SWITCHING NOTES

Note 19

AN ANALYTICAL MODEL FOR THE HIGH VOLTAGE ROPE SWITCH

by

Carleton H. Jones, Jr.; Walter F. J. Crewson;
John T. Naff and Joseph S. Granados

Pulsar Associates, Inc.
La Jolla, California

ABSTRACT

A rope switch is a series connection of low-voltage spark gaps to produce a high voltage switch. A 250 kilovolt rope switch was constructed as part of the HAS program and experimental results were reported in AFWL document EMP-HAS 3-3, September 1970, pp. 51-78. At that time, preliminary efforts to construct a mathematical model that would reproduce the observed results were only partly successful. In this note, a statistical model is developed and tested which reproduces the earlier rope switch time jitter distribution data. This model now permits projections of the performance of still larger rope switch assemblies, of the multi-megavolt class, for example.

FOREWORD

My original intent in establishing this effort was to attempt to tie up one of the loose ends of the HAS program. As an ex-HAS program manager I still retain most of my technical and romantic ties to the program. Although HAS, per se, is dead, it lives on in other programs and many of the efforts initiated and technical predictions made in HAS have now been fulfilled: Multi-megavolt, elevated systems have been fielded; satellite simulators are being constructed; 100 joule per pound capacitors are standard items; atmospheric pressure, gas-insulated Marx generators with stresses exceeding 1 MV/m and densities exceeding 35 joules per pound have been built and operated; distributed pulser arrays producing tens of megavolts are being constructed; and numerous other impacts of HAS are being felt.

One of the major HAS concepts which has not been sufficiently exploited is the airborne dipole, RES-N. One of the major technical limitations in the past was the requirement for lighter-than-air support systems. However, the recent developments in high density capacitors and compact, high-density Marx generators have significantly improved the feasibility of building a 10 to 15 MV system which would not require lighter-than-air. The main unsolved problem is reduction of the output switch weight. The initial HAS effort on the "rope" switch was aimed at this problem. If the "rope" switch is feasible it will allow drastic reductions in output switch weights.

This report represents what I feel is an outstanding effort by the authors to analyze the HAS "rope" switch data. It demonstrates that the "rope" switch is still a viable concept which offers a shot-in-the-arm to RES-N and significant advantages to any system which requires the pulsers to be either flown or supported off the ground.

Daniel M. Strickland, Capt. USAF
Air Force Weapons Laboratory
May 1973

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I. Introduction

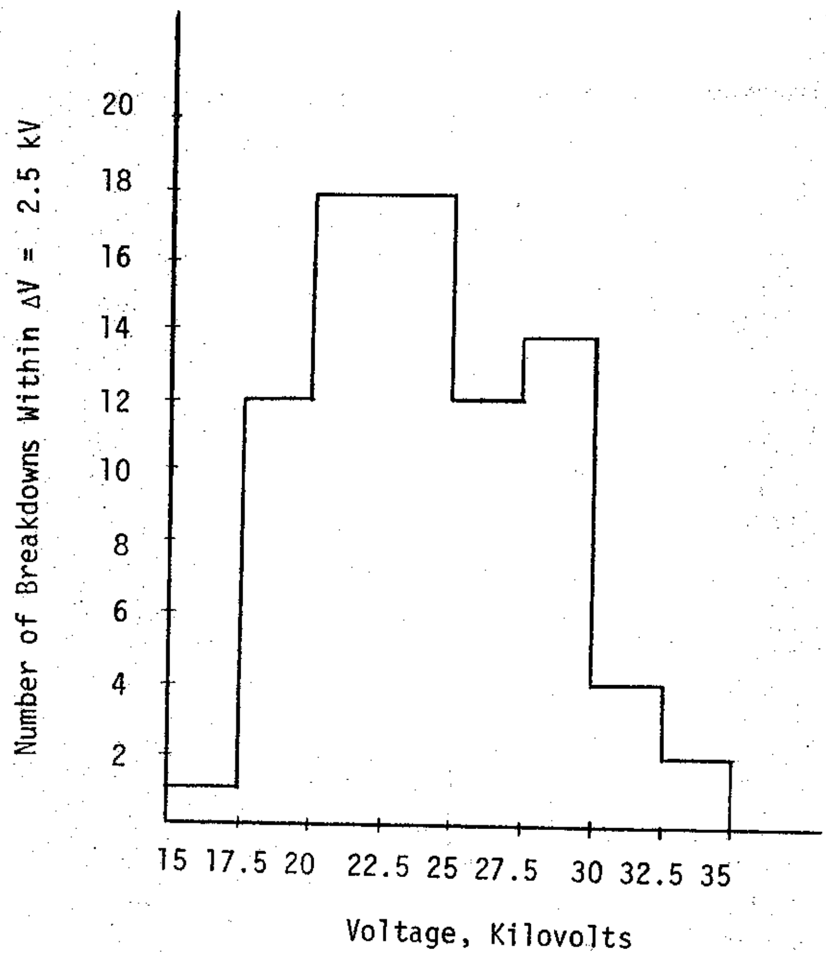
One of the major design challenges of large practical high speed EMP generators is the main high voltage switch. This device must switch the entire pulser voltage of several million volts with high speed and reliability. As EMP pulser voltage specifications have gone higher, the design of these switches has increased substantially in difficulty. To keep pulse risetime at a reasonable value the electrode gap spacing must be kept down. This requires high gas pressures. Furthermore, at higher voltages the outside ends of the main switch must also be capable of withstanding the applied electopotentials. This requires longer and bigger switches. High gas pressures and large volumes do not mix well because much stronger materials are required to contain them. This mechanical requirement leads to even larger switch housings. Because of these factors, in recent years these single channel high voltage switches in EMP systems have gotten dramatically larger and heavier. A special consideration of increasing importance is the practical one of the degree of safety associated with using very high pressure vessels in any EMP generator. This is particularly a problem because these vessels are made of dielectric material which can be attacked or cracked by high voltage sparks which are always present.

Recognizing these difficulties and moved by the requirement to further reduce the bulk and weight of EMP generators, the Air Force Weapons Laboratory several years ago initiated efforts toward studying the feasibility of a practical cascade switch sometimes referred to as a "Rope" switch. This switching technique consists of replacing a single large main gap with a series of smaller gaps. For example, a one megavolt switch could be replaced by a series string of twenty 50 kV switches. These first tests were successful in demonstrating the practical operation of a rope switch. It appeared that such a device might be light in weight and at least equal the performance of conventional uniform field gaps. The results of this first experimental effort were reported in 1970 in the "EMP High Altitude Simulation Technology Report: Volume 3-3, Report No. 2, Section 3," entitled 'The Rope Switch.' This report describes an experimental test and gives performance data for a ten-stage cascade rope switch operating at 250,000 volts. No extended attempt was undertaken at that time to explain the operation of the rope switch. In fact, various members of the high voltage community differed as to the interpretation and meaning of the published results.

In January, 1973, the effort reported here was undertaken to review the data of the 1969 and 1970 experiments to determine if a theoretical model could be elicited which would explain the data and indicate the practical feasibility of this switch for use in larger EMP generators. This review of the data resulted in a tentative model which yields the experimental results. This analysis has indicated that a practical rope switch is feasible and would have important advantages over more conventional single channel switches. These advantages would be most importantly improved performance, reliability and safety.

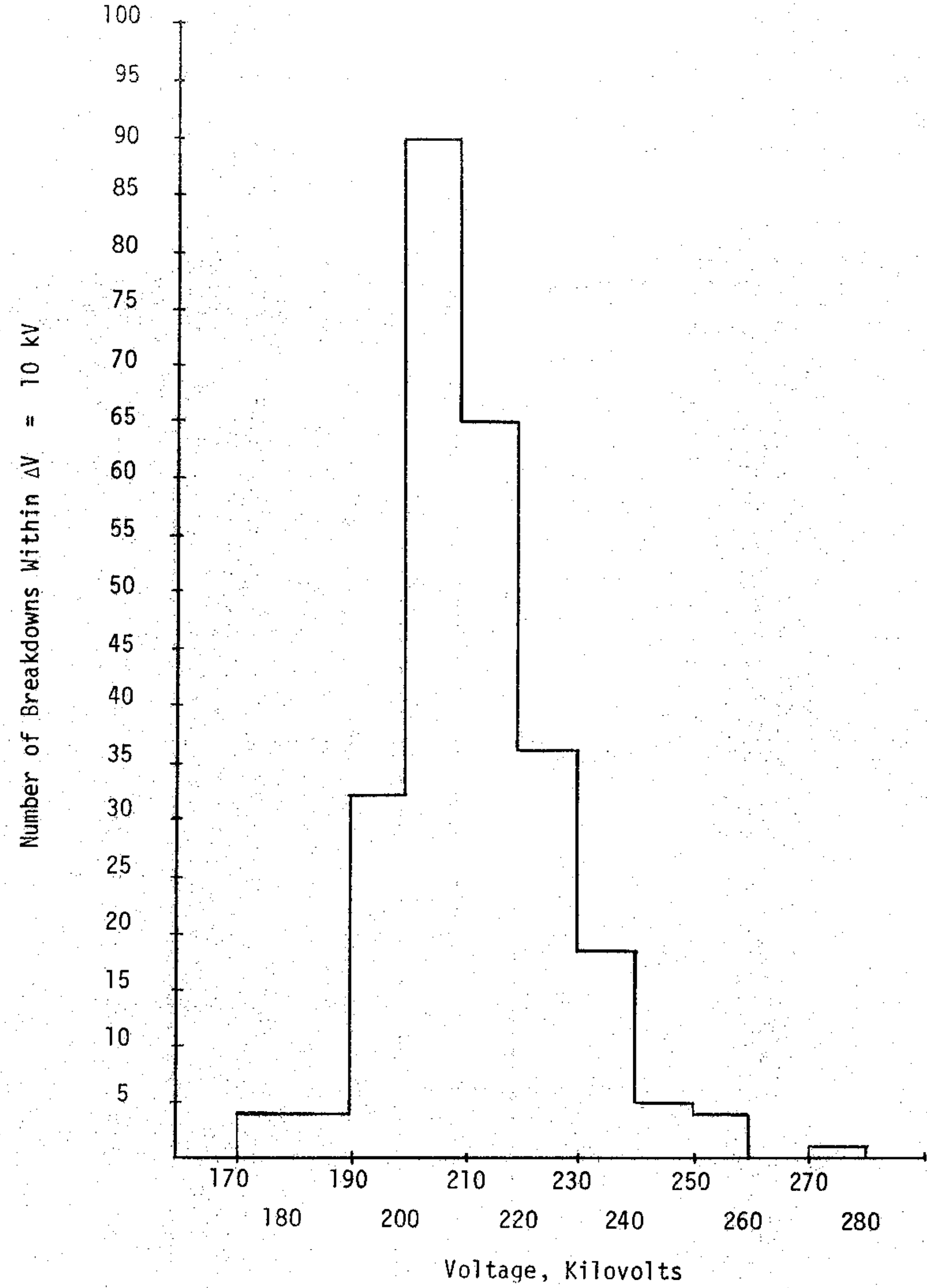
II. The HAS Program Experimental Results

