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Consequences of the Electron Equation of Motion

Pt I
Impulsive and Oscillatory Motions

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ABSTRACT
The equation of motion for a classical charged particle was developed in Physics Note No 5, and solutions were obtained for linear motion in a constant electric field, and for motion in a constant magnetic field. Further solutions to the non-linear equations are obtained by inverting the problem and seeking forces that result in a specified motion. Transformations of these solutions then lead to solutions for forces of more direct interest, in particular for impulsive and oscillatory motion. It is found that the radiation term introduces a phase delay between the force and the resultant acceleration and some forces result in discontinuous motion.

I. INTRODUCTION

An equation of motion for a classical charged particle that takes into account the radiation from the accelerating particle was developed in Physics Note No 5[1]. A few analytic results were obtained, in particular for linear motion in a constant electric field, and for motion in a constant magnetic field. A principal result for linear motion under a constant force is that the radiated energy is a factor of ~ 34 higher than would be calculated by the standard approach of ignoring the radiation in calculating the motion, and then using this motion to calculate the radiation. The factor of 34 was obtained for both relativistic and non-relativistic motion, the greatest contribution to the radiation coming from the initial low velocity motion. The basis of the standard approach is the time scale associated with the radiation process ($\tau \sim 6.266\ 10^{-24}$ secs), and the results show, as many other researchers have demonstrated, that liberties are not to be taken with non-linear equations.

The possibility of obtaining further explicit analytic solutions for given forces is small, and so the process is reversed. Selecting a particular motion leads to a specification of the force required to generate that motion. The avoidance of singularities in the force so determined requires that the motions that are considered have accelerations that vanish with the velocity. This method is applied to both impulsive and oscillatory motions. These solutions lead to a general result limiting the energy that can be imparted to an electron.

The equation for linear motion has implications of a fundamental nature for electrons undergoing negative acceleration, a quantum-like discontinuity in the motion being predicted. Though this effect and the other restrictions on the motion would be exceedingly hard to detect there are circumstances where the effects would become significant, and these are explored in the final section of this paper.

II IMPULSIVE MOTIONS

The non-relativistic equation for rectilinear motion is[1]

$$\dot{v} + \tau \frac{\dot{v}^2}{v} = \frac{f}{m}$$

If we assume an acceleration of the form

$$\dot{v} = a e^{-\alpha t}$$

we have

$$v = \frac{a}{\alpha}(1 - e^{-\alpha t})$$

and substituting these into the n-r equation

$$a e^{-\alpha t} + \frac{\alpha \tau a e^{-2\alpha t}}{1 - e^{-\alpha t}} = \frac{f}{m}$$

Simplifying this result by choosing $\alpha\tau = 1$, the required force is given by

$$\frac{f}{m} = \frac{a e^{-\frac{t}{\tau}}}{1 - e^{-\frac{t}{\tau}}}$$

It is to be observed that the force is required to tend to infinity at t = 0, and that the impulse given by

$$I = \int_0^\infty f\,dt$$

also tends to infinity. Despite this the electron acquires only a finite velocity. The kinetic energy is

$$\mathcal{E}_k = \frac{1}{2} m a^2 \tau^2$$

The energy radiated during this motion is

$$\mathcal{E}_r = m\tau \int_0^\infty v^2\,dt = \frac{1}{2} m a^2 \tau^2$$

Selecting an impulsive acceleration that that leads to an impulsive force equivalent to a finite strength δ-function impulse, we choose

$$v = v_o e^{-\frac{\alpha}{t}}$$

$$\dot{v} = \frac{v_o \alpha}{t^2} e^{-\frac{\alpha}{t}}$$

Substitution leads to

$$\frac{f}{m} = v_o \left[\frac{\alpha}{t^2} + \frac{\alpha^2 \tau}{t^4} \right] e^{-\frac{\alpha}{t}}$$

The ultimate kinetic energy is clearly

$$\mathcal{E}_k = \frac{1}{2} m v_o^2$$

while the radiated energy is

$$\mathcal{E}_r = m\tau \int_0^\infty v^2\,dt = \frac{1}{4} \frac{m\tau}{\alpha} v_o^2$$

The impulse is

$$I = \int_0^\infty f\,dt = m\,v_o [1 + \frac{2\tau}{\alpha}]$$

In the limit of the acceleration tending to a δ-function we observe that the impulse tends to a δ-function of infinite strength, the radiated energy tends to infinity and the electron is accelerated to v_0 as before. The inference from these examples is that arbitrarily strong impulses result in a finite increase in kinetic energy for the electron while the radiated energy tends to infinity.

