, which are generated by squaring the magnitude data. The next step is to extract the poles from the sample auto-correlated sequence. The system model (5) can be written as

$$H(z) = A(z)/B(z)$$

$$= \frac{\sum_{i=0}^{K-1} a_i z^{-i}}{\sum_{i=0}^K b_i z^{-i}} \quad \text{with } b_0 = 1. \quad (9)$$

The expression for $H(z)H(1/z)$ directly from (9) can be decomposed into the right-sided and the left-sided sequences, i.e., there exists a unique polynomial $G(z) = \sum_{i=1}^K g_i z^{-i}$ such that

$$H(z)H(1/z) = \frac{G(z)}{B(z)} + R_0 + \frac{G(1/z)}{B(1/z)}. \quad (10)$$

Comparing (8) with (10), one obtains

$$\frac{G(z)}{B(z)} = \sum_{m=1}^{M-1} R_m z^{-m}. \quad (11)$$

The sequence $\{R_m, m=1,2,3,\dots\}$ can be thought of as an impulse response of an unknown system whose poles are identical to those which are to be found. Finding the system poles is equivalent to identifying the set of coefficients $\{b_i\}$. Equation (11) can be written in terms of the difference equation

$$\sum_{i=0}^{j-1} R_{j-i} b_i = g_j, \quad j=1,2,\dots,K \quad (12a)$$

$$\sum_{i=0}^K R_{j-i} b_i = 0, \quad M+K-1 \geq j \geq K+1. \quad (12b)$$

The homogeneous equation (12b) can be used for extracting the poles. Note that $G(z)$ does not have any direct role in finding the poles. Not all M data points are used for pole extraction. Instead, the first L points are used for the following reasons: a) considerable reductions are achieved in computations, b) relative errors due to truncation of the infinite sequence are in general smaller in the first part of the sequence. Using the first L samples $\{R_i, i=0,1, \dots, L-1\}$, the equation (12b) can be rewritten in the matrix equation form

$$\begin{bmatrix} R_1 & R_2 & \dots & R_{K+1} \\ R_2 & R_3 & \dots & R_{K+2} \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ R_{L-K} & R_{L-K+1} & \dots & R_L \end{bmatrix} \cdot \begin{bmatrix} b_K \\ b_{K-1} \\ \cdot \\ \cdot \\ b_1 \\ 1 \end{bmatrix} = \underline{0}. \quad (13)$$

Matrix equation (13) can be solved with respect to $\{b_i\}$ using least squares method [20,21] or SVD-based methods [22,23,24].

The next phase is to find the coefficients of $A(z)$, $\{a_i\}$. Let

$$\alpha_i = \sum_{k=0}^{K-i-1} a_k a_{i+k}, \quad 0 \leq i \leq K-1, \quad (14a)$$

$$\beta_i = \sum_{k=0}^{K-i} b_k b_{i+k}, \quad 0 \leq i \leq K. \quad (14b)$$

Then from (8) and (9), the following set of equations are obtained:

$$\sum_{k=-K}^K R_{k+i} \beta_k = \alpha_i \quad \text{for } i=0,1,\dots,K-1 \quad (15)$$

where $\beta_i = \beta_{-i}$.

Once $\{\beta_i\}$ are identified, $\{\alpha_i\}$ can be calculated using the equation (15). The roots of $\sum_{i=-K+1}^{K-1} \alpha_i z^{-i}$, where $\alpha_i = \alpha_{-i}$, consist of zeros of the unknown system and their conjugate reciprocals. The contribution to the spectral magnitude by the zero at $z=z_0$ is the same as that by the zero at $z=1/z_0^*$. Quantitatively, $||-z_0 z - 1|| = ||-z_0^* z||$ when these are evaluated on the unit circle. If one selects a zero ($z=z_0$) as a root of $A(z)$, the conjugate reciprocal zero ($z=1/z_0^*$) is the root of $A(1/z)$ and vice versa. The ambiguities in selecting the zeros result from the insufficient spectral