The original work on the rope switch was accomplished in late 1969 and early 1970 and was reported in one of the HAS reports. In this early experiment, a 10 stage rope switch was constructed using 10 individual switch stages connected in series. Each stage was a 25 kilovolt field enhanced spark gap. These stages were joined in series so that with a total of ten, a rope switch with a nominal firing voltage of 250 kilovolts was constructed. The firing statistics of an individual gap were determined and reported in the histogram reproduced here as Figure 1. Using substantially the same ramp driving voltage of about 420 volts per nanosecond for each individual gap, a rope switch of ten stages was test fired for a number of shots. The breakdown voltage was again plotted with the results shown reproduced in Figure 2. These results were immediately interesting from a number of standpoints. First, it should be noted that enhanced rather than uniform field gaps were deliberately used for the indivdual 25 kilovolt gaps. Because such gaps have inherently high jitter, it was believed that their use would make it easier to investigate whether there was any improvement in the statistical jitter of the entire rope switch as compared to the individual gaps. The results reported in Figure 2 showed that there was approximately a factor of 5 improvement over the voltage jitter of a single gap when taken on a relative basis. Another feature which was at first unexplained was that while the individual mean firing voltage of the separate gaps was approximately 25 kilovolts, the mean firing voltage of the entire rope switch was about 210 kilovolts rather than the expected 250 kilovolts. At the



BREAKDOWN WITHIN ΔV VS VOLTAGE, SINGLE ELEMENT ROPE SWITCH AS REPORTED FROM "HAS" WORK.

FIGURE 1



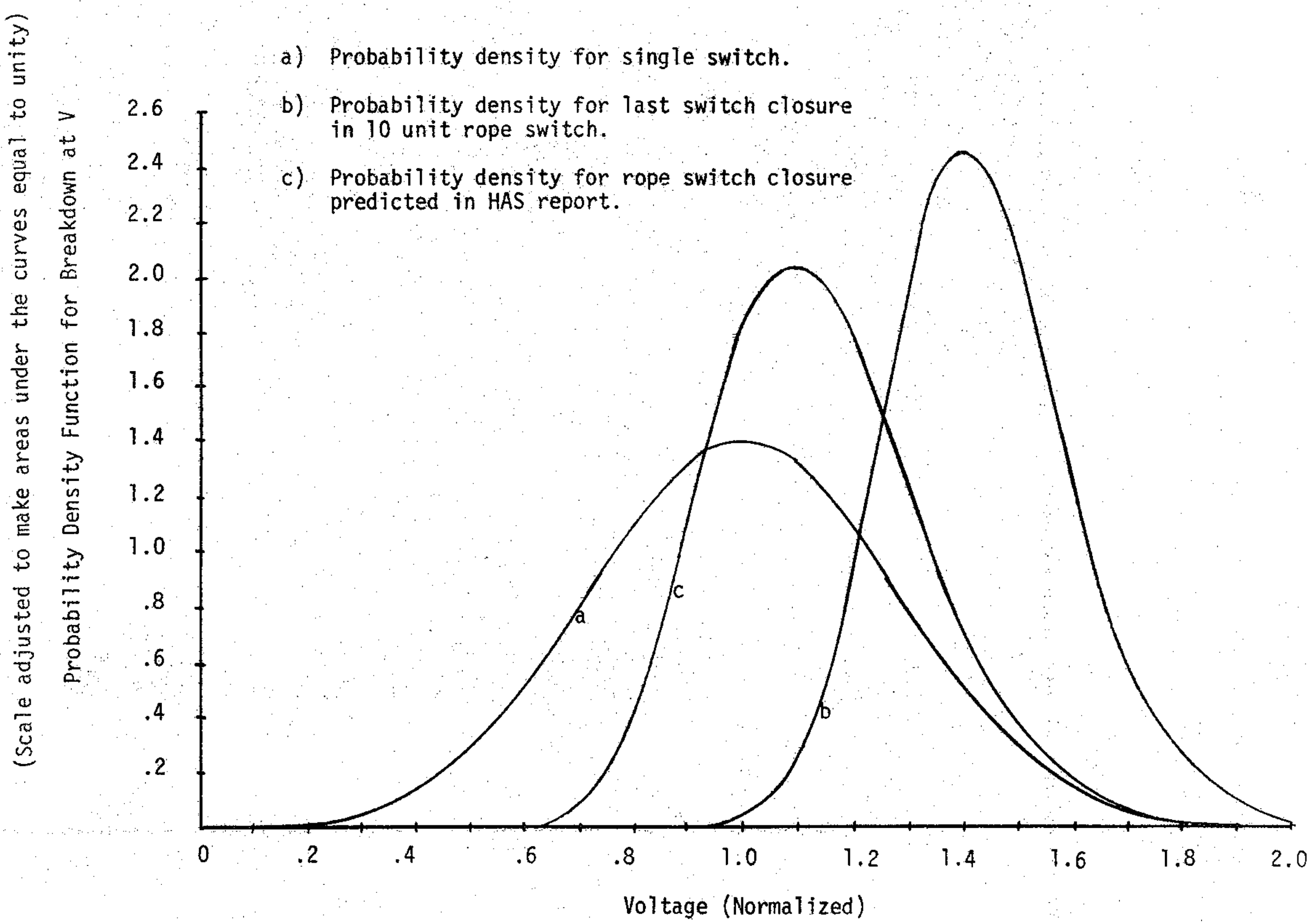
BREAKDOWN WITHIN ΔV VS VOLTAGE, 10 ELEMENT ROPE SWITCH AS REPORTED FROM "HAS" WORK.

FIGURE 2

time the experimental work was done, a brief statistical analysis was also done in an attempt to explain the experimental results. The results of this first effort are shown in Figure 3. This figure is a plot of several probability density functions versus normalized voltage. Curve A is a normal distribution fitted to the single switch distribution of Figure 1. Curve B is a calculated probability density function for the last switch closure in a ten unit rope switch made up of statistically independent gaps. Curve C is the probability density function prediction for the entire rope switch as reproduced from the HAS report. The difficulties noted at the time of the report were that whereas in the experimental results the final peak was shifted down from the normalized voltage of 1.0, the predicted results showed a peak shifted up. Some improvement as expected was noted in the standard deviation of the predicted peaks but it was not nearly as sharp as was observed experimentally. These ambiguities still remained when the experimental work was completed in 1970.

The key issues in a summary evaluation of this experiment were that the jitter of the individual section was deliberately made large by choice of enhanced gaps for the individual sections. The results of the experiment then showed a relative improvement in the overall jitter of the entire rope switch. In addition, the rope switch did act, even with its enhanced gaps, as a satisfactory high voltage switch. Because the theoretical model reported in the paper did not appear to adequately explain the experimental results, it left uncertain the best course of action which should be followed to optimize the performance of larger practical rope switches. Since that time, single gap high voltage switches have gotten larger and heavier. All of the original advantages first attributed to the rope switch have become

FIGURE 3



PROBABILITY OF BREAKDOWN AT V VS VOLTAGE

even more attractive. Hence, in 1973 a decision was made at AFWL to go back and reexamine the data with a view to establishing a theoretical model which would provide a guideline for the design of a practical, inexpensive low jitter high voltage switch.

III. The Analytical Models

The effort to determine a good analytical model for the rope switch as tested in the earlier HAS work was begun entirely anew with a view toward establishing the best model. It was hoped that such a model might be extended by further experimentation. This effort was undertaken with the main objective of providing the foundation for the development of a practical rope switch for high voltage use.

A number of different logical approaches were attempted. The first was to assume that the entire main rope switch closes when the last individual gap closes. It should be pointed out that this is in fact what must happen in the switch. However, the key question that any model must answer is what is the sequence of events leading to this last closure. As a first choice then, a model was taken whereby it was assumed that all of the individual gaps closed independently in a matter determined only by their individual statistics. The question then can be asked, if this simple model is correct, what is the statistical distribution of the last closure? This particular approach ignores the influences on adjacent switches of any switch closing. The change of voltage on the last of the series gaps as a result of earlier gap closures is also not considered. But taking a view that there may be virtue in simplicity we can proceed along these lines.

Consider first the statistical distribution of the individual switch gap.

(1) $$F(v) = \int_{-\infty}^{v} f(v)dv$$

In equation (1) the function represented by $f(v)$ is the probability density function associated with the firing of an individual gap at voltage v. The function represented by $F(v)$ is the probability that the gap has fired by the time a voltage v is reached across the gap. The probability that all n gaps in a rope switch consisting of n gaps have closed at a voltage v is given by equation (2).