III 'HARMONIC' MOTION

We now consider a particle moving under a restoring force proportional to displacement obtaining the equation, for rectilinear motion

$$\dot{v} + \tau\frac{\dot{v}^2}{v} = -\frac{k}{m}x = -\omega^2 x$$

Noting that the equation is homogeneous, we insert a trial solution of the form

$$x = \exp\{\lambda t\}$$

and there results

$$\lambda^2 + \tau\lambda^3 + \omega^2 = 0$$

Expressing this equation in dimensionless form by introducing

$$u = \lambda\tau$$

we obtain

$$u^3 + u^2 + a^2 = 0$$

where in addition we make the replacement

$$a = \omega\tau$$

The discriminant of the cubic is[2]

$$\frac{a^4}{4} + \frac{a^2}{27} > 0$$

and so there is one real root and a pair of complex conjugates. Introducing

$$s_1 = \left\{\left[\frac{a^4}{4} + \frac{a^2}{27}\right]^{1/2} - \left[\frac{a^2}{2} + \frac{1}{27}\right]\right\}^{1/3}$$

$$s_2 = -\left\{\left[\frac{a^4}{4} + \frac{a^2}{27}\right]^{1/2} + \left[\frac{a^2}{2} + \frac{1}{27}\right]\right\}^{1/3}$$

the real root is

$$u_1 = s_1 + s_2 - \frac{1}{3}$$

while the complex roots become

$$u_{1,2} = -\frac{1}{2}(s_1 + s_2) - \frac{1}{3} \pm \frac{i\sqrt{3}}{2}(s_1 - s_2)$$

The real root just means that the motion decays very rapidly. Noting that a^2 is generally very small, it can be shown by expansion to second order in a that the approximate motion is given by

$$x \sim x_0 \exp\left\{-\frac{(1+\omega_0^2\tau^2)}{\tau}t\right\}$$

Additionally we have only one arbitrary constant and so we cannot specify position and velocity independently. Specifically we have

$$\dot{x}_0 = \frac{-(1+\omega_0^2\tau^2)}{\tau}x_0$$

The non-relativistic equation has been used and so we have the restriction

$$x_0 \ll \frac{c\tau}{1+\omega_0{}^2\tau^2}$$

This is a very severe restriction as $c\tau$ is of the order of a classical electron radius.

The non-linear nature of the differential equation means that we cannot combine the solutions to obtain a further solution, in particular one that eliminates the imaginary component. As the solution is exact, this is indicating that free oscillations of the trial form under the classic simple harmonic force are not possible for an electron! It can in fact be shown that no solution of the very general form

$$x = a(t)\cos\lambda(t)$$

exists.

The complex roots in fact indicate 2-D motion. Expansion of the roots gives

$$\lambda_{1,2} = -\omega_0^2\tau \pm i\omega_0$$

and the resulting motion is either positive or negative decaying spirals

$$r = r_0 e^{-\omega_0^2\tau t}$$

$$v_\theta = -\omega_0 x_0[1+\omega_0^2\tau^2]^{1/2}e^{-\omega_0^2\tau t}$$

The relativistic limitation now transfers to the frequency and we have the condition

$$\omega_0 \ll \frac{c}{x_0[1+\omega_0^2\tau^2]^{1/2}}$$

It is clear on physical grounds that 1-D oscillatory solutions must exist. The trial solutions are continuous together with their derivatives, and so we are led to assume that the solution must have discontinuities in at leastone of their first two derivatives, it being a physical requirement that the displacement is continuous. Reformulating the problem as an integral allows the solution to contain discontinuities, but the solution is implicit, leading to an approximation based on iteration. To do this we regard the differential equation as a quadratic in $\dot{v}$ and we obtain

$$\dot{v} = -\frac{v}{2\tau}\left\{1 - \sqrt{1 - \frac{4\tau k}{mv}x}\right\}$$

We can then write

$$v = v_0 \exp\left\{-\frac{1}{2\tau}\int_0^t \left[1 \pm (1 - \frac{4\omega_0^2 x\tau}{v})^{1/2}\right]\right\}dt$$