information originally given. The minimum phase condition can be imposed to resolve the ambiguities. That is, not only poles but zeros of $H(z)$ are restricted to lie inside the unit circle. For more discussions see [6]. Once $\{\alpha_j, j=0,1,\dots,K-1\}$ and $\{z_j, j=1,2,\dots,K-1\}$ are in hand, the scale factor A_0 can be found by comparing the highest order in the numerator of $H(z)H(1/z)$. That is,

$$(-1)^{K-1} \left\{ \prod_{i=1}^{K-1} z_i \right\}^2 A_0 = \alpha_{K-1}. \quad (16)$$

The positive A_0 is chosen by the same argument as above. The residues can be obtained using the equation (6) after $H(z)$ is identified.

Now consider the problem of noise in measuring the spectral magnitudes, one has

$$D_m = H_m + v_m \quad \text{for } 0 \leq m \leq M-1, \quad (17)$$

where H_m is the true spectral magnitude as expressed in (4) and v_m is a zero-mean uncorrelated Gaussian noise sequence with variance σ^2 . Then

$$D_m^2 = H_m^2 + 2 H_m v_m + v_m^2 \quad \text{for } m=0,1,2,\dots,M-1$$

$$\text{and } H_j = H_{N-j}, \quad v_j = v_{N-j} \quad \text{when } M \leq j \leq N-1. \quad (18)$$

Here one sets $N = 2M$. Let the inverse DFT of the corrupted

data $\{D_m^2\}$ be $\{P_m\}$ and denote $W = \exp(-j2\pi/N)$. Then

$$P_j = R_j + n_j, \quad (19)$$

where $n_j = \epsilon_{1j} + \epsilon_{2j}$ (20a)

and $\epsilon_{1j} = \frac{2}{N} \sum_{m=0}^{N-1} H_m v_m W^{-jm},$ (20b)

$$\epsilon_{2j} = \frac{1}{N} \sum_{m=0}^{N-1} v_m^2 W^{-jm} \quad (20c)$$

where ϵ_{1j} is the first noise factor and ϵ_{2j} is the second noise factor. If M is sufficiently large, it can be shown that the noise $\{n_j\}$ which contaminates the sample auto-correlated sequence $\{R_j\}$ is jointly Gaussian with second moments

$$\begin{aligned} E\{n_j^2\} &\cong E\{\epsilon_{1j}^2\} \\ &= \frac{2\sigma^2}{M} (R_0 + R_{2j}) \quad \text{for } j > 0 \end{aligned} \quad (21a)$$

and

$$\begin{aligned} E\{n_j n_m\} &\cong E\{\epsilon_{1j} \epsilon_{1m}\} \\ &= \frac{2\sigma^2}{M} (R_{j+m} + R_{j-m}) \quad \text{for } j \neq m. \end{aligned} \quad (21b)$$

The second noise factor has little effect on $\{R_j\}$, and $E\{n_j^2\}$ is relatively much greater than $E\{n_j n_m\}$. It is safe to assert that for large enough M the noise is approximated to be uncorrelated Gaussian. Thus the estimation of the system poles from the noisy transformed data described in

the equation (19) can be put into the general category of maximum likelihood (ML) estimation. The Iterative Preprocessing Algorithm (IPA) [15], which is based upon ML estimation may be used, for extracting the poles from the noisy auto-correlated sequence.