$$J(v) = [F(v)]^n \tag{2}$$

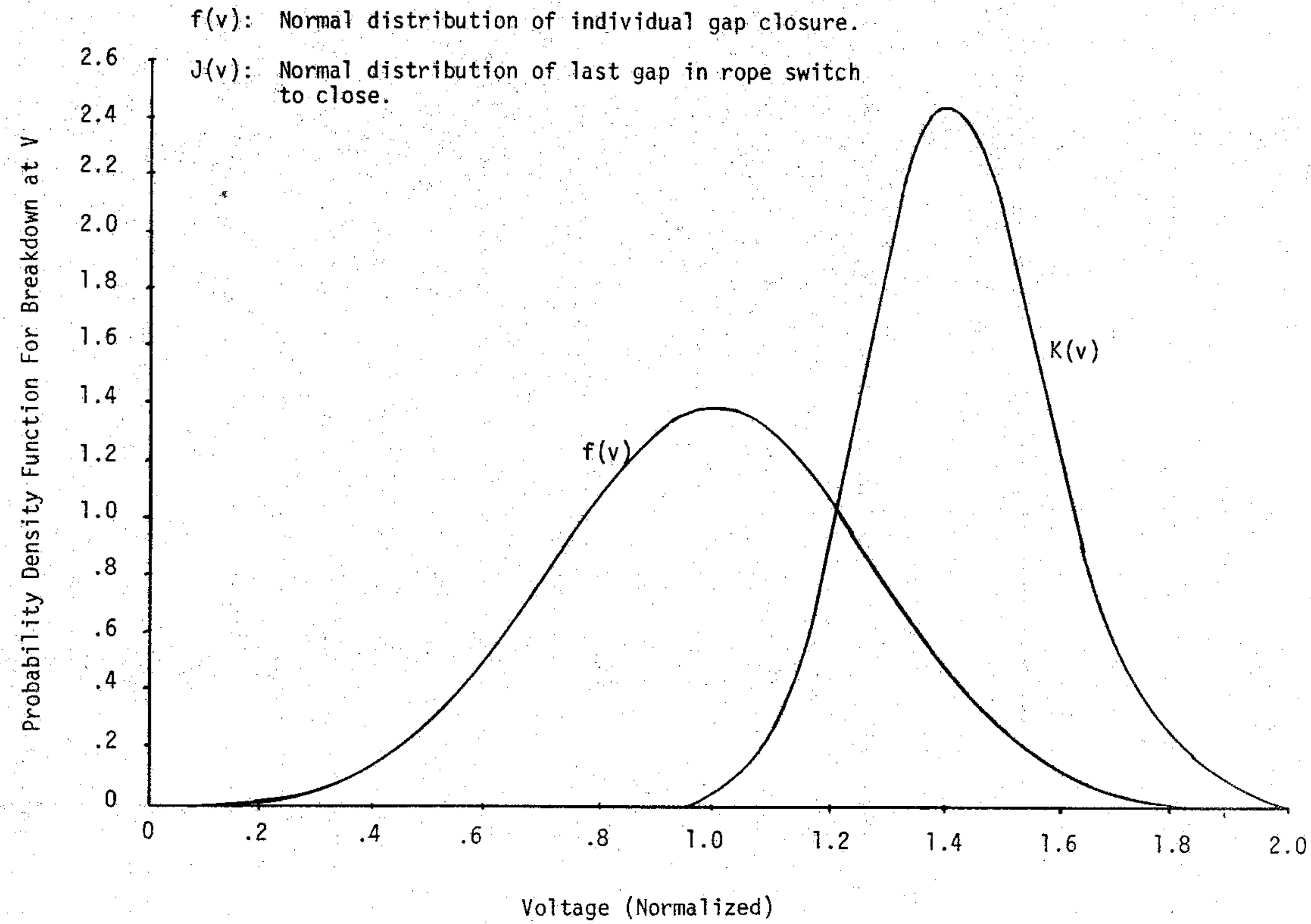
We consider the case $n = 10$ with a normal distribution about a normalized voltage of 1.0. Equation (2), when differentiated to obtain the corresponding density function in equation (3), predicts a curve shifted to a higher voltage and with a somewhat narrower full width at half maximum as shown in Figure 4.

$$K(v) = \frac{dJ}{dv} = nf(v)\,[F(v)]^{n-1} \tag{3}$$

This curve $K(v)$ is identical to the curve in the original paper noted in Figure 3 as Curve B. The difficulty is that while it predicts a somewhat narrower distribution, it is not narrow enough when compared with the experimental data. In addition, the mean firing voltage of the rope switch is predicted to increase from a simple n times the breakdown of the single switch, whereas experimental data demonstrates a decrease.

It is true that from a logical standpoint the rope switch must close when the last switch closes. However, our first assumption of independent closure of the individual gaps has serious flaws. It can be demonstrated that under normal circumstances after several of the

FIGURE 4



PROBABILITY OF BREAKDOWN AT V VS VOLTAGE

individual gaps in a rope switch have closed, the applied voltage stress on the remaining gaps will be so high as to result in their closure in a manner which might be quite different than would be expected for individually operating gaps. This leads us to the next model to be considered.

In this model we postulate that the main switch closes when the first gap closes and that the delay between the first switch closure and the closure of all the other switches is so short as to be unmeasurable. Equation (4) represents the probability that at least one switch has closed by the time voltage v is reached.

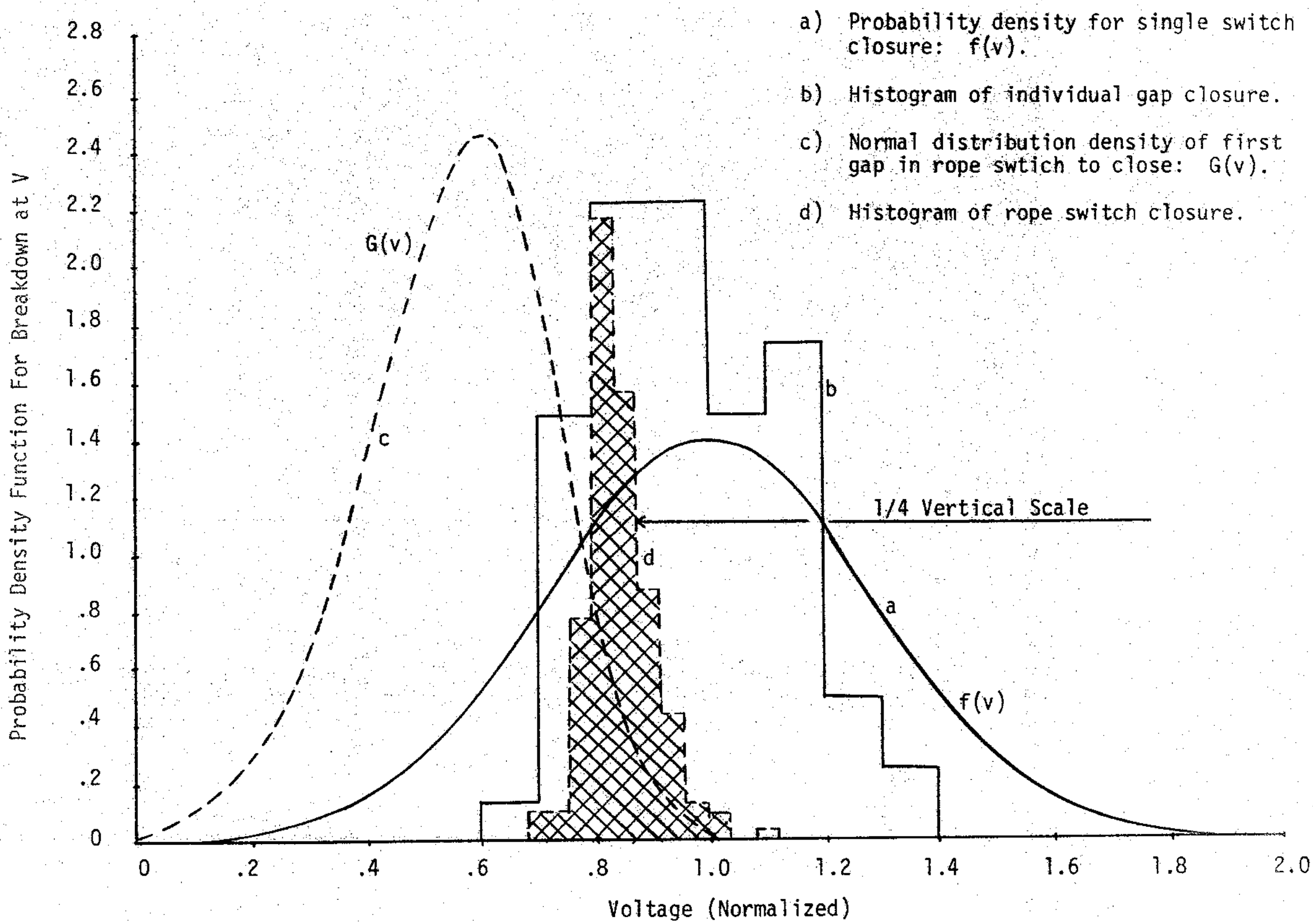
(4) $$P(v) = 1-[1-F(v)]^n$$

In equation 4 the quantity in brackets on the right hand side of the equation represents the probability that no switches have closed when voltage v is reached. Raising this quantity to the power (n) brings it to the form where it represents the probability that no switches have closed in a set of N switches. One minus this quantity then is our function P(v) which is the integral form of the probability that at least one switch has closed by voltage v. This yields equation (5) which gives the probability density function that at least one switch has closed by voltage v.

(5) $$G(v) = \frac{dP}{dv} = nf(v)\ [1-F(v)]^{n-1}$$

Figure 5 shows a summary of our results to this point. We have here the two data histograms normalized to the same area which show the individual switch distribution density and the density distribution

FIGURE 5



PROBABILITY OF BREAKDOWN AT V VS VOLTAGE

of the entire rope switch. Also shown is the normal statistical distribution corresponding to the histogram distribution of a single switch. Finally, G(v) is shown [from equation (5)] which is the distribution of the first event for a ten element idealized rope switch.

It is clear from the inspection of these graphs that the choice of the first event has resulted in a distribution density function which has shifted in the direction which is desired. In fact, it is shifted too far by approximately a factor of two. Also, it is noted that our function G(v) has a smaller standard deviation which is also expected for a correct model. However, except for these factors we are not much closer with this model than with our simplest model which considered only last switch closure. At this point we make another assumption in order to carry us further.

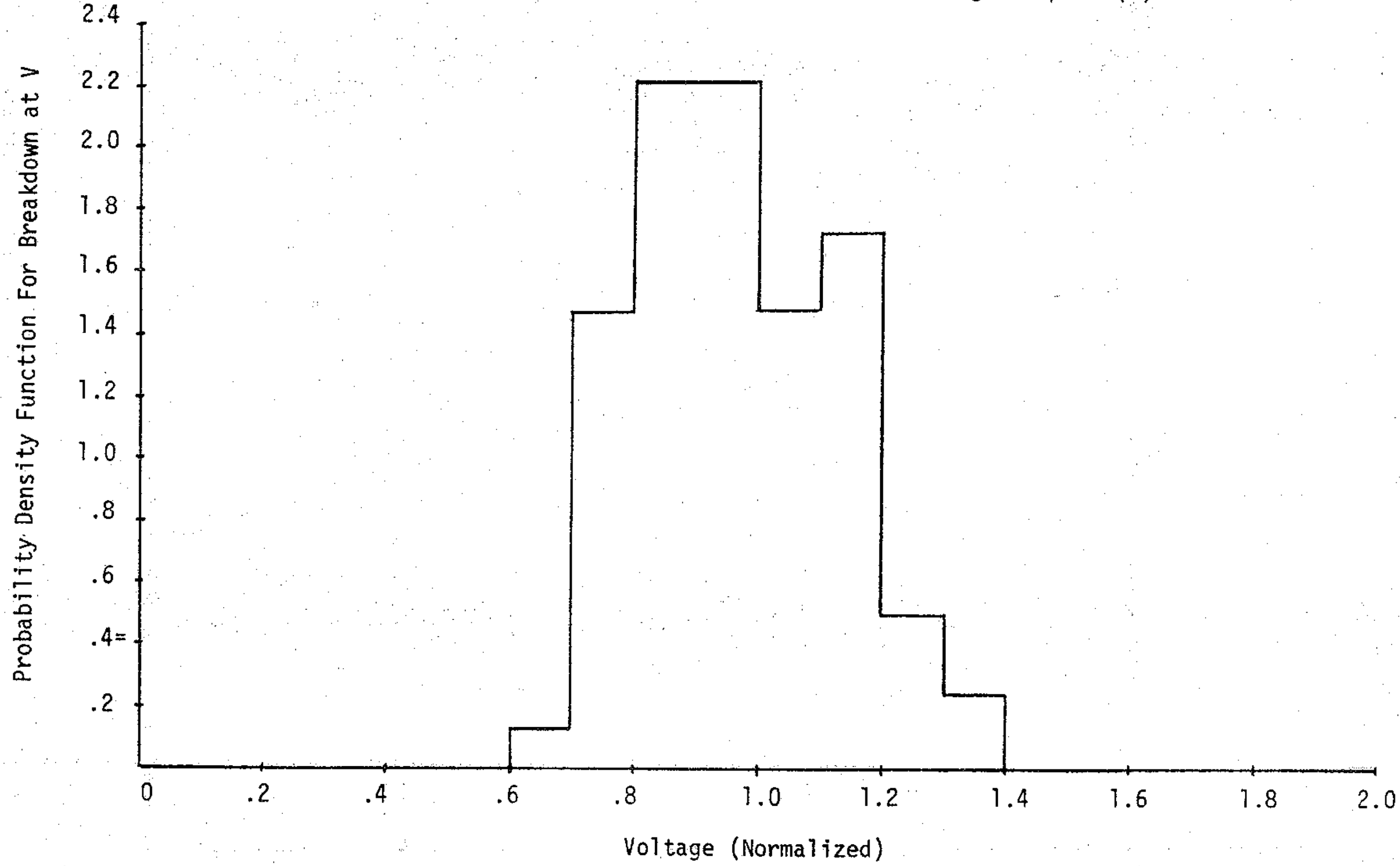
We shall intorduce the hypothesis that the main rope switch is closed by some process which is started by the first gap closure. That is to say, we will treat the first gap closure as an initiating event. This initiating event is to be handled statistically as an individual switch; that is, the first switch is to fire in a set of N switches. After that we will say, for the purposes of our model, that a second process, the details of which we will leave undetermined for the present, closes the final switch.