This solution must correspond to the radiationless case as $\tau \to 0$, and this requires that we adopt the negative sign. If we now consider an electron passing through x=0 with a velocity v_0 at t=0 the radiation rate will initially be very small and the solution will approximate to

$$v \sim v_0 \cos(\omega_0 t)$$

$$x \sim \frac{v_0}{\omega_0}\sin\omega_0 t$$

Substituting these approximate solutions into the implicit solution, there results

$$v \sim v_0 \exp\left\{-\frac{1}{2\tau}\int_0^t \left[1 - (1 - 4\omega_0\tau\tan\omega_0 t)^{1/2}\right] dt\right\}$$

Expanding the integrand to second order in τ, the solution reduces to

$$v \sim v_0 \exp\left\{-\int_0^t \tan(\omega_0 t)\, d(\omega_0 t) - \omega_0\tau\int_0^t \tan^2(\omega_0 t)\, d(\omega_0 t)\right\}$$

Carrying out the integrations and simplifying

$$v \sim v_0 \cos(\omega_0 t) \exp\left\{-\omega_0\tau[\tan(\omega_0 t) - \omega_0 t]\right\}$$

and we observe that this solution drops very rapidly to zero as $\omega_0 t$ approaches $\pi/2$. the limit of validity of the solution is somewhat less than $\pi/2$. Tthe radical in the implicit slotionimplies that the maximumvalue of $\omega_0 t$ is given by

$$4\omega_0\tau\tan(\omega_0 t) = 1$$

which has an approximate solution

$$\omega_0 t \sim \frac{\pi}{2} - 4\omega_0\tau$$

and for all practical frequencies will be very close to $\pi/2$.

Setting x_0 equal to the maximum displacement of the electron, and taking this position to correspond to a new time zero, we can calculate the return motion in a similar manner. The result is

$$v \sim -x_0\,\omega_0 \sin(\omega_0 t)\,\exp\left\{-\,\omega_0\tau[\,\cot(\omega_0 t) + \omega_0 t]\right\}$$

We can use this result to estimate the radiation loss over a quarter cycle. The maximum velocity is given by

$$v_m = -\,\omega_0 x_0 \exp[-\frac{\pi}{2}\omega_0\tau]$$

while the maximum velocity in the abscence of radiation would be

$$v_m' = \omega_0 x_0$$

The fractional energy loss due to radiation is then

$$\frac{E_{rad}}{E} = 1 - \exp(-\pi\,\omega_0\tau) \sim \pi\,\omega_0\tau$$

The conventional approach is to calculate the motion ignoring the radiation, and then to compute the radiation loss from the calculated acceleration

$$\frac{E_{rad\ conv}}{E} = \frac{2\tau\int_0^{\pi/2} \dot{v}^2\,dt}{v^2} = \frac{\pi}{2}\omega_0\tau$$

giving

$$\left(\frac{E_{rad}}{E_{rad\ conv}}\right)_{1/4\ cycle} \sim 2$$

for non-relativistic motion

IV FORCED OSCILLATORY MOTION

We can ask what force is required to maintain an electron in sinusoidal motion, and we proceed as before by writing

$$v = v_o \sin\,\omega t$$

$$\dot{v} = \omega\,v_o \cos\,\omega t$$

The required force is then given by

$$\frac{f}{m} = \omega\, v_o \cos \omega t [\, 1 + \omega \tau \cot \omega t]$$

A periodically infinite force is needed to maintain the motion, and it is readily seen that the impulse integral diverges. Electrons cannot oscillate with precisely simple harmonic motion.

We can obtain oscillatory motion and at the same time avoid the requirement for infinite forces by choosing an oscillatory motion that has the velocity vanishing with the acceleration. Such a form is

$$v = v_o \sin^2 \omega t$$

we have

$$\dot{v} = 2\, v_o\, \omega \sin \omega t \cos \omega t = \omega\, v_o \sin 2\omega t$$

Substituting into the equation of motion yields

$$\frac{f}{m} = 2\omega\, v_o \cos \omega t\, [\, \sin \omega t + 2\omega\tau \cos \omega t\,]$$

Using elementary trigonometry, this latter result can be written

$$\frac{f}{m} = \omega\, v_o [1 + \omega^2 \tau^2]^{\frac{1}{2}} \sin (\, 2\omega t + \phi\,) + \omega^2 \tau v_o$$

$$\phi = \tan^{-1} \omega\tau$$

and the effect of the radiation term is seen to be a phase difference between the driving force and the resultant oscillations together with the need for a constant force. The energy radiated in one oscillation is given by

$$\mathcal{E}_r = m\, \tau\, \omega^2 v_o^2 \int_0^{\frac{\pi}{\omega}} \sin^2 2\, \omega t\, dt = \pi\, \omega\, \tau . \frac{1}{2} m\, v_o^2$$