Section 4 Residue Estimation

An important property of the ML estimator is the invariance property. Formally,

$$f(\hat{a}) = \hat{f}(a) \quad (22)$$

where " $\hat{\cdot}$ " denotes the ML estimate. Now the coefficients of the denominator polynomial of the transfer function, $\{\hat{b}_i\}$ are estimated using the IPA. The pole estimates $\{\hat{p}_i\}$ are the roots of the polynomial $\sum_{j=0}^K \hat{b}_j z^{-j}$ by the invariance property of the ML estimator. To find the estimate of the residues, one begins with the pole-residue expression of equation (11)

$$\frac{zG(z)}{B(z)} = \sum_{i=1}^K \frac{\xi_i}{1-p_i z^{-1}} \quad (23)$$

where ξ_i is the residue associated with p_i in the rational function $zG(z)/B(z)$. The estimation of $\{\xi_j\}$, $\hat{\xi}_j$, can be found by combining equation (23) with equation (11).

$$\sum_{j=1}^K R_j z^{-j} = \sum_{j=1}^K \frac{\xi_j z^{-1}}{1-p_j z^{-1}} \quad (24)$$

Comparing the coefficients on both sides of equation (24) and using the vector notation, one can simplify it in a matrix form:

$$\underline{R} = C(\underline{p}) \underline{\xi} \quad (25)$$

where

$$\underline{R} = [R_1 \ R_2 \ \dots \ R_L]^t,$$

$$\underline{\xi} = [\xi_1 \ \xi_2 \ \dots \ \xi_K]^t$$

and

$$C(\underline{p}) = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ p_1 & p_2 & p_3 & \dots & p_K \\ \vdots & \vdots & \vdots & \dots & \vdots \\ p_1^{L-1} & p_2^{L-1} & \dots & p_K^{L-1} \end{pmatrix}.$$

Equation (19) in a vector form can be written as

$$\underline{n} = \underline{p} - C(\underline{p}) \underline{\xi}. \quad (26)$$

Selecting the value of $\hat{\underline{\xi}}$ which minimizes the L_2 -norm of (26) leads to

$$\hat{\underline{\xi}} = \{ C(\hat{\underline{p}})^t C(\hat{\underline{p}}) \}^{-1} C(\hat{\underline{p}})^t \underline{p}. \quad (27)$$

Equating (12) to (25) and applying the invariance property of ML estimator

$$\sum_{j=1}^K \hat{g}_j z^{-j} = z^{-1} \sum_{j=1}^K \hat{\xi}_j \prod_{\substack{i=1 \\ i \neq j}}^K (1 - \hat{p}_i z^{-1}). \quad (28)$$

Using the equation (10), (12) and (15), one can show that

$$\hat{R}_0 = - \hat{g}_K / \hat{b}_K, \quad (29)$$

$$\hat{\alpha}_0 = \hat{R}_0 \hat{\beta}_0 + 2 \sum_{i=1}^K \hat{b}_i \hat{g}_i, \quad (30a)$$

and

$$\hat{\alpha}_j = \hat{R}_0 \hat{\beta}_j + \sum_{i=0}^{K-j-1} \hat{b}_{i+j+1} \hat{g}_{i+1} + \sum_{i=0}^{K-j} \hat{b}_{K-j-1-i} \hat{g}_{K-j-i} \quad (30b)$$

for $j=1, 2, \dots, K-1$

where

$$\hat{\beta}_i = \sum_{j=0}^{K-i} \hat{b}_j \hat{b}_{i+j}. \quad (31)$$

The estimates of zeros are selected as those located inside the unit circle on the complex z-transform plane among the roots of $\sum_{j=-K+1}^{K-1} \hat{\alpha}_j z^{-j}$ by imposing the minimum-phase condition. The estimate of the scale factor A_0 can be found from (16):

$$\hat{A}_0^2 = \hat{\alpha}_{K-1} / (-1)^{K-1} \left(\prod_{i=1}^{K-1} \hat{z}_i \right) \quad (32)$$

The denominator of the right hand side of the equation (32) has the same sign as $\hat{\alpha}_{K-1}$ so that \hat{A}_0 is real. There exists also an ambiguity in selecting A_0 . We choose the positive value of A_0 again by imposing a minimum-phase condition. Once the estimates of poles and zeros are obtained, the residue estimates can be found from equation (5),

$$\hat{r}_i = \hat{H}(z) (1 - \hat{p}_i z^{-1}) \Big|_{z=\hat{p}_i}. \quad (33)$$