Referring again to Figure 5, a second point is suggested by these graphs. The use of a standard normal distribution function for f(v) with its infinite tailoffs may introduce changes in our function G(v) from equation (5) which are significant to our purpose here. Figure 6 is another representation of the histogram made from the experimental data for single switch closure. To determine the effect of the functional shape of f(v) on G(v) we shall arbitrarily take this histogram without any smoothing whatsoever as our f(v) and see what is yielded by equation (5). The integral function F(v), obtained by graphical integration of Figure 6, is shown in Figure 7. Note that the units of amplitude have been chosen so that F(v) is normalized to a final value of 1 for large V. Substituting F(v) as indicated in Figure 7 into equation (5) gives us a G(v) which is shown in Figure 8. Also shown in Figure 8 in dashed lines on the same normalized scale is the histogram representing the experimental results of the HAS experiments. Comparison should now be made between Figure 5 and Figure 8. In Figure 5 the normalized smooth distribution function was used as the starting point in equation 5 to calculate G(v). Using precisely the same mathematics but instead starting with the histogram function of the experimental data and then using equation (5) a new function G(v) was calculated as shown in Figure 8. It seems quite clear that the blocky shape of our histogram function and most critically the fact that it goes to zero for finite values of voltage has an important effect on the shape of our final distribution function G(v).

It should be noted at this point that the histogram function shown in Figure 8 has been corrected for time of flight measurement discrepancies that were discovered during our analysis of the experimental data. In the original experiment a voltage probe was attached to the Marx generator cone at a distance back from the rope switch which resulted in a 2 nanosecond delay in the indication of rope switch closure. Because the voltage on the 10-section rope switch was rising at approximately 420 volts per nanosecond per section, this resulted in an indication

FIGURE 6

Normalized Histogram Distribution for Single Gap: f(v)



PROBABILITY OF BREAKDOWN AT V VS VOLTAGE

INTEGRAL DISTRIBUTION FUNCTION FROM HISTOGRAM

$$F(v) = \int_{-\infty}^{v} f(v)dv$$

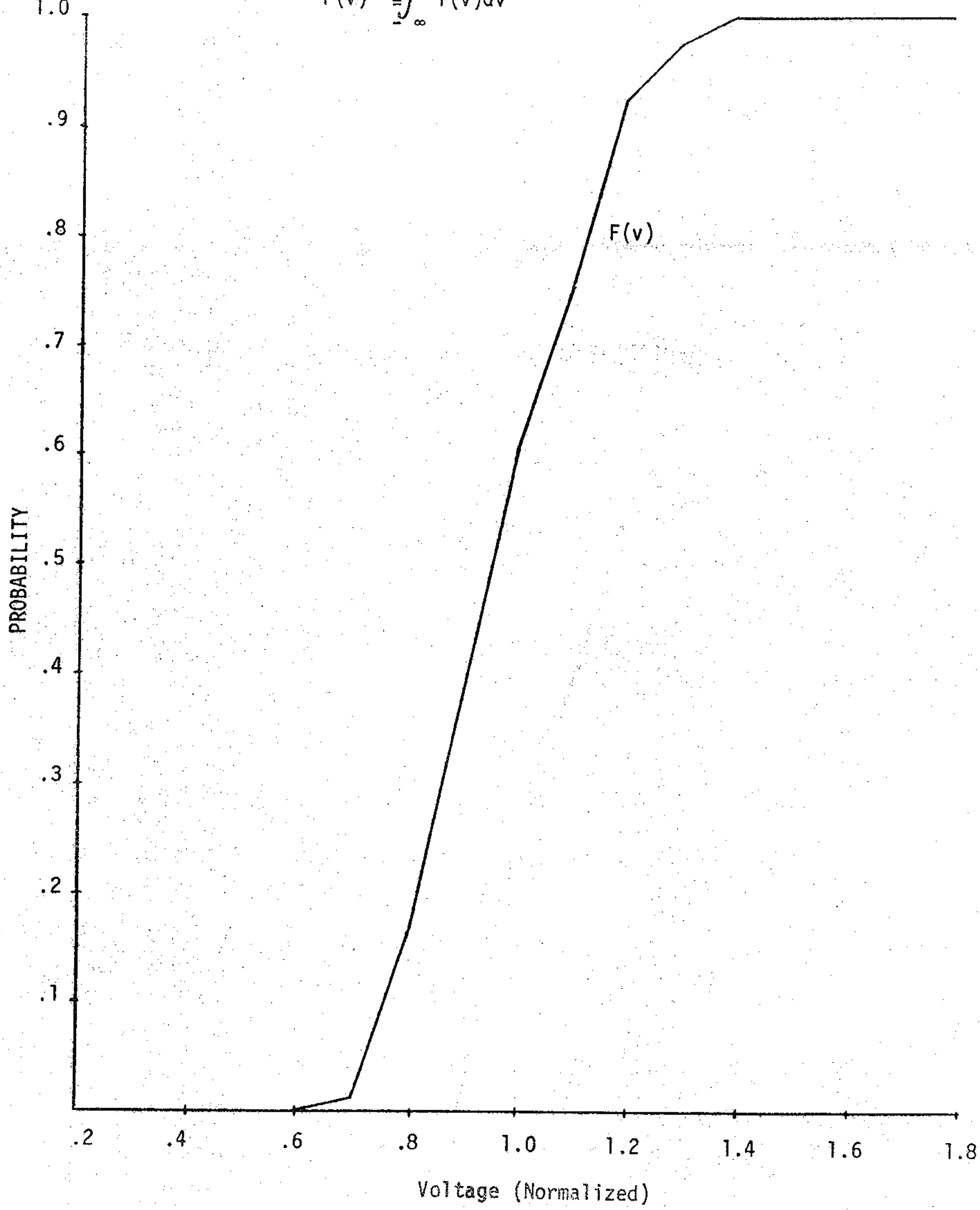


FIGURE 7

COMPARISON OF G(v) TO ROPE SWITCH CLOSURE HISTOGRAM

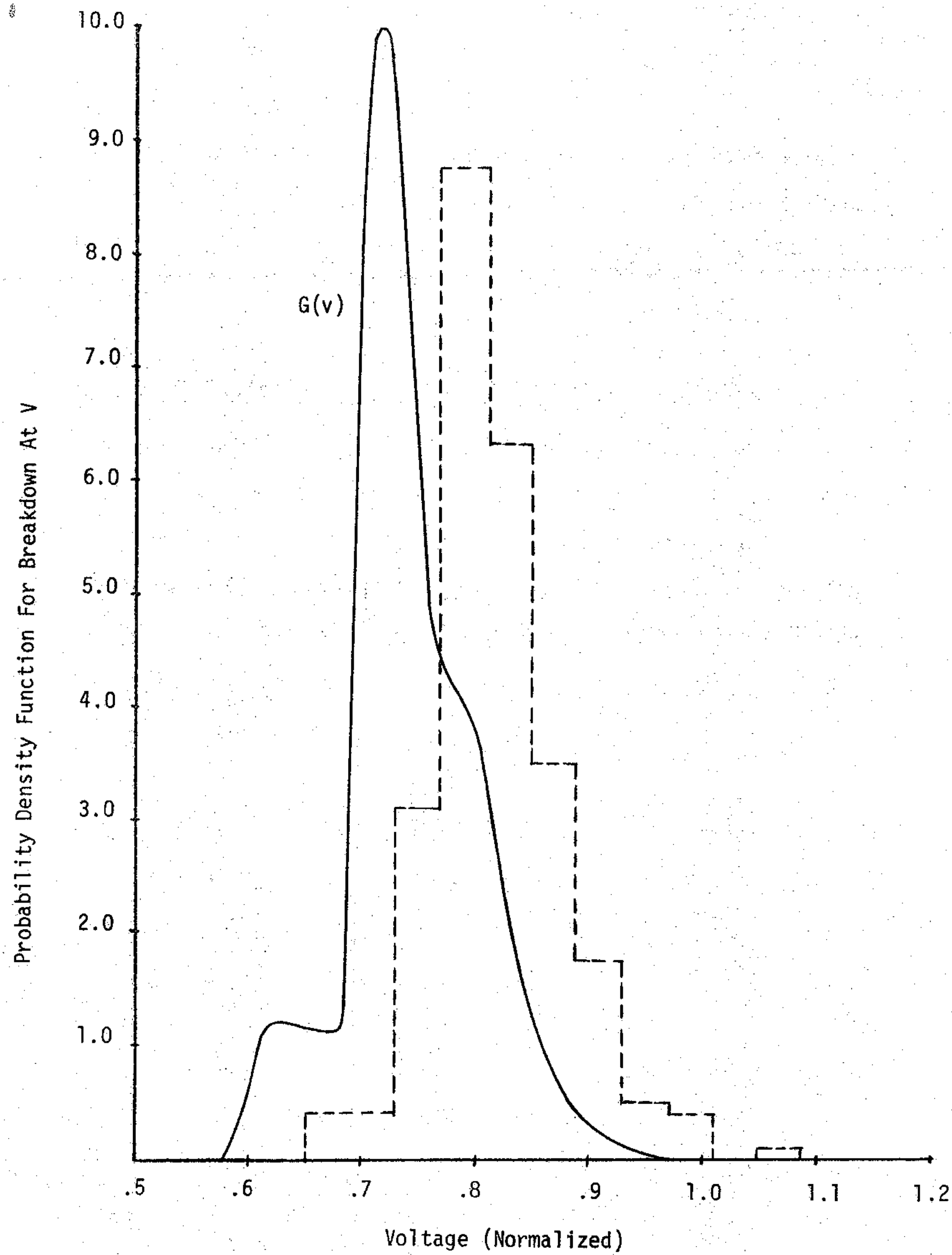


FIGURE 8

of closure of the rope switch which was too high by 8400 volts for each measurement. When normalizing the scale in Figure 8, this 8400 volts was subtracted before normalization was conducted to correct for this effect. Even with this correction, which had the effect of moving the histogram slightly to the left, we can see that our predicted curve still lies noticeably below the experimental histogram.