The velocity is always positive and so the electron will have a mean drift velocity

$$\bar{v} = \frac{\omega}{\pi} v_o \int_0^{\frac{\pi}{\omega}} \sin^2 \omega t\, dt = \frac{v_o}{2}$$

The more fundamental solution to the equation of motion is the response to a simple sinusoidal force and so we attempt to transform the above solution. We make the replacements

$$2\omega \rightarrow \omega$$

$$\omega t \rightarrow \omega t - \tan^{-1}(\,\omega\tau)$$

$$f_o = [m \omega v_o][\, 1 + (\omega\tau)^2]^{1/2}$$

with the result that the equation

$$\dot{v} + \tau \quad \dot{v}^2/v = \{f_o/m\} \{\sin \omega t + \sin\phi\,\}$$

has a solution

$$v = v_o[\,1 - \cos(\omega t - \phi)]$$
$$\phi = \tan^{-1}\omega\tau$$
$$v_o = [f_o/\omega m]\cos\phi$$

and the radiated energy is

$$\mathcal{E}_r = m\tau\omega^2 v_o^2 \int_0^{\frac{2\pi}{\omega}} \sin^2\omega t\,dt = \pi\omega\tau . m v_o^2$$

The additional constant force is the force required to provide the energy for the radiation, and this provides a check on the calculations. The net distance moved in one period is

$$x = v_o \int_{\tau}^{\frac{2\pi - \omega\tau}{\omega}} [\,1 - \cos(\omega t - \phi)]\,dt = \frac{2\pi}{\omega} v_o$$

giving for the work done by the constant force

$$\mathcal{E} = f_o \sin\phi . x = \pi\omega\tau m v_o^2$$

We may compare this calculation with the result obtained using the standard procedure of ignoring the radiation while calculating the motion. The equation of motion would be assumed to be

$$\dot{v} = f_o/m . \sin(\omega t)$$

and the radiated energy would be calculated as

$$\mathcal{E}' = m\tau \int_0^{2\pi} \dot{v}^2\,dt = \pi\omega\tau m v_o^2$$

as before. However if the electron is undergoing sinusoidal motion the actual force has the additional term $f_o/m.\sin\phi$. This modifies the result to

$$\mathcal{E}' = m\tau \int_0^{2\pi} \dot{v}^2\,dt = \pi\omega\tau m v_o^2 \left[\frac{1 + 3\omega^2\tau^2}{1 + \omega^2\tau^2}\right]$$

V. CIRCULAR MOTION

We now consider 2-dimensional motion and ask what force is necessary to maintain circular motion at constant speed. The n-r equation of motion is

$$\dot{\mathbf{v}} + \tau\frac{\dot{v}^2}{v^2}\mathbf{v} = \frac{\mathbf{f}}{m}$$

Making the substution

$$r = R_o e^{i\omega t}$$

there results

$$\frac{f}{m} = -R_o \omega^2 \sqrt{1 + \omega^2 \tau^2}\, e^{i(\omega t - \phi)}$$

$$\tan\phi = \omega\tau$$

Resolving the force into radial and tangential components

$$f_r = m\omega v_o$$
$$f_\theta = m\omega^2 \tau v_o$$

VI. MOTION UNDER A GENERAL FORCE

The previous sections have demonstrated that the effect of the radiation on the motion is to introduce a phase or time delay between the force and the resulting motion. In note No 5 the solution for linear motion under a constant force was obtained for both the relativistic and the non-relativistic case by treating the equation as a quadratic in the acceleration. In this special case the acceleration was integrable leading to times and distances expressed as functions of velocity. The expression for the acceleration takes the form

$$\dot{v} = \frac{v}{2\tau}\left\{\pm\sqrt{1 + \frac{4f\tau}{mv}} - 1\right\}$$

The validity of this expression does not depend on any assumptions apart from those in the derivation of the original equation. The choice of sign depends on the nature of the functions involved. In the previous study of oscillatory motion this expression requires the positve sign for times greater than τ. It is to be observed that retarding forces imply a limiting lower velocity, the radical becoming imaginary for

$$v < -4f\tau/m = v_m$$

This implies that should this condition be approached the velocity jumps discontinuously from v_m to zero as the electron slows down. The further implication is that an amount of energy equal to this loss in kinetic energy is radiated.