Section 5 Numerical Examples

5.1 Two simple examples

To illustrate the fitness of the proposed technique this algorithm is applied to the couple of examples. As the first example the system is assumed to be described by

$$H(z) = \frac{2-1.8/z}{1-1.8/z+.97/z^2} .$$

The system has two poles $p_{1,2} = .9 \pm j .4$ which are located close to the unit circle (radius = .984886). The noiseless spectral magnitude data is taken by sampling the $H(z)$ uniformly in angle at the points on the unit circle, $z = \exp(jm\pi/M)$, $m=0,1,2,\dots,M-1$. Poles are extracted using the covariance method from the transformed auto-correlated sequence. The results are shown in Table 1. The estimated values of poles are shown as the data length M varies.

M	pole (estimated)	order of error
16	.91165 ± j .38162	10 ⁻²
32	.89981 ± j .38533	10 ⁻²
64	.90063 ± j .40013	10 ⁻⁴
128	.90000 ± j .40001	10 ⁻⁵
256 or more	.90000 ± j .40000	<10 ⁻⁵

Table 1. Pole estimation from the auto-correlated sequence

The errors of the estimated poles for the smaller M are due to truncation. In fact it has been observed that the poles are accurately found regardless of the pole locations whenever the spectral data do not contain noise.

As the second example the fourth order transfer function is chosen

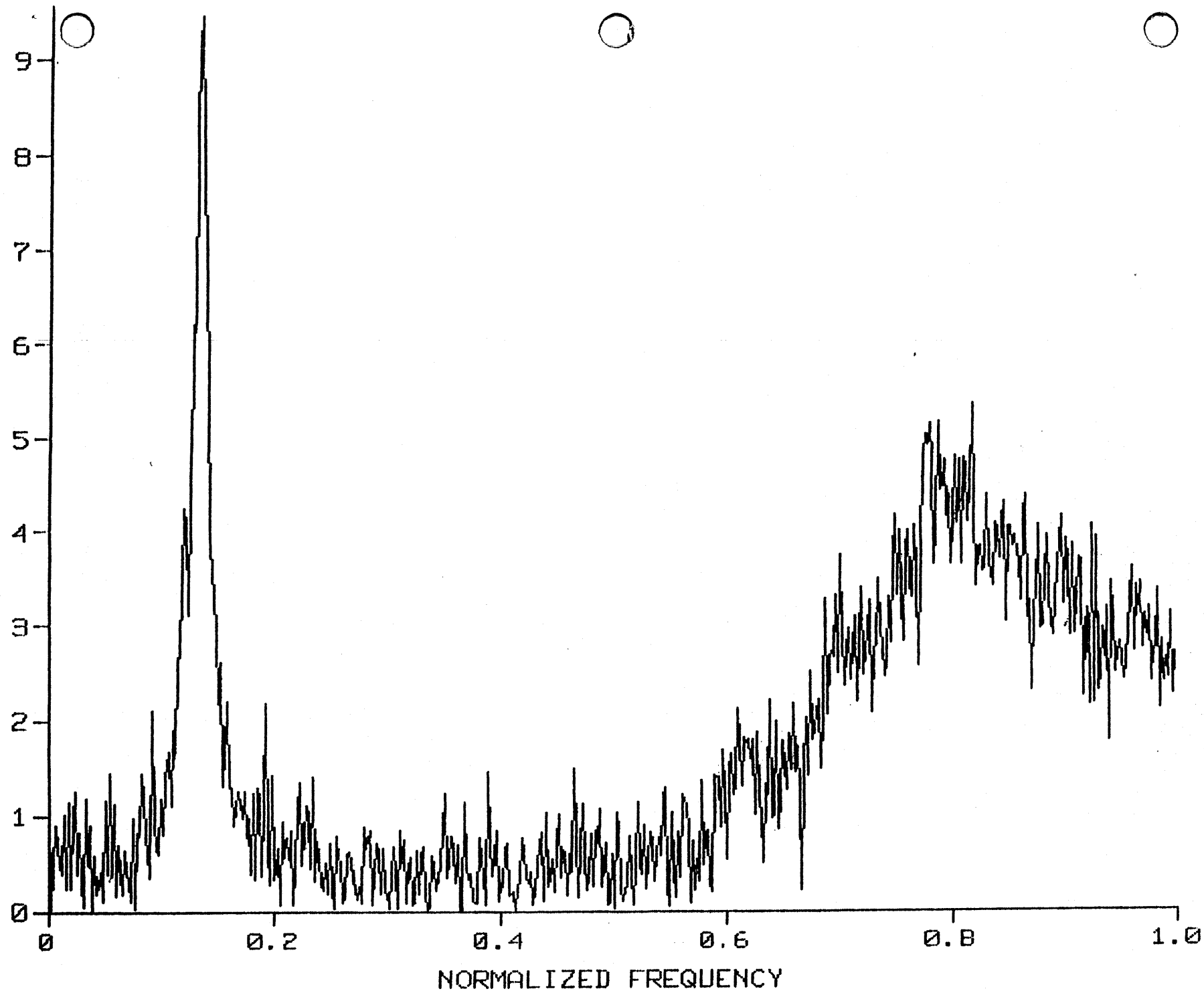
$$H(z) = \frac{(1-.8z^{-1})(1-.8z^{-1} + .65z^{-2})}{(1-1.8z^{-1} + .97z^{-2})(1+1.2z^{-1} + .61z^{-2})}$$