One would perhaps be justified in leaving the model at this point and attributing the differences to uncertain experimental factors. The curve G(v) is certainly much closer than any previous model. However, at this point we introduce a new function h(v) which for this experiment we take to be a constant representing the delay in time from the closure of the first switch until the complete main switch closes. This has the effect of saying that if there is a finite delay between the time the first switch fires (the density distribution given by G(v) of Figure (8)) and the time when the main switch is in fact closed, then the curve G(v) of Figure 8 should be shifted to the right by a finite voltage corresponding to the time delay. Figure 9 represents an arbitrary plotting of the distance from the function G(v) toward the right to the histogram indicated by the dotted lines in Figure 8. It will certainly be noted that this process is quite arbitrary and has an uncertainty which is probably at least of the order of the histogram widths. However, it is instructive to do this as it will give us a measure of the supposed delay between the first switch closure and final switch closure for the rope switch. If the points in Figure 9 are replotted on probability paper as shown in Figure 10, it is seen that they form a close fit to a normal distribution. Such a fit yields a mean delay corresponding to .058 normalized volts, with a standard h(v) VS NORMALIZED VOLTAGE

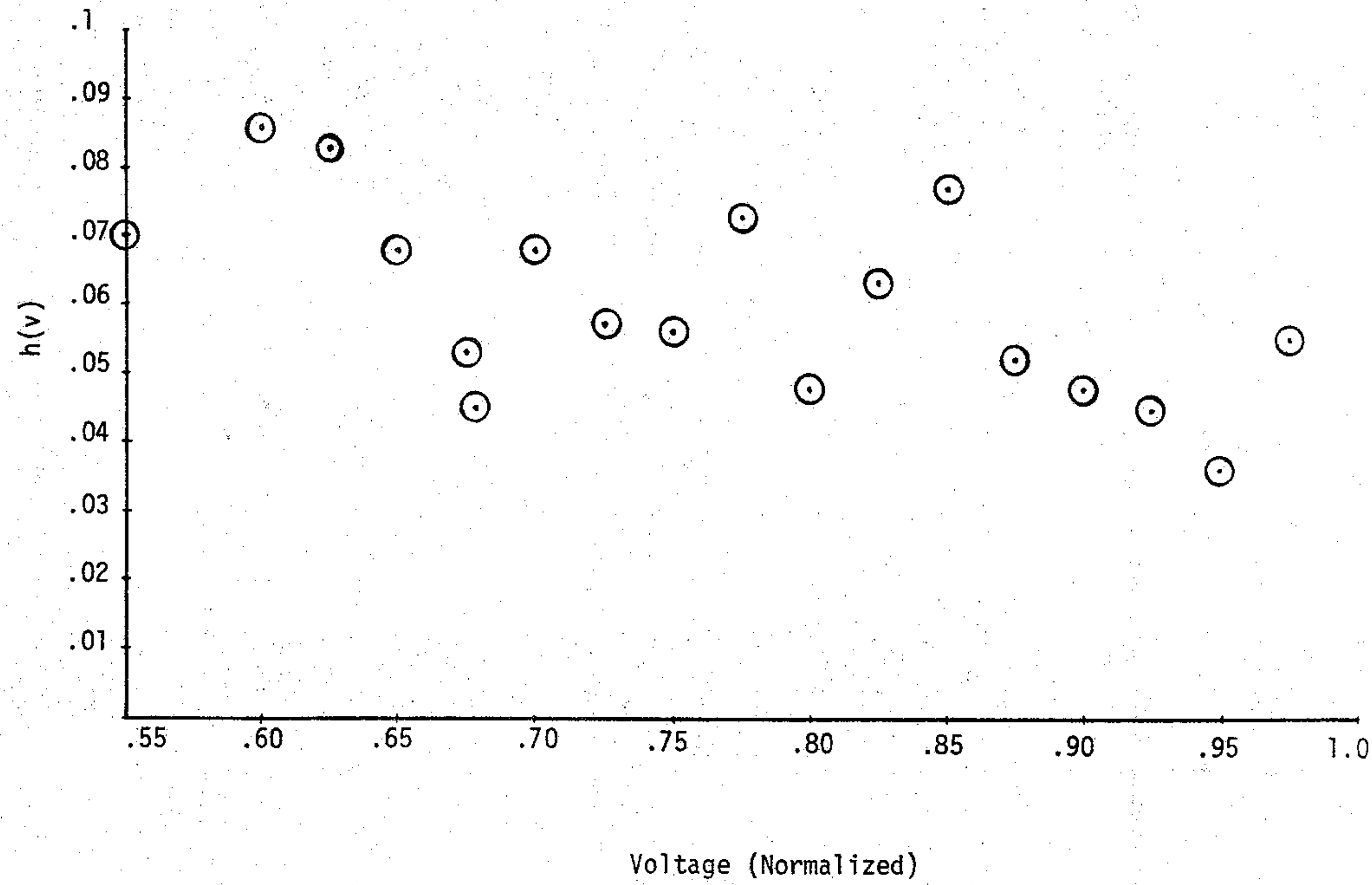
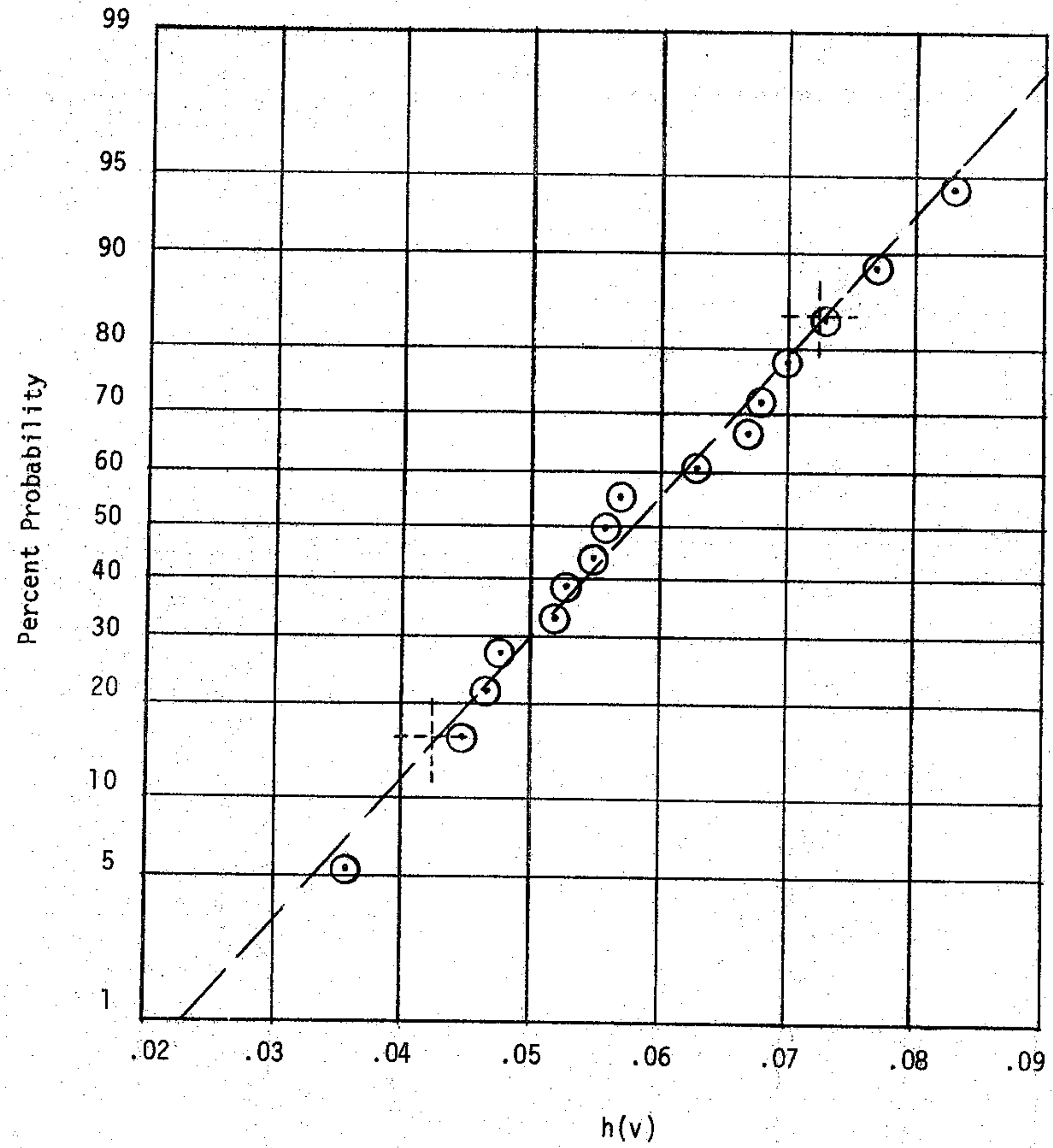


FIGURE 9



h(v) PLOTTED AS NORMAL DISTRIBUTION

FIGURE 10

deviation of approximately .015 normalized volts. This last figure is encouraging because from the very nature of the method of our fit to the histogram in Figure 8 we can note that the histogram automatically introduces an uncertainty of approximately $\pm$ .02 normalized volts.