The relativistic equation of motion is

$$\frac{\dot{v}}{(1 - \frac{v^2}{c^2})^{\frac{3}{2}}} + \tau \frac{\dot{v}^2}{v(1 - \frac{v^2}{c^2})^3} = \frac{f}{m_o}$$

and this leads to the result

$$\dot{v} = \frac{v}{2\tau}\left(1 - \frac{v^2}{c^2}\right)^{3/2}\left\{\sqrt{1 + \frac{4f\tau}{m_0 v}} - 1\right\}$$

and the minimum velocity is again

$$v_m = \frac{-4f\tau}{m_o}$$

The radiated energy is

$$\mathscr{E}_r = m_o c^2\left\{\left(1 - \left(\frac{4f\tau}{m_o c}\right)^2\right)^{-1/2} - 1\right\}$$

This result suggests that for a more general force law a similar effect is to be expected. The equation of motion would have to be solved for the force law under consideration to confirm this suggestion.

Before demonstrating discontinuous motion in a more general case, it is interesting to draw one further conclusion from the above result. The maximum value of the minimum positive velocity is c

$$f = -\frac{m_o c}{4\tau}$$

If this force is due to the presence of a single electron, we can write

$$f = -\frac{q^2}{4\pi\epsilon_o r^2}$$

Equating these two expressions and solving for r we obtain the distance of closest approach of two electrons

$$r_m = \frac{q^2}{4\pi\epsilon_o m_o c^2}.\sqrt{\frac{8}{3}} = \sqrt{\frac{8}{3}}.r_e$$

where r_e is the classical electron radius. This is not to be taken as indicative of any structure to the electron. At this distance the radiation loss is equal to the increase in potential energy for a virtual reduction in the separation.

We may consider a spatial variation of the force by making the substitution

$$\dot{v} = v\frac{dv}{dr}$$

and the equation for linear motion becomes

$$v\frac{dv}{dr} + \tau v\left(\frac{dv}{dr}\right)^2 = a$$

We may find solutions as before by imposing a motion and determining the force law for such a motion. To find a suitable motion to investigate we choose the approach of two electrons, and determine the motion ignoring the radiation term. That is we initially consider the equation

$$v\frac{dv}{dr} = -\frac{k}{r^2}$$

where r is the separation of the electrons, and

$$k = \frac{q^2}{4\pi\epsilon_o m_o}$$

Integrating and solving for the velocity

$$v = \left[v_o^2 + \frac{2k}{s} - \frac{2k}{r} \right]^{\frac{1}{2}}$$

where s is the initial separation and v_0 the initial velocity. Imposing this solution on the differential equation, the force law to maintain this motion with the radiation taken into account is

$$a = -\frac{k}{r^2}\left[1 - \tau\frac{k}{r^2}\left(v_o^2 + \frac{2k}{s} - \frac{2k}{s} \right)^{-\frac{1}{2}} \right]$$

The solution for v predicts that the electrons will come to rest at a separation given by

$$r_r = \frac{2k}{v_o^2 + \frac{2k}{s}}$$

However returning to the differential equation and regarding it as a quadratic in dv/dr, we have

$$\frac{dv}{dr} = \frac{-1 \pm \sqrt{1 + \frac{4a\tau}{v}}}{2\tau}$$

For retarding forces this expression becomes imaginary for

$$\frac{4a\tau}{v} < -1$$

and the limiting condition is given by equality. Expressing a in terms of the velocity, this condition becomes

$$\frac{4\tau k}{r^2}\left[1 - \frac{\tau k}{r^2 v} \right] = v$$

Solving this equation for $r^2 v$, we find that the acceleration becomes imaginary for

$$r^2 v < 2\tau k$$

that is we have the relation

$$v_c = \frac{2\tau k}{r_c^2}$$

where the suffix c refers to critical. Introducing the solution for v there results an expression connecting the critical range with the initial velocity

$$\left[v_o^2 + \frac{2k}{s} - \frac{2k}{r_c}\right]^{\frac{1}{2}} = \frac{2\tau k}{r_c^2}$$