where the poles are: $p_{1,2} = .9 \pm j .4$, $p_{3,4} = -.6 \pm j .5$ and zeros are: $z_1 = .8$, $z_{2,3} = .4 \pm j .7$. The magnitude spectrum is shown in Fig.2 when the number of sampling data points M are equal to 2^{12} with noise level $\sigma = .5$. The results for the various values of the noise level are summarized in Table 2 for the same M. It is seen that the poles are less sensitive to the noise than the residues.

5.2 Validity of minimum phase signal

The two examples which have been chosen in the previous experiment happened to have minimum-phase transfer function. One may have a question at this point; is it valid in general to impose the minimum-phase condition over the practical situations? To the authors' knowledge, it is not possible to explain anything about the validity analytically. Literature seems to assume the minimum-phase condition whenever there is an ambiguity about the phase.

MAGNITUDE SPECTRUM



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Table 2. Pole-residue estimation for the noises

Noise(σ)	pole	zero	residue
.2	(.89994, \pm .40003)	.80534	(.13785, \pm .01265)
	(-.59776, \pm .50076)	(.39690, \pm .69045)	(.36881, \pm .82410)
.5	(.89979, \pm .40008)	.81499	(.13901, \pm .00932)
	(-.59452, \pm .50238)	(.39550, \pm .68903)	(.37187, \pm .82860)
.7	(.89966, \pm .40011)	.82254	(.13994, \pm .00803)
	(-.59245, \pm .50378)	(.39660, \pm .69639)	(.36995, \pm .83467)
1.	(.89940, \pm .40015)	.83548	(.14161, \pm .00771)
	(-.58949, \pm .50636)	(.40226, \pm .72153)	(.36009, \pm .84918)
1.2	(.89919, \pm .40017)	.84509	(.14292, \pm .00912)
	(-.58763, \pm .50840)	(.41056, \pm .75119)	(.34697, \pm .86401)
1.5	(.89882, \pm .40020)	.86080	(.14461, \pm .02120)
	(-.58500, \pm .51194)	(.45541, \pm .86453)	(.29364, \pm .91421)

** Only L=32 points are used for parameter estimation. The true residues are (.13717, \pm .01584) and (.36284, \pm .82417).