Following these considerations, we can then estimate the value for h by which G(v) should be shifted to the right. This process yields our final density distribution function H(v) shown in equation (6).

$$(6) \qquad H(v) = G(v-h)$$

For the present experimental results we estimate for h the value shown in equation (7).

$$(7) \qquad h = 0.058 \pm 0.02 \text{ normalized volts}$$

In this experiment the slope of the voltage wave was approximately 420 volts per nanosecond per gap. The normalized voltage scale is 1 = 250,000 volts. We can convert the units of equation (7) by the methods shown in equation (8).

$$(8) \qquad h(\text{normalized volts}) \frac{2.5 \times 10^5 \text{ volts}/\text{normalized volts}}{4.2 \times 10^3 \text{ volts}/\text{nanosecond}} = h(\text{nanoseconds})$$

By applying this operation to equation (7) we are able to estimate the time required for switch closure after initiation of the events by the closure of the first switch.

$$(9) \qquad h = 3.5 \pm 1.2 \text{ nanoseconds}$$

FINAL RESULT: COMPARISON OF H(v) AND HISTOGRAM

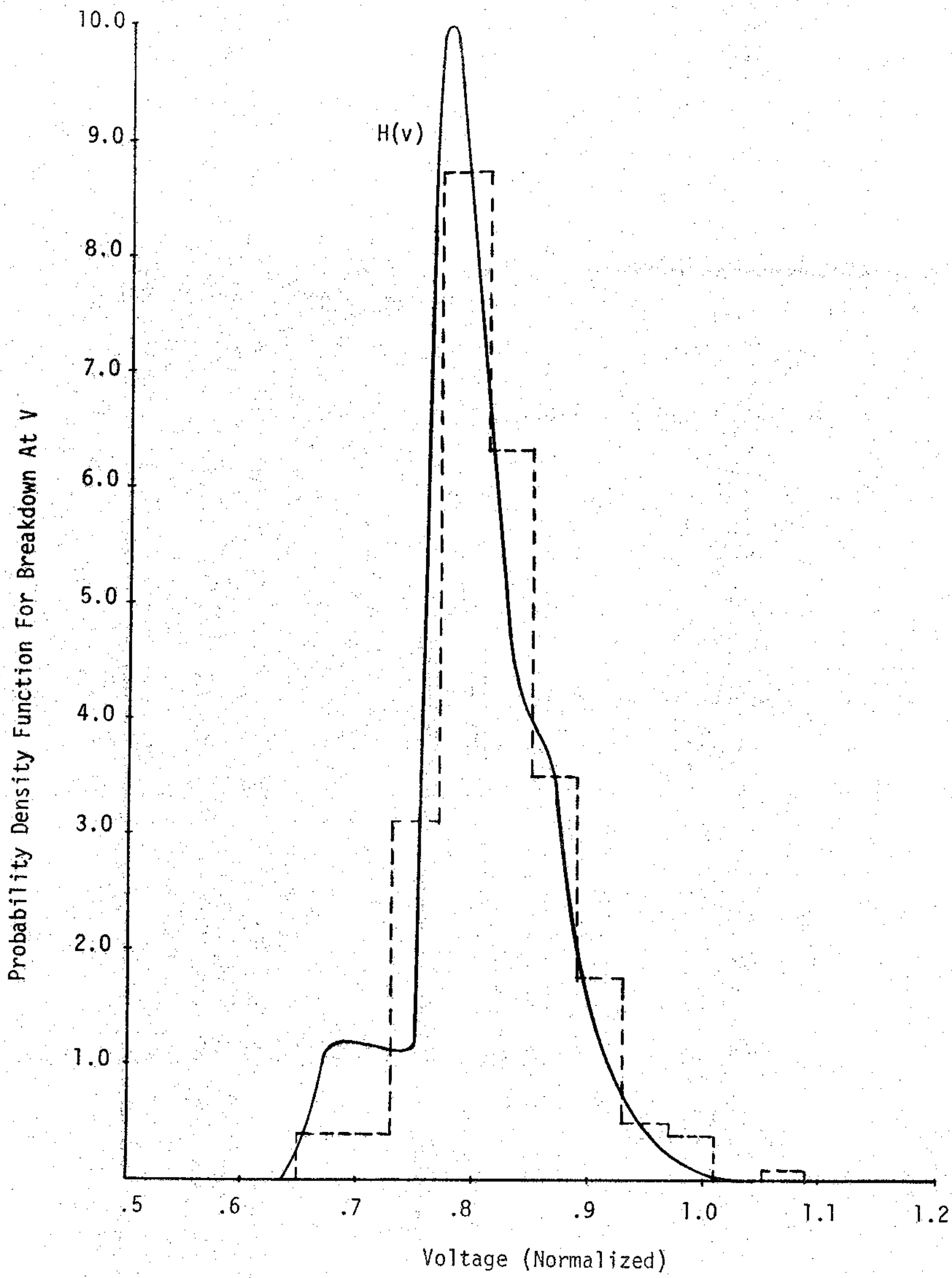


FIGURE 11

Using the numerical value of 3.5 nanoseconds for h and substituting in equation (6) we can now plot our final predicted result which is shown in Figure 11. It is noted here that we now have a fit between H(v) and the histogram representing the experimental output which is well inside the experimental uncertainty, which can be taken as represented by the histogram bars.

IV. Some Considerations Resulting From the Model

We now have a model which is rather straightforward and direct in its application and prediction. That is to say, if the model is correct, the output of the rope switch can be predicted by assuming that the rope switch operates in the following manner. First of all, a first switch gap closes as a result of a random statistical process which is going on in each individual gap. After this switch closes, the entire switch closes in a process which takes a finite period of time. Nothing else need be introduced to explain the experimental data as reported.

It is useful at this point to ask a number of questions about the direction in which to proceed further. First of all, it is quite clear that the shape of the distribution density of the firings of the individual switches is very important in determining how well the final rope switch operates. In this area, for once, nature seems to be working in our favor. The most desirable situation indicated from the model is that the individual switch sections should have a sharp lower cutoff in their density distribution functions so that there is a voltage below which there is virtually no probability that the switch will fire. It is this fact which cuts off the lower distribution of the complete rope switch. An interesting direction to extend the analysis is to ask the practical engineering question of how many sections (N) should be used in a given situation to make an optimal rope switch. We can give a partial indication by considering the final form of our distribution function given in its complete analytic form here:

$$H(v) = nf(v-h)\,[1 - \int_{-\infty}^{v-h} f(v)\,dv]^{n-1} \qquad (10)$$

It is useful to consider the effect on H(v) of N using several different simple functions f(v). For purposes of discussion, we shall consider an interval of v of -1 to +1 which contains virtually all of the switch closures of the individual switches. The simplest function would be a square wave for f(v) as given in equation (11).

$$(11) \qquad f(v) = \begin{cases} 0 & v<-1 \\ 1/2 & -1\leq v\leq 1 \\ 0 & v>1 \end{cases}$$

The effect of N will be to change the full width at half maximum of the final distribution H(v). For this discussion we will consider h = 0, because the full width at half maximum of H(v) is the same as G(v), that is, with no change due to h. With this in mind the values given in equation (11) can be substituted in equation (10) with h = 0 and we obtain equation (12).

$$(12) \qquad H(v) \begin{cases} = 0 & v<-1 \\ = n \frac{1}{2} [1 - \frac{1}{2} (1+v)]^{n-1} & -1\leq v\leq 1 \\ = 0 & v>1 \end{cases}$$

$$\text{where } h = 0$$

These curves of H(v) are plotted in Figure 12 for N equals 5, 10 and 20 together with the original probability density distribution function. It can be noted that this function has a step discontinuity with an infinite slope at the left hand boundary of the original density distribution function. The width at half max of the function gets noticeably narrower with increasing N. The peak value of the function is N/2 for this simple case and the full width at half max is given in equation (13).

H(v) VS NORMALIZED VOLTAGE (h = 0)

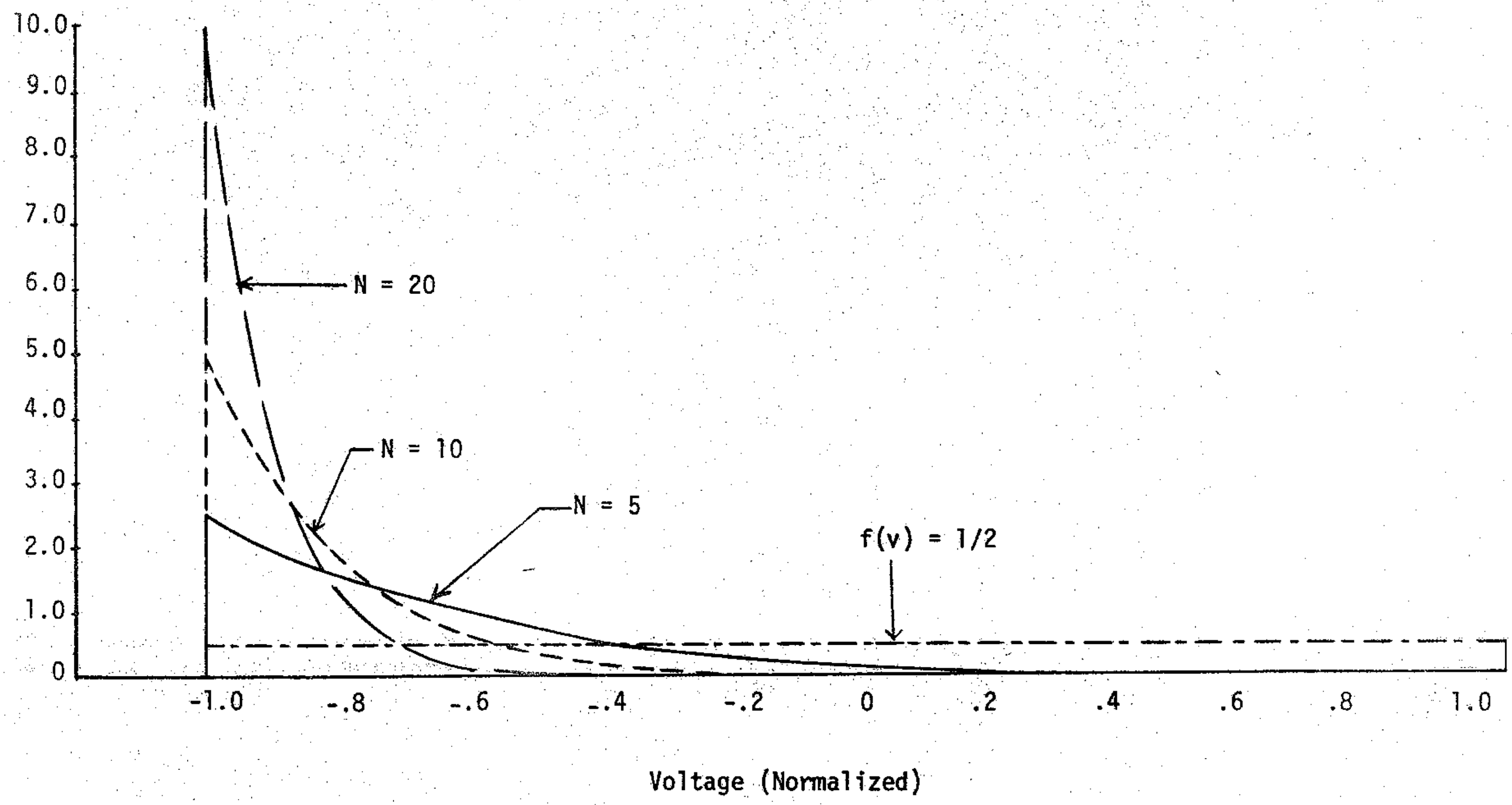


FIGURE 12

(13) $$w = 2\left(1 - \frac{1}{2^{\frac{1}{n-1}}}\right)$$

If the function representing the individual switch distribution is approximated by a half period of a sine wave, we get the results shown in Figure 13.