Rearranging we have a quartic equation for r_c

$$r_c^4\left(v_o^2 + \frac{2k}{s}\right) - 2kr_c^3 - (2k\tau)^2 = 0$$

From the definitions and the accepted values of the various constants

$$(2k\tau)^2 = 1.0075.10^{-41}$$

At a range of a classical electron radius

$$2kr_e^3 = 1.13343.10^{-41}$$

At significantly greater ranges the first two terms of the quartic are much greater than the constant term and we obtain the first approximation to a root as

$$r_{co} = \frac{2k}{v_o^2 + \frac{2k}{s}}$$

This corresponds to the position of zero velocity as given by the solution for velocity. We now write the root of the equation as

$$r_c = r_{co} + \delta$$

and expanding to first order in δ

$$\frac{\delta}{r_{co}} = \left[\frac{\tau}{2k}\right]^2 \left(v_o^2 + \frac{2k}{s}\right)^3$$

For n-r electrons at a large separation

$$\frac{\delta}{r_{co}} = O\left[\left(\frac{6.10^{-24}}{500}\right)^2 (9.10^{14})^3\right] = O(10^{-7})$$

and the approximation is justified. We now determine the critical velocity by calculating the velocity at $r_{co} + \delta$, with the result

$$v_c = \frac{\tau}{2k}\left(v_o^2 + \frac{2k}{s}\right)^2$$

assuming a large, essentially infinite initial separation, the critical velocity becomes

$$v_c = \frac{\tau v_o^4}{2k}$$

To ensure that thr n-r assumptions are valid, we set $v_o = 0.1c$, yielding $v_c = 9.993.10^3$ m/s this being equivalent to $2.84.10^{-4}$ eV. For predictions compatible with the possibility of making observations it is clear that relativistic energies must be considered.

To make an estimate of the critical energy for a relativistic electron we note that we only need to consider relativistic motion in the initial slowing down to non-relativistc velocities so as to obtain a consistent pair of values for v_o and s. Accordingly the scalar equation of motion ignoring radiation becomes

$$\frac{v\frac{dv}{dr}}{\left[1-\frac{v^2}{c^2}\right]^{\frac{3}{2}}} = -\frac{k}{r^2}$$

Rearranging and integrating

$$\int_{v_o'}^{v_o} \frac{vdv}{(1-\frac{v^2}{c^2})^{\frac{3}{2}}} = -\frac{k}{r}\Big]_{\infty}^{s}$$

Carrying out the remaining integration

$$\frac{c^2}{\left[1-\frac{v_o'^2}{c^2}\right]^{\frac{1}{2}}} - \frac{c^2}{\left[1-\frac{v_o^2}{c^2}\right]^{\frac{1}{2}}} = \frac{k}{s}$$

Taking the initial velocity to be 0.8c and the velocity at which we may use n-r equations to be 0.1c we obtain the estimate

$$\frac{k}{s} = 5.94642.10^{16}$$

giving for the critical velocity $v_c = 1.77634.10^8$ m/s. This corresponds to an energy of 89.7 keV, while the initial velocity corresponds to an energy of 340.67 keV.

The foregoing represents the classical theory of electron-electron Bremsstrahlung for a head-on collision. Such a collision is a relatively unlikely event and it is necessary that the analysis be extended to two dimensional motion see if the discontinuous nature of the motion persists.

VII. SUMMARY AND DISCUSSION

A number of solutions have been obtained by inverting the problem of finding the motion for a specified force to finding the force for a specified motion. Modification of these results show that for sinusoidal motion to be maintained a small constant force has to be applied to provide the energy lost in the radiation process. It has been shown that for certain limiting forms of impulse, impulses of infinite strength lead to a finite velocity of the electron, while the radiated energy tends to infinity. If we regard a collision as an impulse that gives the initial conditions for oscillations of atomic or molecular systems, the implication is that the 'ultraviolet catastrophe'[3] predicted by the Rayleigh -Jeans formula for the thermal spectrum would not have occurred.

The main result, however, is that discontinuities in velocity can occur, and this has led to a classical model of electron-electron Bremmstrahlung, and the conclusion is that classical physics is not only shown to be consistent down to atomic dimensions but also is not as far removed from quantum mechanics as perhaps we have been led to believe.

VIII. REFERENCES

[1] The Equation of Motion for a Classical Charged Particle I.L.GALLON Physics Note No 5

[2] Handbook of Mathematical Functions ABRAMOWITZ and STEGUN Dover

[3]Heat and Thermodynamics ROBERTS and MILLER Blackie & Son