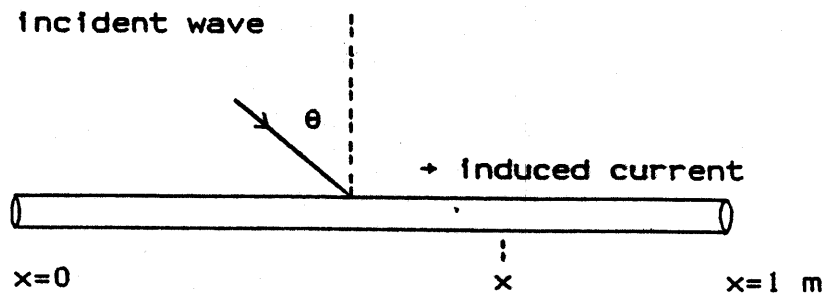


Fig.3 Incident wave to the thin wire

Now consider the electromagnetic interaction problem shown in Fig.3; the incident wave impacts on the thin wire with the incident angle θ . The induced current can be measured at some points on the wire. If the incident wave is of CW (Continuous Wave) type, one can obtain the corresponding spectral amplitude accurately from the steady state response. For a given angle of incidence and an observation point, the amplitude spectral data is collected over the frequency range of interest. The poles should be the same regardless of the incident angle and the observation point, but the residues are determined by the product of the natural mode, the normalization factor and the coupling coefficients. The reader may refer to [15] and its references for more discussions. It is interesting to see how much the true residues are different from the residues estimated from the minimum phase solution. The same poles used in [15] are chosen here, i.e.,

$$p_{1,2} = .5589 \pm j.7325,$$

$$p_{3,4} = -.2831 \pm j.8396,$$

$$p_{5,6} = -.8320 \pm j.2237.$$

The magnitude of the complex residues are shown in Table 3. From the algorithm described in Section 3 and 4, the poles and the residues of the minimum phase solution were found. Table 4 illustrates whether the true residues for the specified incident angle(θ_i) and observation point(x_j) are of minimum phase. If they are of nonminimum phase, the table also shows the locations where the relative error between the minimum phase solution and the true one is within 5%. If the error is greater than 5%, an 'X' has been placed on the corresponding blank. $\{\theta_i\}$ and $\{x_j\}$ are chosen 0, 20, 35, 48, 70 degree and .16, .245, .5, .755, .84 m respectively (the wire length = 1m).

Table 3. Residue magnitude

		x_1	x_2	x_3	x_4	x_5
θ_1	res(1)	3.81401	5.20688	7.33327	5.20688	3.81401
	res(2)	3.06062	2.17525	2.93820	2.20582	3.06062
	res(3)	0.00869	0.09755	0.00390	0.09755	0.08585
θ_2	res(1)	3.50894	4.79040	6.74671	4.79040	3.50894
	res(2)	0.08270	0.05877	0.07939	0.05960	0.08270
	res(3)	0.33033	3.70711	0.14828	3.70711	3.26267
θ_3	res(1)	2.89880	3.95744	5.57359	3.95744	2.89880
	res(2)	3.14411	2.23458	3.01835	2.26600	3.14411
	res(3)	0.43460	4.87736	0.19509	4.87736	4.29262
θ_4	res(1)	2.28886	3.12475	4.40084	3.12475	2.28886
	res(2)	4.13475	2.93865	3.96936	2.97996	4.13475
	res(3)	0.39994	4.48834	0.17953	4.48834	3.95024
θ_5	res(1)	1.06860	1.45885	2.05463	1.45885	1.06860
	res(2)	2.48395	1.76540	2.38459	1.79021	2.48395
	res(3)	0.20896	2.34511	0.09380	2.34511	2.06396

Table 4. Comparison of the minimum phase solution to the true one

	x_1	x_2	x_3	x_4	x_5
θ_1	0	0	5%	0	0
θ_2	0	X	0	0	0
θ_3	5%	X	0	X	X
θ_4	5%	X	0	X	5%
θ_5	5%	X	0	5%	5%

The 11 cases out of 25 different combinations of (θ_i, x_j) are of minimum phase and 7 cases are close to the true residues with less than 5% error. The 18 cases out of 25 observations (11 minimum-phase cases plus 7 observations within the 5% error) show the closeness to the true residues. This is equivalent to saying that the residues can be estimated with the reliability of .7 ($\cong 18/25$) in this example.