A normal distribution gives us the distribution shown in Figure 14. Figure 15 combines these results and is the representation of the full width at half maximum on normalized coordinates for various f(v) as a function of N.

It should be emphasized that with Figure 15 the normalization is taken so that 2 represents the interval which contains virtually all of the shots of the single gap. The corresponding interval from Figure 6 is (1.4 - .6) or 0.8. This means that to convert our experimental normalized scale, say in Figure 11, to the scale of Figure 15 we multiply by 2/.8 or 2.5. Referring to Figure 11, the approximate full width at half max of the experimental results are .08. The fitted curve H(v) yields .07 so that we will take our uncertainty as going between .07 and .08. Converting these values by multiplying by 2.5, the points are then plotted on Figure 15. Note that they lie very close to the curve corresponding to the initial square distribution. This is not surprising when one considers that the initial experimental distribution is very close to a square distribution. Perhaps, this is primarily because of the histogram technique used.

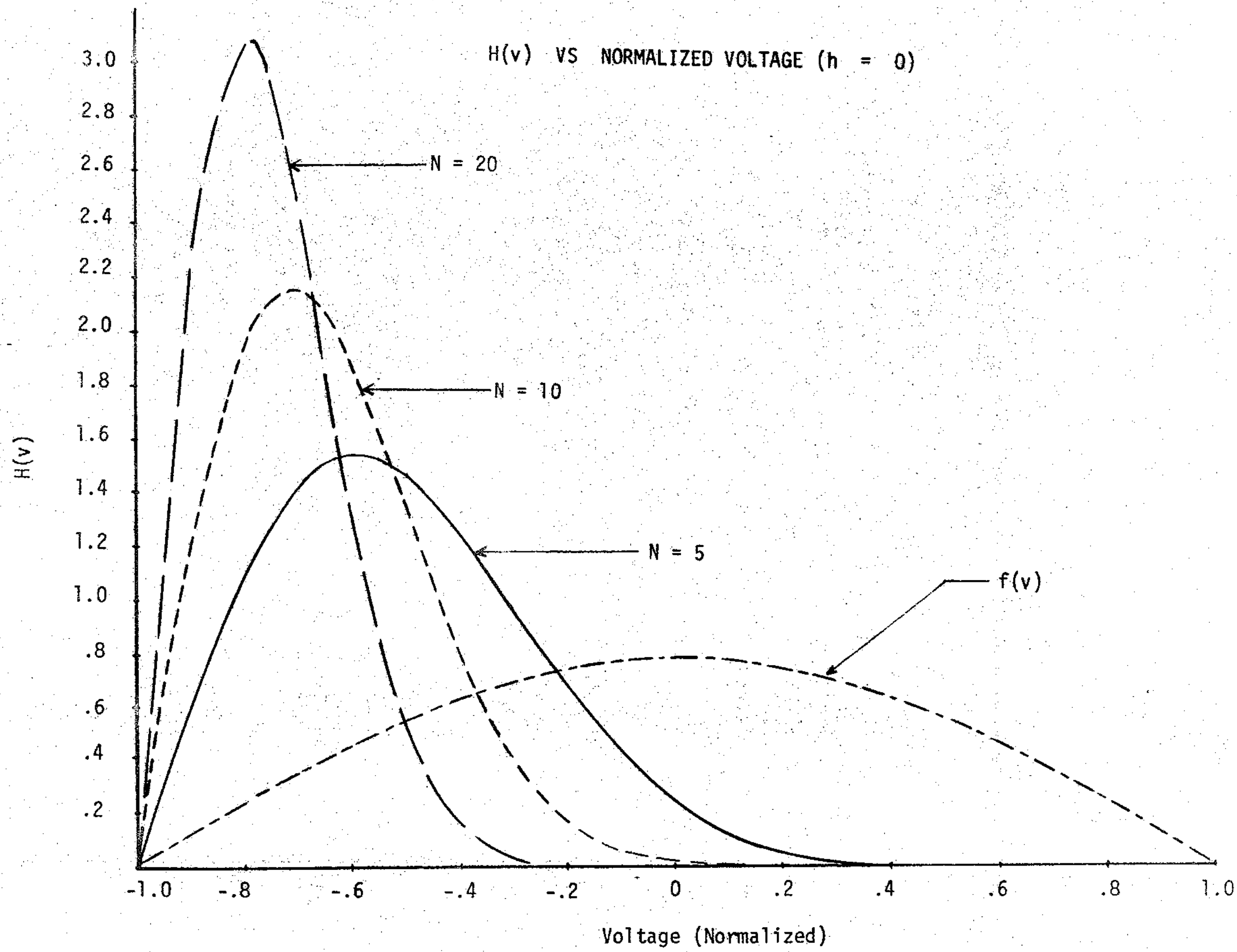

H(v) VS NORMALIZED VOLTAGE (h = 0)
N = 20
N = 10
N = 5
f(v)
H(v)
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
Voltage (Normalized)

FIGURE 13

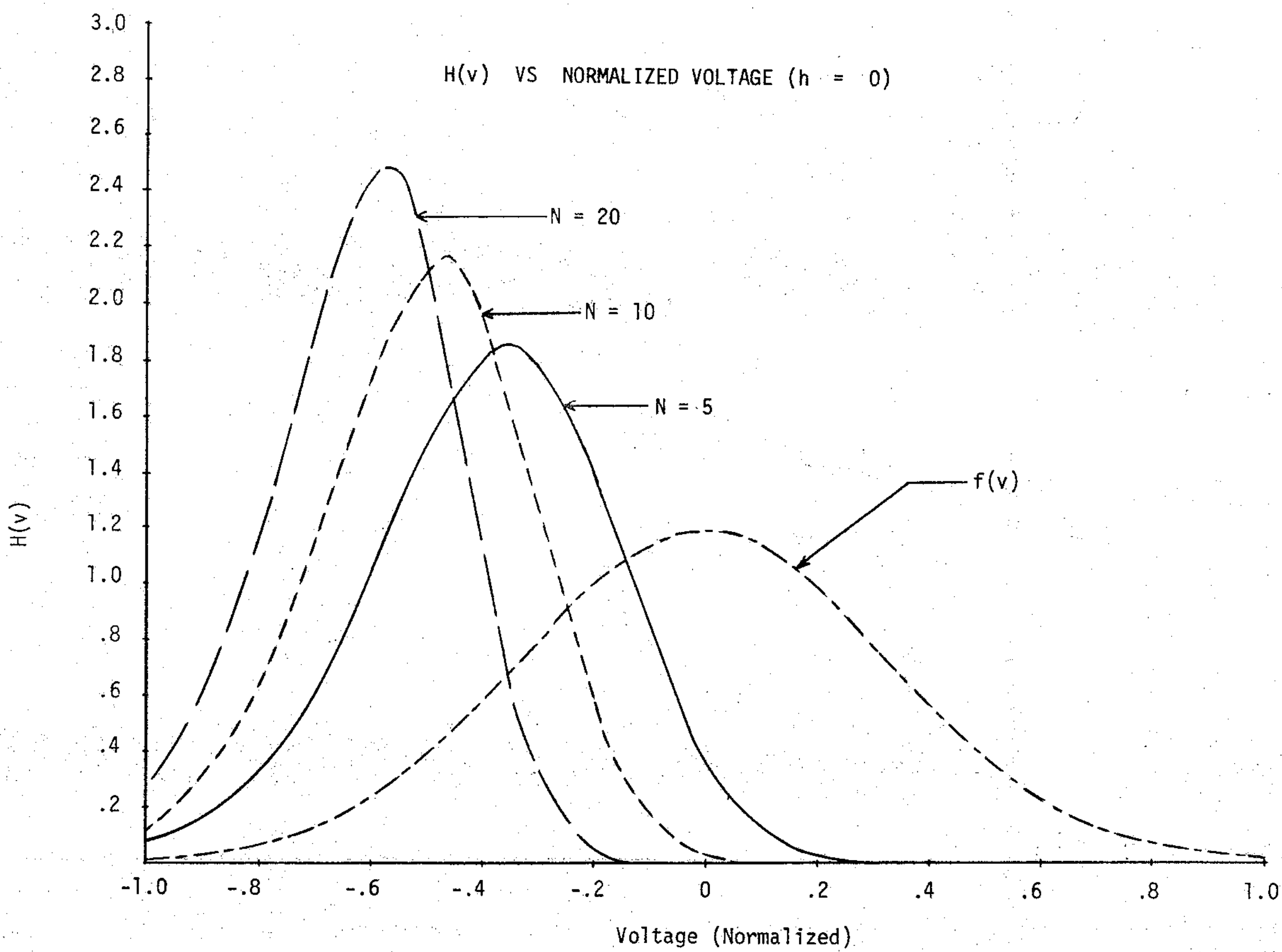

H(v) VS NORMALIZED VOLTAGE (h = 0)
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
H(v)
N = 20
N = 10
N = 5
f(v)
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
Voltage (Normalized)

FIGURE 14

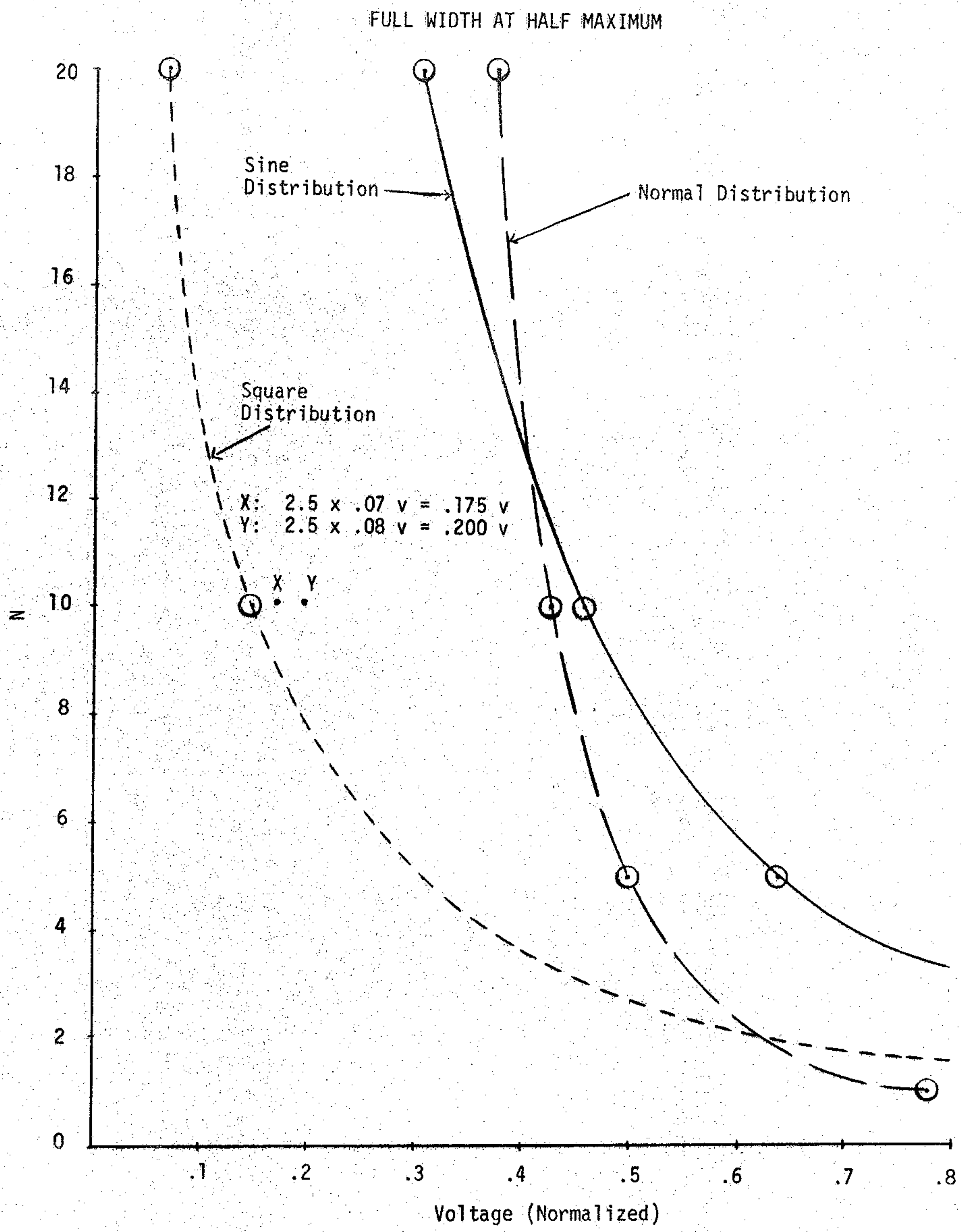

FULL WIDTH AT HALF MAXIMUM
20
18
16
14
12
10
8
6
4
2
0
N
Sine Distribution
Normal Distribution
Square Distribution
X: 2.5 x .07 v = .175 v
Y: 2.5 x .08 v = .200 v
X
Y
.1
.2
.3
.4
.5
.6
.7
.8
Voltage (Normalized)

FIGURE 15

V. Summary.

In summary then, this review of the rope switch data and model yields the following points:

1. Insofar as the data indicates, the action of the rope switch can be modeled by considering that the closure of the entire switch is initiated by one gap going statistically at random in the string.
2. Closure of the full rope switch takes place at a time after the firing of the first switch which is approximately constant and perhaps corresponding to roughly twice the transit time of the main rope switch at the speed of light.
3. To optimize the performance of the rope switch and minimize jitter, one should design the individual switch gaps so that they have two features. First, that their individual voltage jitter is itself minimized to the maximum practical amount. Secondly, it is exceedingly important, if important gains are to be made, that there exist a sharp threshold of voltage below which it is highly improbable that any single switch will fire. It can be said that initial evidence obtained from the model seems to indicate that where there is a sharp threshold below which individual switches do not fire, increasing the number of sections will improve the overall jitter of the rope switch. However, if there is a soft distribution and a fuzzy lower threshold, there may be little or no improvement with increasing numbers of sections. In fact, the jitter may get worse.

Further work will have to be aimed at testing the validity of these conclusions by attempting to design a rope switch made out of individual sections which function as well as possible. It must be remembered that the data discussed here was taken initially with switches which were deliberately made ragged in performance. The second point which has to be investigated is the nature of the assumed delay indicated by h in equation (7) above. It is possible that this delay is not constant and that the firing voltage may have other functional dependencies which are not clear due to the uncertainty of the data that was examined here.

VI. Appendix.

An Interesting Extension of the Model.

It is interesting to note that the curve shown for G(v) in Figure 8 can be reproduced almost identically from another more complicated function which introduces an additional element to the model.

Consider a rope switch of n sections. We will assume that the first switch section closes indicated in equation (4) above. This then leaves n-1 switches not fired. We shall assume that the voltage dropped across the first switch to fire is instantaneously redistributed across the remaining switches. This means that the second switch to fire sees a voltage of:

(14) $$V + \frac{V}{n-1} = V\left(1 + \frac{1}{n-1}\right) = \frac{Vn}{n-1}$$

The third switch to fire sees a voltage of:

(15) $$v_3 = \frac{vn}{n-2}$$

The probability that all switches have closed is then the product as follows:

(16) $$Q(v) = \left[1-(1-F(v))^{n}\right]\left[1-\left(1-F\left(\tfrac{nv}{n-1}\right)\right)^{n-1}\right]\left[1-\left(1-F\left(\tfrac{nv}{n-2}\right)\right)^{n-2}\right]\cdots$$

If we take the experimental values of n = 10 and F(v) from Figure 7 and then differentiate Q(v) in equation (16), we obtain a function which is extremely close to the G(v) shown in Figure 8. These two functions are compared in Figure 16.

The close similarity of the two functions indicates that with the distribution density function present in this experiment statistics and voltage alone would produce a virtually instantaneous closure after the first switch closes. The chief element in this model which produces this is the assumption of instantaneous redistribution of voltage after each gap closes. This, of course, cannot be in accord with experimental fact and therefore leaves open the question of closure delay.

This model is best thought of as a more complex extension of the steps represented by equation (4) in the main paper. This extension (at least its first factors) will be useful or even required for adequate modeling in situations where the redistribution of voltages after each separate switch closure takes place in a time which is short when compared with the time between individual switch closures. This was not, however, believed to be the case for the experiment reviewed here.

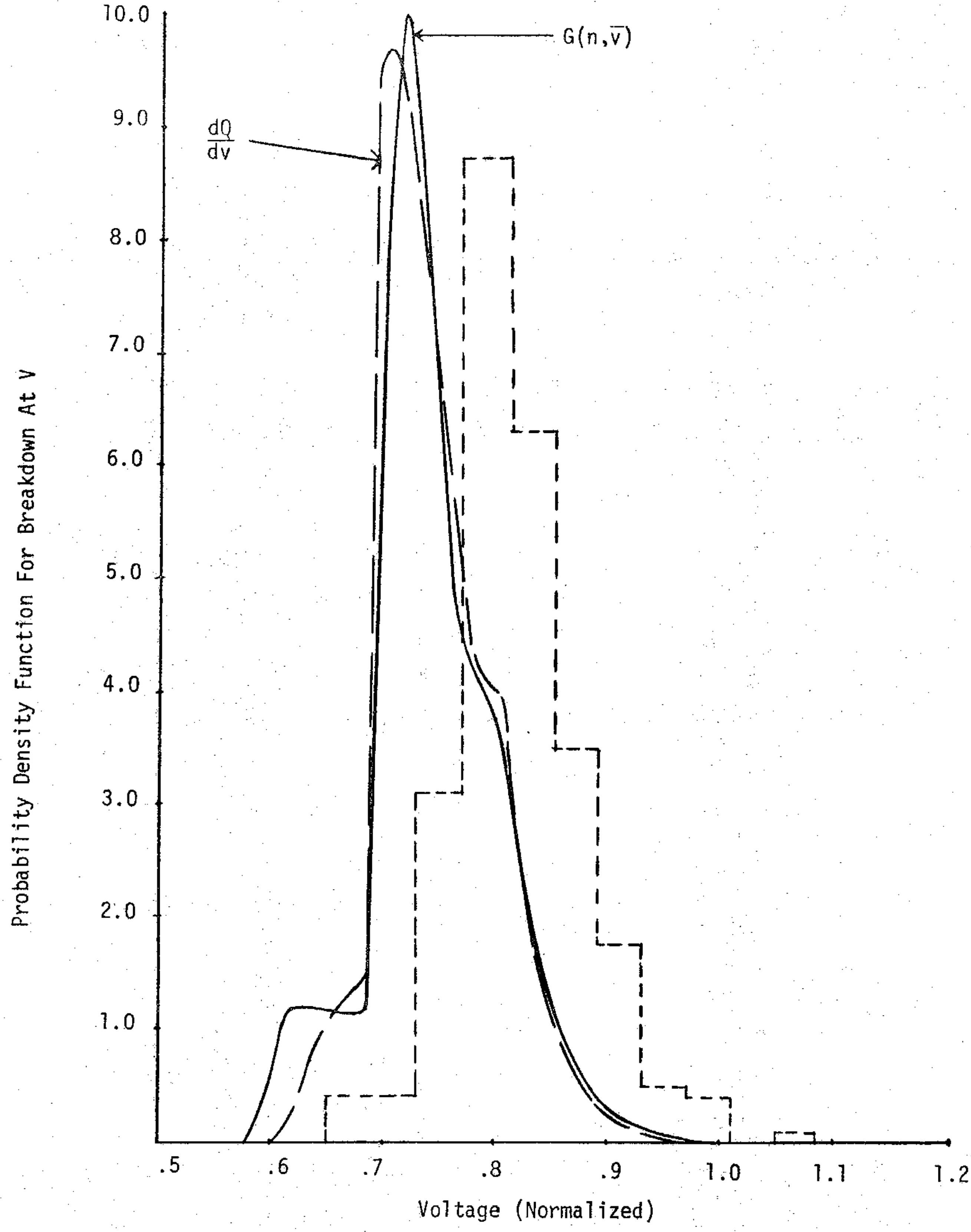

10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
Probability Density Function For Breakdown At V
G(n,v̄)
dQ/dv
.5
.6
.7
.8
.9
1.0
1.1
1.2
Voltage (Normalized)

FIGURE 16

VII. List of Symbols.

f(v): The probability density function for firing of an individual gap.

h(v): The delay from closure of the first gap to the closure of the entire switch. May be expressed as normalized voltage or time using the assumed voltage ramp: $V = \alpha t$.

v: Normalized voltage.

W: Full width at half maximum.

F(v): The probability distriubtion function of an individual gap firing by the time voltage v is reached across the gap.

$[\frac{dF}{dv} = f(v)]$

G(v): The probability density function of the event that at least one gap in an N-gap rope switch has closed by voltage v.

$[\frac{dP}{dv} = G(v)]$

H(v): The final probability density function of rope switch closure as predicted from the model.

J(v): The probability distribution function that all gaps in an N-gap rope switch have closed by voltage v.

K(v): The probability density function of the event that all gaps in an N-gap rope switch have closed by voltage v.

$[\frac{dJ}{dv} = K(v)]$

P(v): The probability distribution function of at least one gap in an N-gap rope switch closing by voltage v.

Q(v): The probability distribution function of rope switch closure as predicted by the alternate model discussed in the Appendix.