5.3 Residue magnitude extraction using an approximation technique

The impulse response $\{h_i\}$ can be expressed as

$$h_i = \sum_{k=1}^K r_k p_k^i. \quad (34)$$

Similarly the sample auto-correlated sequence can be expressed as

$$R_n = \sum_{k=1}^K C_k p_k^n. \quad (35)$$

where C_k can be found from equation (11),

$$C_k = \frac{G(z)}{B(z)} (1 - p_k z^{-1}) \Big|_{z=p_k}. \quad (36)$$

Thus $R_n = \sum_{i=0}^{\infty} h_i^* h_{i+n}$ can be simplified to

$$R_n = \sum_{k=1}^K \sum_{j=1}^K \frac{r_j^* r_k}{1 - p_j^* p_k} p_k^n. \quad (37)$$

Comparing equation (35) with equation (37) we find

$$\begin{aligned} C_k &= \sum_{j=1}^K \frac{r_j^* r_k}{1 - p_j^* p_k} \\ &= \frac{|r_k|^2}{1 - |p_k|^2} + \sum_{\substack{j=1 \\ j \neq k}}^K \frac{r_j^* r_k}{1 - p_j^* p_k}. \end{aligned} \quad (38)$$

It is not possible to solve equation (38) explicitly with respect to $\{r_k\}$. The first term of the left hand side of the equation (38) is dominant if the poles are well separated from one another. Thus under this condition C_k can be approximated to

$$|C_k| \cong \frac{|r_k|^2}{1 - |p_k|^2}. \quad (40)$$

Hence

$$|r_k| \cong [(1 - |p_k|^2) |C_k|]^{1/2}. \quad (41)$$

Table 5 shows how close the solutions based on the approximation are to the true ones.

Table 5. Comparison of the 1st order approximation to the true ones

	x_1	x_2	x_3	x_4	x_5
θ_1	5%	5%	5%	5%	5%
θ_2	5%	X	5%	10%	10%
θ_3	5%	X	5%	X	X
θ_4	5%	X	5%	X	X
θ_5	5%	X	5%	X	X

Although only 13 cases out of 25 are close to the true residue magnitudes with less than 5% error, all the rest of the observations have the errors bounded to 20%.

Section 6 Conclusion

This paper presents an efficient method of determining the system parameters from spectral magnitude data. Uniformly sampled energy spectral data, which is obtained by squaring the magnitude of the measured data, is transformed into its auto-correlated sequence in the time domain. This permits the suppression of noise which contaminates the magnitude data and allows the use of existing time-domain estimation techniques. Although ambiguities are inherently involved in finding the residues due to the insufficient data originally available, they can be resolved by increasing the number of sampled points in frequency.

Several numerical examples were considered to test the new signal parameter estimation method developed in this paper. It is found that relatively small error can be obtained for signal parameters with a relatively small M , the number of data points, and the error can be reduced as M is increased. It is also found that poles are less sensitive to noise in the data than the residues. For the numerical example chosen in this paper, even significant noise did not result in error greater than 20% for the real part of the residues. And finally, the minimum phase solution was used to compute the residues of an electromagnetic interaction problems with less than 5% error in 18 cases out of 25.

Acknowledgement

Research for this paper was sponsored by the office of

Energy Storage and Distribution of the Department of Energy (DOE), under Contract DE-AC-5-84OR21400 with Martin Marietta Energy Systems, Inc. as operator of the Oak Ridge National Laboratory.

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